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**A SPLITTING CRITERION FOR RANK 2 VECTOR BUNDLES
ON P^n**

EDOARDO BALLICO

A SPLITTING CRITERION FOR RANK 2 VECTOR BUNDLES ON \mathbf{P}^n

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This is an addendum to a recent paper of V. Ancona, T. Peternell and J. Wisniewski. Here we prove (using heavily their paper) two criteria for the splitting of rank 2 algebraic vector bundles (one on \mathbf{P}^n and one on certain algebraic complete manifolds).

More precisely, the aim here is to show why the proofs of [1, Th. 10.5], and [1, Th. 10.13], give the following two theorems.

THEOREM 1. *Let E be a rank 2 algebraic vector bundle on \mathbf{P}^n which satisfies the assumptions of [1, Th. 10.5]. Then E splits.*

THEOREM 2. *Let E be a rank 2 algebraic vector bundle on a projective manifold X with (X, E) satisfying the assumption of [1, Th. 10.13]. Then E splits.*

The assumptions on X in Theorem 2 are very restrictive (e.g. X is a Fano manifold with $\text{Pic}(X) \cong \mathbf{Z}$). We only remark that the assumptions of Theorem 1 are satisfied if there is a two dimensional projective family, S , of lines in \mathbf{P}^n such that the splitting type of $E|L$ is the same for all $L \in S$.

Proof of Theorem 1. By the statement of [1, Th. 10.13], E numerically splits, i.e. it has the same Chern classes of a direct sum of 2 line bundles, i.e. there are integers a_1, a_2 with $a_1 \leq a_2$ such that $c_1(E) = a_1 + a_2$ and $c_2(E) = a_1 a_2$. The key remark is that the proof of [1, Th. 10.5], shows the existence of a line L such that $E|L \cong \mathbf{O}_L(a_1) \oplus \mathbf{O}_L(a_2)$. Since $4c_2(E) - c_1(E)^2 \leq 0$, E is not stable. Hence there is an integer $t \geq (a_1 + a_2)/2$ such that $H^0(\mathbf{P}^n, E(-t)) \neq 0$; take as t the minimal one; the corresponding section s of $E(-t)$ will vanish on a codimension 2 subscheme, Z , with $\deg(Z) = c_2(E(-t))$. Since $c_2(E(-x)) < 0$ if $a_1 < x < a_2$, we have $t \geq a_2$. If $t = a_2$ we obtain $Z = \emptyset$; hence E splits. Hence we

may assume $t > a_2$. This implies that $s|D = 0$ for every line D such that $E|D \cong \mathcal{O}_D(b_1) \oplus \mathcal{O}_D(b_2)$ with $a_1 \leq b_1 \leq b_2 \leq a_2$; in particular by semicontinuity this is true for a general line of \mathbf{P}^n . Hence $s = 0$, contradiction. \square

The proof of Theorem 2 is simply the remark (following [1], Remark 10.12) that, having Theorem 1 instead of the statement of [1, Th. 10.5], we obtain the stronger assertion of Theorem 2 instead of the numerical splitting asserted by [1, Th. 10.13].

The proof of Theorem 1 (i.e. of the small part of [1] needed) works in positive characteristic. The same remark applies to Theorem 2 if we assume $\text{Pic}(X) \cong \mathbf{Z}$ instead of making the assumptions on X which by [2] imply in characteristic 0 that $\text{Pic}(X) \cong \mathbf{Z}$.

We think that [1, Remark 10.12], (on the extension of [1, §10], to other manifolds) is potentially very interesting and we hope that some reader will be able to use it.

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