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# NILPOTENT CHARACTERS

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### In this note we study modular characters of finite psolvable groups which are induced from p-nilpotent subgroups and its $\pi$ -version.

1. Introduction. There is at least one reason to study such characters. In [2], for any block B of a finite group G, Alperin and Broué found a successful and natural Sylow B-theory which synthesized local group theory with several results on blocks by Brauer. This approach led to the Broué-Puig idea of nilpotent blocks. From the local representation point of view, therefore, the nilpotent blocks are the most natural blocks.

It is well known that theorems on p-blocks, in general, become far more accessible when we restrict our attention to the p-solvable groups. Sometimes, as it happens with the cyclic defect theory, they almost become trivial. This is not the case with the nilpotent blocks. Puig described the block algebra of a nilpotent block of a p-solvable group in [11].

Here we focus ourselves with the characters of the block. If  $\varphi$  is a modular character lying in a *p*-block *B* of a finite *p*-solvable group, we show that *B* is nilpotent if and only if  $\varphi$  is induced from a *p*-nilpotent subgroup. With this approach and applying Isaacs  $\pi$ -theory we are able to introduce nilpotent  $\pi$ -blocks ( $\pi$ -blocks have been studied by Robinson, Staszewski, Slattery and others) and to describe them satisfactorily: they only have a unique modular character  $\varphi$  (which is induced from a subgroup *K* with a normal Hall  $\pi$ -subgroup), and its  $|\operatorname{Irr}(D)|$  ordinary characters are also induced from convenient characters of *K* (*D* is defect group of the block). Finally, we will find the Fong characters associated with  $\varphi$  (the characters  $\alpha$  of a Hall  $\pi$ -subgroup with  $\alpha^G = \Phi_{\varphi}$ ).

Of course, when the set of primes  $\pi$  is just the complement of a prime  $p, \pi$ -blocks are just the ordinary blocks.

2. Subpairs and nilpotent blocks. If B is a p-block of a finite group G, a B-subpair is a pair  $(P, b_P)$  where P is a p-subgroup of G and  $b_P$  is a block of  $PC_G(P)$  inducing B (we are using Alperin's book notation [1]). If P is a defect group of B, then  $(P, b_P)$  is said to be a Sylow B-subpair. It is one of the main results in [2] to show that Sylow B-subpairs are G-conjugate and that each B-subpair is contained in one Sylow B-subpair (a natural but not obvious definition of containment is given in [1]). It is worth to mention that if the block B is the principal block, local block theory is just Sylow theory.

Inspired by Frobenius Theorem, Broue and Puig defined nilpotent blocks: a block *B* is said to be nilpotent if whenever  $(P, b_P)$  is a *B*-subpair then  $N_G(P, b_P)/C_G(P)$  is a *p*-group.

We begin with a Lemma. It is not in general true that if  $b^G$  is defined and nilpotent, then b is nilpotent (we will give some example below). However, in some special conditions more is true. (We recall that notation used in [2] and [4], is entirely equivalent to that in [1]: just apply V. 3.5 of [5]).

LEMMA 1. Let B be a block of a p-solvable group G. Let  $\theta \in Irr(O_{p'}(G))$  be covered by B and let  $b \in Bl(T)$  cover  $\theta$  and induce B, where T is the inertia group of  $\theta$  in G. Then B is nilpotent if and only if b is nilpotent.

*Proof.* Suppose that B is nilpotent and let  $(P, b_P)$  be a b-subpair. We wish to show that  $N_T(P, b_P)/C_T(P)$  is a p-group. Let us denote by  $* : \operatorname{Irr}_P(O) \to \operatorname{Irr}(C_O(P))$  the Glauberman Correspondence (see Chapter 13 of [6]), where  $O = O_{p'}(G)$ .

By applying, for instance, Lemma (4.4) of [13] to T, we have that if  $b_P$  covers  $\psi^*$ , where  $\psi \in \operatorname{Irr}_P(O)$ , then b covers  $\psi$ . Therefore, we have that  $b_P$  lies over  $\theta^*$ . We observe that  $N_T(P)$  is the inertia subgroup of  $\theta^*$  in  $N_G(P)$ . This is because  $N_G(P)$  acts on O fixing the P-invariant characters and commuting with the correspondence (see Theorem (13.1) (c) of [6]).

By Theorem (1.2.4) of [4], we know that  $b_P^{PC_G(P)}$  is nilpotent; so let  $\delta$  be the unique Brauer character in  $b_P^{PC_G(P)}$ . Since  $\delta$  lies over  $\theta^*$  and  $PC_T(P)$  is the inertia group of  $\theta^*$  in  $PC_G(P)$ , let  $\mu \in \operatorname{IBr}(PC_T(P)|\theta^*)$  such that  $\mu^{PC_G(P)} = \delta$ . By Fong-Reynolds (Theorem V.2.5 of [5]), we know that  $\mu$  is the only modular character in  $b_P$ . Therefore, if  $x \in N_T(P, b_P)$  then  $\mu^x = \mu, \delta^x = \delta$  and consequently  $x \in N_G(P, b_P^{PC_G(P)})$ . Then  $N_T(P, b_P)/C_T(P)$  is isomorphic to a subgroup of  $N_G(P, b_P^{PC_G(P)})/C_G(P)$ , which is a *p*-group by hypothesis.

Now assume that b is nilpotent and let  $(P, b_P)$  be a B-subpair. We want to prove that  $N_G(P, b_P)/C_G(P)$  is a p-group. Let  $H = PC_G(P)$ . We note that  $b_P^{HO}$  covers  $\theta^x$ , for some  $x \in G$ . This can be seen, for instance, by taking an irreducible character of  $b_P^{HO}$  lying under some irreducible character of B (by Theorem B of [3]). Since P is contained in a defect group of  $b_P^{HO}$ , it follows that some O-conjugate of P, say  $P^o$ , stabilizes  $\theta^x$ , by Fong-Reynolds. Therefore, P stabilizes  $\theta_1$  and  $b_P^{HO}$  covers  $\theta_1$ , where  $\theta_1 = \theta^{xo^{-1}}$ . Let  $T_1$  be the stabilizer of  $\theta_1$  in G. If we denote by  $\theta_1^* \in \operatorname{Irr}(C_O(P))$  the Glauberman correspondent of  $\theta_1$  with respect to P, by an earlier argument we have that  $N_{T_1}(P)$  is the stabilizer of  $\theta_1^*$  in  $N_G(P)$ .

Now let  $\gamma^* \in \operatorname{Irr}(C_O(P))$  be covered by  $b_P$ . Then  $\gamma$  is covered by  $b_P^{HO}$ , and therefore  $\gamma = \theta_1^c$ , for some  $c \in C_G(P)$ . Thus  $\theta_1^* = (\gamma^*)^{c^{-1}}$  is also covered by  $b_P$ . Since  $PC_{T_1}(P)$  is the stabilizer in  $PC_G(P)$  of  $\theta_1^*$ , we find  $e \in Bl(PC_{T_1}(P)|\theta^*)$  such that  $e^{PC_G(P)} = b_P$ . By an earlier argument,  $e^{T_1}$  lies over  $\theta_1$ , and, since it induces B, it follows that  $e^{T_1}$  is a G-conjugate of b. Therefore, it is nilpotent. By Theorem (1.2) of [4], e is also nilpotent and thus it contains a unique modular character, say  $\delta$ . By Fong-Reynolds,  $\delta^{PC_G(P)}$  is the unique modular character in  $b_P$ .

Suppose now that  $y \in N_G(P, b_P)$ . Then y fixes P and  $\delta^{PC_G(P)}$ . By Clifford Theory,  $(\theta_1^*)^y = (\theta_1^*)^c$ , for some  $c \in C_G(P)$ . Thus  $yc^{-1} \in N_{T_1}(P)$  and by the uniqueness in the Clifford Correspondence,  $\delta^{yc^{-1}} = \delta$ . Then  $yc^{-1} \in N_{T_1}(P, e)$ . Consequently,  $N_G(P, b_P) \subseteq N_{T_1}(P, e)C_G(P)$ . Thus  $N_G(P, b_P)/C_G(P)$  is isomorphic to a subgroup of  $N_{T_1}(P, e)/C_{T_1}(P)$ , which is a p-group.

LEMMA 2. Let B be a nilpotent block of a p-solvable group G and let  $\theta \in Irr(O_{p'}(G))$  covered by B. If  $\theta$  is G-invariant then G is p-nilpotent.

*Proof.* We argue by induction on |G|. Write  $O = O_{p'}(G)$ .

By Fong Theory, (see, for instance, Theorem (2.1) of [13]), we know that the Sylow *p*-subgroups of G are the defect groups of B. Fix P a Sylow *p*-subgroup of G and let  $(P, b_P)$  be a Sylow B-

subpair. By Frobenius Theorem, it suffices to show that if Q is any *p*-subgroup of P then  $N_G(P)/C_G(P)$  is a *p*-group. By Theorem (16.3) of [1], let  $(Q, b_Q) \leq (P, b_P)$ . Since  $b_Q$  is nilpotent, let  $\delta$  be the unique Brauer character in  $b_Q$ . By earlier arguments in Lemma 1, if  $\theta^* \in \operatorname{Irr}(C_O(Q))$  is the Q-Glauberman correspondent of  $\theta \in \operatorname{Irr}_Q(O)$ , then  $b_Q$  lies over  $\theta^*$  and  $\theta^*$  is  $N_G(Q)$ -invariant. By local group theory, it is well known that  $C_O(Q) = O_{p'}(N_G(Q))$ . If  $QC_G(Q) < G$ , by induction, we have that  $QC_G(Q)/O_{p'}(N_G(Q))$  is a *p*-group. Therefore, by Green's Theorem (see, for instance, (3.1) of [8]),  $\delta_{O_{p'}(N_G(Q))} = \theta^*$ , and since  $\delta$  is the only Brauer character lying over  $\theta^*$ , we have that  $\delta$  and  $\theta^*$  determine one each other uniquely. Therefore,  $\delta$  is  $N_G(Q)$ -invariant, and so it is  $b_Q$ . Thus,  $N(Q, b_Q)/C_G(Q) = N_G(Q)/C_G(Q)$  is a *p*-group in any case, and Frobenius Theorem applies.

**3.**  $\pi$ -characters. If G is a  $\pi$ -separable group, we denote by  $I_{\pi}(G)$  the set of Isaacs  $\pi$ -characters of G. Of course, when  $\pi = p', I_{\pi}(G)$  is just the set of Brauer characters of G. We refer the reader to [7] and [8], for definitions, notation and basic properties of the set  $I_{\pi}(G)$ . We recall that there exists a canonical subset of the irreducible characters of  $G, B_{\pi}(G)$ , such that restriction to  $\pi$ -elements gives a bijection from  $B_{\pi}(G)$  onto  $I_{\pi}(G)$  (Theorem (9.3) of [7]).

We certainly will use that any  $\pi$ -character is induced from a  $\pi$ degree  $\pi$ -character (Huppert's Theorem, see (3.4) of [8]), and other fact proved recently in [9]. If  $\varphi \in I_{\pi}(G)$  and  $\varphi = \delta^G = \mu^G$ , where  $\delta \in I_{\pi}(K)$  and  $\mu \in I_{\pi}(J)$  have  $\pi$ -degree, then the Hall  $\pi'$ -subgroups of K and J are G-conjugate: this invariant is the vertex of a  $\pi$ character.

We say that  $\varphi \in I_{\pi}(G)$  is *nilpotent* if  $\varphi = \delta^{G}$ , where  $\delta \in I_{\pi}(K)$  with  $K = O_{\pi\pi'}(K)$ .

LEMMA 3. Let G be a  $\pi$ -separable group and let  $\varphi \in I_{\pi}(G)$  be nilpotent. If  $\theta \in \operatorname{Irr}(O_{\pi}(G))$  is G-invariant and lies under  $\varphi$  then  $G = O_{\pi\pi'}(G)$ .

Proof. Write  $\varphi = \delta^G$ , where  $\delta \in I_{\pi}(K)$  with  $K = O_{\pi\pi'}(K)$ , and let  $O = O_{\pi}(G)$ . Since OK has a normal Hall  $\pi$ -subgroup, by replacing  $(K, \delta)$  by  $(OK, \delta^{OK})$ , we may assume that  $O \subseteq K$ . Now, by (3.4)

of [8], let  $\beta \in I_{\pi}(R)$  with  $\pi$ -degree be such that  $\beta^{K} = \delta$ . Since  $\beta^{OR}$  has also  $\pi$ -degree (because |OR : R| is a  $\pi$ -number), we also may assume that  $\delta$  has  $\pi$ -degree.

By comments above, observe that if P is a Hall  $\pi'$ -subgroup of K, then P is a vertex of  $\varphi$ .

Let  $U = O_{\pi\pi'}(G)$ . We claim that  $\varphi_U = e\eta$ , where  $\eta \in I_{\pi}(U)$  and  $\eta_O = \theta$ . To see this, let  $\chi \in B_{\pi}(G)$  be a lifting of  $\varphi$  (see Theorem (9.3) of [7]), and let  $\psi \in B_{\pi}(U)$  be under  $\chi$  ((7.5) of [7]). Then, by (6.3) and (6.5) of [7],  $\psi_U = \theta$  and  $\psi$  is the only  $B_{\pi}$ -character lying over  $\theta$ . Therefore,  $\psi$  is G-invariant and so it is  $\psi^o = \eta \in I_{\pi}(U)$ , its restriction to  $\pi$ -elements. This proves the claim.

Now, since  $\psi$  has  $\pi$ -degree, by (5.4) of [7],  $\psi$  is  $\pi$ -special and therefore,  $(U, \psi)$  is a subnormal  $\pi$ -factorable pair in the sense of [7]. Therefore,  $(U, \psi) \leq (W, \alpha)$ , where  $(W, \alpha), \alpha$  a  $\pi$ -special character of W, is a nucleous of  $\chi$  (definition (4.6) of [7]). Thus  $\alpha^{o^G} = \varphi$ , and by Theorem B of [9], it follows that  $P^x$  is a Hall  $\pi'$ -subgroup of W, for some  $x \in G$ . Then  $P^x \cap U$  is a Hall  $\pi'$ -subgroup of U, and thus  $U \subseteq OP \subseteq K$ .

Now, since U/O and  $O_{\pi}(K)/O$  are normal subgroups of K/Oof coprime order it follows that  $O_{\pi}(K)/O \subseteq C_{G/O}(U/O) \subseteq U/O$ , by Lemma 1.2.3. Therefore, we conclude that  $O_{\pi}(K) = O$ . Let  $V = O_{\pi\pi'\pi}(G)$ . Since K/U and V/U have coprime orders it follows that  $V \cap K = U$ . Observe that  $\delta_U = \eta$ , by (3.1) of [8], and that  $\delta^{KV}$  has  $\pi$ -degree. Therefore,  $\eta^V = (\delta^{KV})_V \in I_{\pi}(V)$ . Since  $\psi$  lifts  $\eta$ , necessarily  $\psi^V \in \operatorname{Irr}(V)$ . Since  $\psi$  is G-invariant, by problem (6.1) of [6], for instance, it follows that U = V = G, as wanted.

LEMMA 4. Let G be a  $\pi$ -separable group and let Y be a normal  $\pi$ -subgroup of G. Let  $\varphi \in I_{\pi}(G)$ , let  $\theta \in \operatorname{Irr}(Y)$  under  $\varphi$  and let  $\delta \in I_{\pi}(T|\theta)$  with  $\delta^{G} = \varphi$ , where T is the stabilizer of  $\theta$  in G. Then  $\varphi$  is nilpotent if and only if  $\delta$  is nilpotent.

**Proof.** By the definition, certainly  $\varphi$  is nilpotent if  $\delta$  is nilpotent. So assume that  $\varphi$  is nilpotent and write  $\varphi = \psi^G$ , where  $\psi \in I_{\pi}(K)$ , with K having a normal Hall  $\pi$ -subgroup. Since YK has also a normal Hall  $\pi$ -subgroup, we may replace K by YK and assume that K contains Y. Also, by replacing K by some G-conjugate, we may assume that  $\psi$  lies over  $\theta$ . If  $\alpha \in I_{\pi}(K \cap T|\theta)$  induces  $\psi$ , by uniqueness in the Clifford correspondence, (3.2) of [8], it follows

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that  $\alpha^T = \delta$ , and the proof of the Lemma is complete.

Now we prove.

THEOREM 5. Let B be a p-block of a p-solvable group and let  $\varphi \in \text{IBr}(B)$ . Then B is nilpotent if and only if  $\varphi$  is nilpotent.

Proof. We argue by induction on |G|. Let  $\theta \in \operatorname{Irr}(O_{p'}(G))$  be under  $\varphi$ , let  $\delta \in IBr(T|\theta)$  with  $\delta^G = \varphi$ , where T is the stabilizer of  $\theta$  in G, and let  $b \in Bl(T)$  be the block of  $\delta$ . If T = G, by Lemma 2 and Lemma 3, we have that, in both cases, G is p-nilpotent and so every block and every character are nilpotent. If T < G, by induction and Lemma 1 and Lemma 4, we have that  $\varphi$  is nilpotent if and only if  $\delta$  is nilpotent if and only if b is nilpotent if and only if B is nilpotent.  $\Box$ 

4.  $\pi$ -Blocks. Brauer himself considered the idea of generalizing *p*-blocks to  $\pi$ -blocks, for a set of primes  $\pi$ . Later, Robinson and others introduced several definitions of  $\pi$ -blocks. We will follow the Isaacs-Slattery's approach which certainly coincides with Robinson's when the group is  $\pi$ -separable. We refer the reader to [12] and [13], for definition, notation and further comments on the subject.

THEOREM 6. Let G be a  $\pi$ -separable group and let  $\varphi \in I_{\pi}(G)$  be nilpotent. Let B be the  $\pi$ -block of  $\varphi$ . Then

(a)  $\varphi$  is the only modular character in B.

(b) If  $\delta^G = \varphi$ , where  $\delta \in I_{\pi}(K)$  has  $\pi$ -degree and K has a normal Hall  $\pi$ -subgroup, then the map  $\psi \to \psi^G$  from  $\operatorname{Irr}(K|\delta_{O_{\pi}(K)}) \to$  $\operatorname{Irr}(B)$  is a bijection.

(c) With the notation of (b),  $(\delta_{O_{\pi}(K)})^G = \Phi_{\varphi}$ . Thus, if H is a Hall  $\pi$ -subgroup of G containing  $O_{\pi}(K)$ , then  $(\delta_{O_{\pi}(K)})^H \in \operatorname{Irr}(H)$  is a Fong character for  $\varphi$ .

Proof. (a) Let  $\theta \in \text{Irr}(O)$  under  $\varphi$ , where  $O = O_{\pi}(G)$ . Let  $\delta \in I_{\pi}(T|\theta)$  with  $\delta^G = \varphi$ , where T is the stabilizer of  $\theta$  in G, and let b be the  $\pi$ -block of  $\delta$ . If T = G, by Lemma 3, G has a normal Hall  $\pi$ -subgroup. Also by (2.8) of [12], we know that the modular characters in B are the  $\pi$ -characters over  $\theta$ . By (6.3) of [7], it follows

that  $\varphi$  is the only one. On the other hand, if T < G, by Lemma 3, induction and Theorem (2.10) of [12], the result follows.

(b) We argue by induction on |G|.

Since  $\delta$  has  $\pi$ -degree, we have that  $\alpha = \delta_{O_{\pi}(K)} \in \operatorname{Irr}(O_{\pi}(K))$ .

Let  $V = OO_{\pi}(K)$ . Since |OK: V| is a  $\pi'$ -number, we have that  $\alpha^V = (\delta^{OK})_V \in \operatorname{Irr}(V)$ . Since  $\alpha$  is K-invariant, by (4.3) of [7], it follows that the map  $\psi \to \psi^{OK}$  is a bijection from  $\operatorname{Irr}(K|\alpha) \to$  $\operatorname{Irr}(OK|\alpha^V)$ . Now let  $\theta \in \operatorname{Irr}(O)$  be under  $\alpha^V$  and let  $\epsilon \in I_{\pi}(T \cap$  $OK|\theta$  be such that  $\epsilon^{OK} = \delta^{OK}$ , where T is the stabilizer of  $\theta$  in G. If  $\mu = \epsilon^T$ , observe that  $\mu \in I_{\pi}(T|\theta)$  and  $\mu^G = \varphi$ . By Lemma 4, notice that  $\mu$  is nilpotent. If T = G, by Lemma 3, we have that O is a Hall  $\pi$ -subgroup of G. Also,  $\varphi_0 = \theta$ , which forces OK = G. In this case,  $V = O, \alpha^V = \theta$  and we know that  $\psi \to \psi^G$  is a bijection from  $\operatorname{Irr}(K|\alpha) \to \operatorname{Irr}(G|\theta)$ . Since  $\operatorname{Irr}(B) = \operatorname{Irr}(G|\theta)$ , by (2.8) of [12], in this case, we are done. So we may assume that T < Gand by induction we have that the map  $\psi \to \psi^T$  is a bijection from  $\operatorname{Irr}(T \cap OK|_{\epsilon_{T \cap V}}) \to \operatorname{Irr}(b)$ . Since  $\epsilon_{T \cap V}$  is  $T \cap OK$ -invariant and induces  $\alpha^V$ , by (4.3) of [7], it follows that the map  $\psi \to \psi^{OK}$ is a bijection from  $\operatorname{Irr}(T \cap OK|_{\epsilon_{T \cap V}}) \to \operatorname{Irr}(OK|_{\alpha^V})$  (observe that  $(T \cap OK)V = OK$ , because they have coprime indices). By the above and Theorem (2.10) of [12], we have that the map  $\psi \to \psi^G$ is a bijection from  $\operatorname{Irr}(T \cap OK|_{\epsilon_{T \cap V}}) \to \operatorname{Irr}(B)$  and therefore so it is the map  $\psi \to \psi^G$  from  $\operatorname{Irr}(OK|\alpha^V) \to \operatorname{Irr}(B)$ . This proves (b).

(c) By Lemma (2.3) of [10], it suffices to show that  $(\delta_{O_{\pi}(K)})^G = \Phi_{\varphi}$ . If  $\chi \in \operatorname{Irr}(B)$ , by (b), we have that  $\chi^o = (\chi(1)/\varphi(1)) \varphi$ . Then,

$$\Phi_{\varphi} = \sum_{\chi \in \operatorname{Irr}(B)} (\chi(1)/\varphi(1)) \chi = \sum_{\psi \in \operatorname{Irr}(K|\delta_{O_{\pi}(K)})} (\psi(1)/\delta(1)) \psi^{G}$$
$$= \left(\sum_{\psi \in \operatorname{Irr}(K|\delta_{O_{\pi}(K)})} (\psi(1)/\delta(1)) \psi\right)^{G}$$
$$= \left(\left(\delta_{O_{\pi}(K)}\right)^{K}\right)^{G} = \left(\delta_{O_{\pi}(K)}\right)^{G}.$$

It is not difficult to show that all Fong characters associated with  $\varphi$  arise this way.

We think it is worth to remark that if an irreducible character  $\chi$  is induced from a *p*-nilpotent character the *p*-block of  $\chi$  need not

to be nilpotent. For instance, consider  $\chi$  an irreducible character of degree 3 in the symmetric group on four letters and p = 2. The block of  $\chi$  is the principal block which is not nilpotent (because G is not *p*-nilpotent). However,  $\chi$  is induced from a Sylow 2-subgroup of G.

5. An example. We mentioned above that if a block  $b^G$  is defined and nilpotent, then *b* needs not to be nilpotent. More surprisingly, if a block nilpotent *b* covers a block *e*, *e* needs not to be nilpotent (this fact was communicated to the author by L. Puig, and we take this opportunity for thanking him). We give an easy

EXAMPLE 7. Let  $D = \langle x, y \rangle$  be the dihedral group of order 8, with  $C = \langle x \rangle$  of order 4 and  $x^y = x^{-1}$  and let D act on  $P = \langle z \rangle$ of order 3 by  $z^y = z^{-1}$  and C acting trivially. Let G = PD be the semidirect product and put p = 3. Let  $\lambda \in Irr(C)$  of order 4 and  $\hat{\lambda} = \lambda \times 1_P \in Irr(P \times C)$ . Then  $\chi = (\hat{\lambda})^G \in Irr(G)$ . Observe that, by (7.1) of [7],  $\chi \in B_2(G)$  and thus,  $\varphi = \chi^o \in IBr(G)$ . Observe that  $\varphi$  is nilpotent. Let  $J = P\langle y \rangle$  and let  $H = J \times Z \triangleleft G$ , where  $Z = \langle x^2 \rangle$ . Then  $\chi_H = \mu_1 + \mu_2$ , where  $\mu_1 \in Irr(H/P)$ , and  $\mu_i$  is linear. Then,  $\mu_i$ , which is normal constituent of a nilpotent character  $\varphi$ , is not nilpotent (since H is not p-nilpotent). This shows that, in general, nilpotent characters do not lie over nilpotent characters. Also,  $\mu_i^G = \varphi$ , and hence the nonnilpotent block of  $\mu_i$  induces the block of  $\varphi$ .

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