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### ON MODULI OF INSTANTON BUNDLES ON $\mathbb{P}^{2n+1}$

VINCENZO ANCONA AND GIORGIO MARIA OTTAVIANI

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#### ON MODULI OF INSTANTON BUNDLES ON $\mathbb{P}^{2n+1}$

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Let  $MI_{\mathbb{P}^{2n+1}}(k)$  be the moduli space of stable instanton bundles on  $\mathbb{P}^{2n+1}$  with  $c_2 = k$ . We prove that  $MI_{\mathbb{P}^{2n+1}}(2)$  is smooth, irreducible, unirational and has zero Euler-Poincaré characteristic, as it happens for  $\mathbb{P}^3$ . We find instead that  $MI_{\mathbb{P}^5}(3)$  and  $MI_{\mathbb{P}^5}(4)$  are singular.

#### 1. Definition and preliminaries.

Instanton bundles on a projective space  $\mathbb{P}^{2n+1}(\mathbb{C})$  were introduced in **[OS]** and **[ST]**. In **[AO]** we studied their stability, proving in particular that special symplectic instanton bundles on  $\mathbb{P}^{2n+1}$  are stable, and that on  $\mathbb{P}^5$  every instanton bundle is stable.

In this paper we study some moduli spaces  $\mathrm{MI}_{\mathbb{P}^{2n+1}}(k)$  of stable instanton bundles on  $\mathbb{P}^{2n+1}$  with  $c_2 = k$ . For k = 2 we prove that  $\mathrm{MI}_{\mathbb{P}^{2n+1}}(2)$  is smooth, irreducible, unirational and has zero Euler-Poincaré characteristic (Theor. 3.2), just as in the case of  $\mathbb{P}^3$  [Har].

We find instead that  $MI_{\mathbb{P}^5}(k)$  is singular for k = 3, 4 (theor. 3.3), which is not analogous with the case of  $\mathbb{P}^3$  [**ES**], [**P**]. To be more precise, all points corresponding to symplectic instanton bundles are singular. Theor. 3.3 gives, to the best of our knowledge, the first example of a singular moduli space of stable bundles on a projective space. The proof of Theorem 3.3 needs help from a personal computer in order to calculate the dimensions of some cohomology group [**BaS**].

We recall from [OS], [ST] and [AO] the definition of instanton bundle on  $\mathbb{P}^{2n+1}(\mathbb{C})$ .

**Definition 1.1.** A vector bundle E of rank 2n on  $\mathbb{P}^{2n+1}$  is called an instanton bundle of quantum number k if

(i) The Chern polynomial is  $c_t(E) = (1 - t^2)^{-k} = 1 + kt^2 + {k+1 \choose 2}t^2 + \dots$ 

(ii) E(q) has natural cohomology in the range  $-2n - 1 \le q \le 0$  (that is  $h^i(E(q)) \ne 0$  for at most one i = i(q))

(iii)  $E|_r \simeq \mathcal{O}_r^{2n}$  for a general line r.

Every instanton bundle is simple [AO]. There is the following characterization:

**Theorem 1.2** ([**ST**], [**AO**]). A vector bundle E of rank 2n on  $\mathbb{P}^{2n+1}$  satisfies the properties (i) and (ii) if and only if E is the cohomology of a monad

(1.1) 
$$\mathcal{O}(-1)^k \xrightarrow{A} \mathcal{O}^{2n+2k} \xrightarrow{B} \mathcal{O}(1)^k$$
.

With respect to a fixed system of homogeneous coordinates the morphism A (resp. B) of the monad can be identified with a  $k \times (2n + 2k)$  (resp.  $(2n + 2k) \times k$ ) matrix whose entries are homogeneous polynomials of degree 1. Then the conditions that (1.1) is a monad are equivalent to:

A, B have rank k at every point 
$$x \in \mathbb{P}^{2n+1}$$
,  $A \cdot B = 0$ .

**Definition 1.3.** A bundle S appearing in an exact sequence:

(1.2) 
$$0 \to S^* \to \mathcal{O}^d \xrightarrow{B} \mathcal{O}(1)^c \to 0$$

is called a Schwarzenberger type bundle (STB).

The kernel bundle Ker B in the monad (1.1) is the dual of a STB.

**Definition 1.4.** An instanton bundle is called special if it arises from a monad (1.1) where the morphism B is defined in some system of homogeneous coordinates  $(x_0, \ldots, x_n, y_0, \ldots, y_n)$  on  $\mathbb{P}^{2n+1}$  by the matrix

$$B = \begin{bmatrix} x_{0} \\ \vdots & \ddots \\ x_{n} & x_{0} \\ & \ddots & \vdots \\ y_{0} \\ \vdots & \ddots \\ y_{n} & y_{0} \\ & \ddots & \vdots \\ & & y_{n} \end{bmatrix}$$





 $E = \operatorname{Ker} B / \operatorname{Im} A$  is a special instanton bundle.

Property (iii) of the definition 1.1 can be checked by the following:

**Theorem 1.6** [OS]. Let E = Ker B/Im A as in (1.1). Let r be the line joining two distinct points  $P, Q \in \mathbb{P}^{2n+1}$ . Then

distinct points  $P, Q \in \mathbb{P}^{2n+1}$ . Then  $E|_r \simeq \mathcal{O}_r^{2n} \Leftrightarrow A(P) \cdot B(Q)$  is an invertible matrix.

**Example 1.7.** Consider the special instanton bundle E of the example 1.5. Let P = (1, 0, ...; 0, ..., 0), Q = (0, ..., ; 0, ..., 1). Then

$$A(P) = \begin{bmatrix} & & & -1 \\ & & \cdot \\ & & \cdot \\ & & -1 \end{bmatrix} \qquad B(Q) = \begin{bmatrix} 1 \\ 1 \\ & \cdot \\ & 1 \end{bmatrix}$$

and  $A(P) \cdot B(Q) = \begin{bmatrix} -1 \\ \cdot \\ -1 \\ -1 \end{bmatrix}$  is invertible. Hence E is trivial on the line  $\{x_1 = \ldots = x_n = y_0 = \ldots = y_{n-1} = 0\}.$ 

**Proposition 1.8.** Let E be an instanton bundle as in (1.1). Then

 $H^{2}(E \otimes E^{*}) = H^{2}[(\operatorname{Ker} B) \otimes (\operatorname{Ker} A^{t})]$ 

Proof. See [AO] Theorem 3.13 and Remark 2.22.

**Remark 1.9.** If  $E \simeq E^*$ , then

 $H^2(E\otimes E^*)=H^2[(\operatorname{Ker} A^t)\otimes (\operatorname{Ker} A^t)]=H^2[(\operatorname{Ker} B)\otimes (\operatorname{Ker} B)].$ 

**Remark 1.10.** The single complex associated with the double complex obtained by tensoring the two sequences

$$0 \to \operatorname{Ker} A^{t} \to \mathcal{O}^{2n+2k} \xrightarrow{A^{t}} \mathcal{O}(1)^{k} \to 0$$
$$0 \to \operatorname{Ker} B^{t} \to \mathcal{O}^{2n+2k} \xrightarrow{B^{t}} \mathcal{O}(1)^{k} \to 0$$

gives the resolution

$$0 \to (\operatorname{Ker} A^{t}) \otimes (\operatorname{Ker} B) \to \mathcal{O}^{2n+2k} \otimes \mathcal{O}^{2n+2k} \to \mathcal{O}^{2n+2k} \otimes \mathcal{O}(1)^{k} \oplus \mathcal{O}(1)^{k} \otimes \mathcal{O}^{2n+2k} \xrightarrow{\alpha} \mathcal{O}(1)^{k} \otimes \mathcal{O}(1)^{k} \to 0$$

where  $\alpha = (A^t \otimes id, id \otimes B)$ .

Hence

$$H^2(E\otimes E^*) = \operatorname{Coker} H^0(\alpha)$$

and its dimension can be computed using [BaS]. For the convenience of the reader we sketch the steps needed in the computations.

 $A, B^t$  are given by  $k \times (2n + 2k)$  matrices whose entries are linear homogeneous polynomials.

$$A\otimes \mathrm{Id}_k=(a_1,\ldots,a_{k(2n+2k)})$$

 $\operatorname{and}$ 

$$\mathrm{Id}_k \otimes B^t = (b_1, \ldots, b_{k(2n+2k)})$$

are both  $k^2 \times (2n+2k)k$  matrices. Let

$$C = (a_1, \ldots, a_{k(2n+2k)}, b_1, \ldots, b_{k(2n+2k)}).$$

We will denote by  $\operatorname{syz}_m C$  the dimension of the space of the syzygies of C of degree m. Then

$$\begin{aligned} h^2(E\otimes E^*) &= h^0(\mathcal{O}(2)^{k^2}) - (4n+4k)h^0(\mathcal{O}(1)^k) + \operatorname{syz}_1 C \\ &= k(n+1)[k(2n-5)-8n] + \operatorname{syz}_1 C \\ h^1(E\otimes E^*) &= h^2(E\otimes E^*) + 1 - k^2 + 8n^2k - 4n^2 + 3nk^2 - 2n^2k^2 \\ &= 1 - 6k^2 - 8kn - 4n^2 + \operatorname{syz}_1 C. \end{aligned}$$

Note also that  $h^0(E(1)) = \operatorname{syz}_1 B^t - k$  and  $h^0(E^*(1)) = \operatorname{syz}_1 A - k$ . Remark 1.11. In the same way we obtain

$$h^1(E \otimes E^*(-1)) = \operatorname{syz}_0 C$$
  
 $h^2(E \otimes E^*(-1)) = 2k(nk - 2n - k) + \operatorname{syz}_0 C$ 

#### **2.** Example on $\mathbb{P}^5$ .

Let (a, b, c, d, e, f) be homogeneous coordinates in  $\mathbb{P}^5$ . Example 2.1. (k = 3) Let

$$B^{t} = \begin{bmatrix} a \ b \ c & d \ e \ f \\ a \ b \ c & d \ e \ f \\ a \ b \ c & d \ e \ f \end{bmatrix}$$
$$A = \begin{bmatrix} f \ e \ d & -c \ -b \ -a \\ f \ e \ d & -c \ -b \ -a \end{bmatrix}$$

The corresponding monad gives a special symplectic instanton bundle on  $\mathbb{P}^5$  with k = 3. With the notation of remark 1.10, using [**BaS**] we can compute  $\operatorname{syz}_0 C = 14, \operatorname{syz}_1 C = 174$ . Hence  $h^2(E \otimes E^*) = 3$  from the formulas of Remark 1.10. Moreover  $h^0(E(1)) = 4$ .

**Example 2.2.** (k = 3) Let  $B^t$  as in the Example 2.1 and

$$A = \begin{bmatrix} f \ e \ d & -c - b \ -a \\ e \ d & 2f \ -b \ -a & -2c \\ d & f \ e \ -a & -c \ -b \end{bmatrix}$$

We have  $\operatorname{syz}_0 C = 10$ ,  $\operatorname{syz}_1 C = 171$ . Hence  $h^2(E \otimes E^*) = 0$ . We can compute also the syzygies of  $B^t$  and A and we get  $h^0(E(1)) = 4$ ,  $h^0(E^*(1)) = 3$ , hence E is not self-dual.

**Example 2.3.** (k = 4) Let

$$B^{t} = \begin{bmatrix} a \ b \ c & d \ e \ f \\ a \ b \ c & d \ e \ f \\ a \ b \ c & d \ e \ f \\ a \ b \ c & d \ e \ f \end{bmatrix}$$
$$A = \begin{bmatrix} f \ e \ d & -c \ -b \ -a \\ f \ e \ d & -c \ -b \ -a \\ f \ e \ d & -c \ -b \ -a \\ f \ e \ d & -c \ -b \ -a \end{bmatrix}$$

E is a special symplectic instanton bundle with k = 4. We compute

$$h^2(E \otimes E^*) = 12.$$

**Example 2.4.** (k = 4) Let  $B^t$  as in the Example 2.3. Let

$$A = \begin{bmatrix} f e & d & -c - b & -a \\ e & d & 2f & -b & -a & -2c \\ 3d & f & e & -3a & -c & -b \\ f & e & d & -c & -b & -a \end{bmatrix}.$$

In this case  $h^2(E \otimes E^*) = 6$ ,  $h^0(E(1)) = 4$ ,  $h^0(E^*(1)) = 3$ . Example 2.5. (k = 4) Let  $B^t$  as in the Example 2.3. Let

$$A = \begin{bmatrix} f & e & d & -c & -b & -a \\ e & d & 2f & -b & -a & -2c \\ 3d & f & e & -3a & -c & -b \\ 5d & f & e & d + f & e & -5a & -c & -b & -a & -c & -b \end{bmatrix}$$

Now  $H^2(E \otimes E^*) = 0$ ,  $h^0(E(1)) = 4$ ,  $h^0(E^*(1)) = 2$ .

#### 3. On the singularities of moduli spaces.

The stable Schwarzenberger type bundles on  $\mathbb{P}^m$  (see (1.2)) form a Zariski open subset of the moduli space of stable bundles. Let  $N_{\mathbb{P}^m}(k,q)$  be the moduli space of stable STB whose first Chern class is k and whose rank is q. The following proposition is easy and well known:

**Proposition 3.1.** The space  $N_{\mathbb{P}^m}(k,q)$  is smooth, irreducible of dimension  $1 - k^2 - (q+k)^2 + k(q+k)(m+1)$ .

We denote by  $MI_{\mathbb{P}^{2n+1}}(k)$  the moduli space of stable instanton bundles with quantum number k. It is an open subset of the moduli space of stable 2n-bundles on  $\mathbb{P}^{2n+1}$  with Chern polynomial  $(1-t^2)^{-k}$ .

On  $\mathbb{P}^5$  (as on  $\mathbb{P}^3$ ) all instanton bundles are stable by [**AO**], Theorem 3.6.  $\mathrm{MI}_{\mathbb{P}^{2n+1}}(2)$  is smooth ([**AO**] Theorem 3.14), unirational of dimension  $4n^2 + 12n - 3$  and has zero Euler-Poincaré characteristic ([**BE**], [**K**]).

**Theorem 3.2.** The space  $MI_{\mathbb{P}^{2n+1}}(2)$  is irreducible.

*Proof.* The moduli space  $N = N_{\mathbb{P}^{2n+1}}(2, n+2)$  of stable STB of rank 2n+2 and  $c_1 = 2$  is irreducible of dimension  $4n^2 + 8n - 3$  by Prop. 3.1

For a given instanton bundle E there is a STB S associated with E, which is stable ([**AO**], Theorem 2.8) and unique (ibid., Prop. 2.17). It is easy to prove that the map  $\pi : M \to N$  defined by  $\pi([E]) = [S]$  is algebraic, moreover  $\pi$  is dominant by [**ST**]. If  $m = [E] \in M$ , the fiber  $\pi^{-1}(\pi(m))$  is a Zariski open subset of the grassmannian of planes in the vector space  $H^0(\mathbb{P}^{2n+1}, S^*(1))$ , where  $\pi(m) = [S]$ ; by the Theorem 3.14 of [**AO**],  $h^0(\mathbb{P}^{2n+1}, S^*(1)) = 2n+2$ , hence dim  $\pi^{-1}(\pi(m)) = 4n$ .

In order to prove that M is irreducible, we suppose by contradiction that there are at least two irreducible components  $M_0$  and  $M_1$  of M. Then  $M_0 \cap M_1 = \emptyset$  (M is smooth),  $\pi(M_0)$  and  $\pi(M_1)$  are constructible subset of N by Chevalley's theorem. Looking at the dimensions of  $M_0, M_1, N$  and the fibers of  $\pi$  we conclude that both  $\pi(M_0)$  and  $\pi(M_1)$  must contain an open subset of N, which implies  $\pi(M_0) \cap \pi(M_1) \neq \emptyset$  by the irreducibility of N. This is a contradiction because the fibers of  $\pi$  are connected.

For  $n \geq 2$  and  $k \geq 3$ , it is no longer true that  $\mathrm{MI}_{\mathbb{P}^{2n+1}}(k)$  is smooth. In fact on  $\mathbb{P}^5$  we have:

**Theorem 3.3.** The space  $MI_{\mathbb{P}^5}(k)$  is singular for k = 3, 4. To be more precise, the irreducible component  $M_0(k)$  of  $MI_{\mathbb{P}^5}(k)$  containing the special instanton bundles is generically reduced of dimension 54(k = 3) or 65(k = 4), and  $MI_{\mathbb{P}^5}(k)$  is singular at the points corresponding to special symplectic instanton bundles.

Proof. Let  $E_0$  be the special instanton bundle on  $\mathbb{P}^5$  of the Example 2.2(k = 3) or of the Example 2.5(k = 4). Then  $h^2(E_0 \otimes E_0^*) = 0$  and  $M_0(k)$  is smooth at the point corresponding to  $E_0$ , of dimension  $h^1(E_0 \otimes E_0^*) = 54(k = 3)$  or 65(k = 4). In particular,  $M_0(k)$  is generically reduced. If  $E_1$  is a special symplectic instanton bundle on  $\mathbb{P}^5$ , the computations in 2.1 and 2.3 show that  $h^2(E_1 \otimes E_1^*) = 3(k = 3)$  or 12(k = 4), and  $h^1(E_1 \otimes E_1^*) = 57$  or 77 respectively. Hence  $\mathrm{MI}_{\mathbb{P}^5}(k)$  is singular at  $E_1$  for k = 3 and 4.

**Remark 3.4.** It is natural to conjecture that  $MI_{\mathbb{P}^{2n+1}}(k)$  is singular for all  $n \ge 2$  and  $k \ge 3$ .

**Theorem 3.5.** Let E be an instanton bundle on  $\mathbb{P}^{2n+1}$  with  $c_2(E) = k$ . Then

$$h^{1}(E(t)) = 0$$
 for  $t \leq -2$  and  $k - 1 \leq t$ .

*Proof.* The result is obvious for  $t \leq -2$ . It is sufficient to prove  $h^1(S^*(t)) = 0$  for  $t \geq k - 1$ . We have

$$S^*(t) = \bigwedge^{2n+k-1} S(t-k).$$

Taking wedge products of (1.2) we have the exact sequence

$$0 \to \mathcal{O}(t+1-2n-2k)^{\alpha_0} \to \ldots \to \mathcal{O}(t-k-1)^{\alpha_{2n+k-2}}$$
$$\to \mathcal{O}(t-k)^{\alpha_{2n+k-1}} \to \bigwedge^{2n+k-1} S(t-k) \to 0$$

for suitable  $\alpha_i \in \mathbb{N}$  and from this sequence we can conclude.

Ellia proves Theorem 3.5 in the case of  $\mathbb{P}^3$  ([**E**], Prop. IV.1). He also remarks that the given bound is sharp. This holds on  $\mathbb{P}^{2n+1}$  as it is shown by the following theorem, which points out that the special symplectic instanton bundles are the "furthest" from having natural cohomology.

**Theorem 3.6.** Let E be a special symplectic instanton bundle on  $\mathbb{P}^{2n+1}$  with  $c_2 = k$ . Then

$$h^1(E(t)) \neq 0 \text{ for } -1 \leq t \leq k-2.$$

*Proof.* For n = 1 the thesis is immediate from the exact sequence

$$0 \to \mathcal{O}(t-1) \to E(t) \to \mathcal{J}_C(t+1) \to 0$$

where C is the union of k + 1 disjoint lines in a smooth quadric surface. Then the result follows by induction on n by considering the sequence

$$0 \to E(t-2) \to E(t-1)^2 \to E(t) \to E(t)|_{\mathbb{P}^{2n-1}} \to 0$$

and the fact that, for a particular choice of the subspace  $\mathbb{P}^{2n-1}$ , the restriction  $E|_{\mathbb{P}^{2n-1}}$  splits as the direct sum of a rank-2 trivial bundle and a special symplectic instanton bundle on  $\mathbb{P}^{2n-1}([\mathbf{ST}] 5.9)$ .

**Remark 3.7.** In **[OT]** it is proved that if  $E_k$  is a special symplectic instanton bundle on  $\mathbb{P}^5$  with  $c_2 = k$  then  $h^1(\text{End } E_k) = 20k - 3$ .

In the following table we summarize what we know about the component  $M_0(k) \subset \mathrm{MI}_{\mathbb{P}^5}(k)$  containing  $E_k$ .

**Table 3.10** 

	$h^1(E_k\otimes E_k^*)$	$h^2(E_k\otimes E_k^*)$	$\dim M_0(k)$	$\mathrm{MI}_{\mathbf{P}^5}(k)$
k = 1	14	0	14	open subset of $\mathbb{P}^{14}$
k = 2	37	0	37	smooth, irreduc., unirat.
k = 3	57	3	54	singular
k = 4	77	12	65	singular
$k \ge 2$	20k - 3	$3(k-2)^2$	?	?

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Added in proof. After this paper has been written we received a preprint of R. Miró-Roig and J. Orus-Lacort where they prove that the conjecture stated in the Remark 3.4

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