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**CORRECTION TO: “ASYMPTOTIC RADIAL SYMMETRY FOR  
SOLUTIONS OF  $\Delta u + e^u = 0$  IN A PUNCTURED DISC”**

**KAI SENG (KAISING) CHOU (TSO) AND TOM YAU-HENG WAN**

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 FOR SOLUTIONS OF  $\Delta u + e^u = 0$  IN A PUNCTURED DISC”**

K.S. CHOU AND TOM Y.H. WAN

The negative case ( $K < 0$ ) in the Theorem 3 of the above mentioned paper is incomplete. In this case, the authors considered three separate cases (page 273 Pacific J. Math., **163**, No. 2, 1994). After handling the first two, the authors thought a similar argument would work for the third which turns out to be incorrect. Therefore, we need to reconsider the third case, namely, the function  $f$  may take the form

$$f(z) = \frac{e^{i\alpha} (1 + g(z) + \alpha \log z)}{1 - g(z) - \alpha \log z}$$

for some  $\alpha \in \mathbb{R}$  and some single-valued analytic function  $g$  on the punctured disc  $D^* = \{z \in \mathbb{C} | 0 < |z| < 1\}$ . As in this paper, we may assume that  $K = -4$  and  $|f| < 1$ . Then we conclude that

$$\operatorname{Re} g(z) + \alpha \log r < 0, \quad \text{where } r = |z|,$$

and hence

$$r^\alpha \left| e^{g(z)} \right| < 1.$$

Therefore, 0 is not an essential singularity of  $e^{g(z)}$ . It implies that  $g(z)$  analytically extends across 0. So, in the negative case, we have

**Theorem 1.** *Real smooth solutions of  $\Delta u - 8e^u = 0$  in  $D^*$  are of the form*

$$u = \log \frac{|f'|^2}{(1 - |f|^2)^2}$$

with  $f$  a multi-valued locally univalent meromorphic function of the form

$$f(z) = h(z)z^\beta, \quad \beta \geq 0$$

or

$$(1) \quad f(z) = \frac{1 + h(z) + \alpha \log z}{1 - h(z) - \alpha \log z}, \quad \alpha \in \mathbb{R}$$

for some single-valued analytic function  $h(z)$  on the whole disc  $D = \{z \in \mathbb{C} \mid |z| < 1\}$ .

To find the asymptotic formula, observe that (1) gives

$$u = \log \frac{|zh'(z) + \alpha|^2}{4r^2 (\operatorname{Re} h(z) + \alpha \log r)^2},$$

which implies that

$$u = -2 \log \left( r \log \frac{1}{r} \right) + O(1) \quad \text{as } r \rightarrow 0.$$

Therefore, we have

**Theorem 2.** *Let  $u$  be a smooth real solution of  $\Delta u + 2Ke^u = 0$  for  $K < 0$ , then*

$$u(z) = \alpha \log |z| + O(1), \quad \alpha > -2,$$

or

$$u(z) = -2 \log \left( |z| \log \frac{1}{|z|} \right) + O(1)$$

as  $|z| \rightarrow 0$ .

Finally, it is well-known that all such solution  $u$  are bounded by the Poincaré metric (the unique complete constant curvature  $K$  conformal metric) on  $D^*$  which has finite area near the origin. Therefore, all solution  $u$  satisfies  $\int e^u < +\infty$  in any small region containing the origin.

We thank Prof. R. Finn for communicating to us a counterexample by Yamashita which falls into the third case of  $K < 0$ .

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# PACIFIC JOURNAL OF MATHEMATICS

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On $H^p$ -solutions of the Bezout equation	297
ERIC AMAR, JOAQUIM BRUNA FLORIS and ARTUR NICOLAU	
Amenable correspondences and approximation properties for von Neumann algebras	309
CLAIRE ANANTHARAMAN-DELAROCHE	
On moduli of instanton bundles on $\mathbb{P}^{2n+1}$	343
VINCENZO ANCONA and GIORGIO MARIA OTTAVIANI	
Minimal surfaces with catenoid ends	353
JORGEN BERGLUND and WAYNE ROSSMAN	
Permutation model for semi-circular systems and quantum random walks	373
PHILIPPE BIANE	
The Neumann problem on Lipschitz domains in Hardy spaces of order less than one	389
RUSSELL M. BROWN	
Matching theorems for twisted orbital integrals	409
REBECCA A. HERB	
Uniform algebras generated by holomorphic and pluriharmonic functions on strictly pseudoconvex domains	429
ALEXANDER IZZO	
Quantum Weyl algebras and deformations of $U(g)$	437
NAIHUAN JING and JAMES ZHANG	
Calcul du nombre de classes des corps de nombres	455
STÉPHANE LOUBOUTIN	
On geometric properties of harmonic $Lip_1$ -capacity	469
PERTTI MATTILA and P. V. PARAMONOV	
Reproducing kernels and composition series for spaces of vector-valued holomorphic functions	493
BENT ØRSTED and GENKAI ZHANG	
Iterated loop modules and a filtration for vertex representation of toroidal Lie algebras	511
S. ESWARA RAO	
The intrinsic mountain pass	529
MARTIN SCHECHTER	
A Frobenius problem on the knot space	545
RON G. WANG	
On complete metrics of nonnegative curvature on 2-plane bundles	569
DAVID YANG	
Correction to: "Free Banach-Lie algebras, couniversal Banach-Lie groups, and more"	585
VLADIMIR G. PESTOV	
Correction to: "Asymptotic radial symmetry for solutions of $\Delta u + e^u = 0$ in a punctured disc"	589
KAI SENG (KAISING) CHOU (TSO) and TOM YAU-HENG WAN	