

*Pacific
Journal of
Mathematics*

INTERPOLATING BLASCHKE PRODUCTS

DONALD EDDY MARSHALL AND ARNE STRAY

INTERPOLATING BLASCHKE PRODUCTS

DONALD E. MARSHALL AND ARNE STRAY

We prove that any bounded analytic function on the unit disk \mathbb{D} which extends to be continuous on $\partial\mathbb{D} \setminus E$, for some set E of measure 0, can be uniformly approximated by finite linear combinations of interpolating Blaschke products.

Let H^∞ denote the set of bounded analytic functions defined on the unit disk, \mathbb{D} . Each $f \in H^\infty$ has a non-tangential limit at almost all $e^{i\theta} \in \partial\mathbb{D}$ which we call $f(e^{i\theta})$, and

$$\sup_{z \in \mathbb{D}} |f(z)| = \|f\|_\infty$$

where $\|f\|_\infty$ is the $L^\infty \equiv L^\infty(\partial\mathbb{D}, d\theta)$ norm of the non-tangential limit f . Thus we can view H^∞ as a uniformly closed subalgebra of L^∞ . An *inner* function is a function $I \in H^\infty$ with $|I(e^{i\theta})| = 1$, a.e. $d\theta$. For example $S(z) = e^{\frac{z+1}{z-1}}$ is an inner function, though it has radial limit 0 at $\zeta = 1$. Inner functions are of central importance in the study of analytic functions on \mathbb{D} . For example, Beurling used inner functions (and coined the name) to characterize the translation invariant subspaces of the Hardy space H^2 . Rudin and Carleson used inner functions to characterize the ideals of the disk algebra, and Newman used them to characterize the Shilov boundary of H^∞ .

If $\{z_n\} \subset \mathbb{D}$ with $\sum_n (1 - |z_n|) < \infty$ then a *Blaschke product* with zero set $\{z_n\}$ is an inner function of the form

$$B(z) = \prod_{n=1}^{\infty} c_n \frac{z - z_n}{1 - \bar{z}_n z},$$

where c_n are constants, with $|c_n| = 1$, chosen so the the infinite product converges. Blaschke products are important because they characterize the zero sets of bounded analytic functions: if $f \in H^\infty$ then there is a Blaschke product B and a non-vanishing $g \in H^\infty$ with $f = Bg$ and $|f(e^{i\theta})| = |g(e^{i\theta})|$ almost everywhere on $\partial\mathbb{D}$. Frostman proved that if I is inner then $(I - \lambda)/(1 - \bar{\lambda}I)$ is a Blaschke product for λ in a dense subset of \mathbb{D} (the exceptional set in fact has logarithmic capacity 0). Letting $\lambda \rightarrow 0$ shows that every inner function can be uniformly approximated by Blaschke products. Because of

their importance in this subject, the problem of characterizing the linear span of the inner functions arose in 1965. Douglas and Rudin proved that every unimodular function in L^∞ can be uniformly approximated by the quotient of two inner functions. Since every L^∞ function of norm smaller than 1 is the average of two unimodular functions, finite linear combinations of quotients of Blaschke products are dense in L^∞ . Later Hoffman proved that Blaschke products separated the points of the maximal ideal space of H^∞ . In [M1] it was proved that the unit ball of H^∞ is the norm closed convex hull of the set of Blaschke products. See [G] and [H] for these and other background results about inner functions and Blaschke products.

Important developments in function theory on the disk have come from considering a special kind of zero set, called an interpolating sequence. The pseudo-hyperbolic metric on \mathbb{D} is given by

$$\rho(z, w) = \left| \frac{z - w}{1 - \bar{w}z} \right|.$$

A sequence $\{z_n\} \subset \mathbb{D}$ is called an *interpolating sequence* if

$$(1) \quad \inf_n \prod_{j \neq n} \rho(z_j, z_n) > 0.$$

An interpolating sequence is the zero set of a Blaschke product B and the condition (1) can be rephrased in terms of B since

$$(2) \quad (1 - |z_n|^2)|B'(z_n)| = \prod_{j \neq n} \rho(z_j, z_n).$$

Such a Blaschke product is called an *interpolating Blaschke product*. Carleson proved that if $\{z_n\}$ is an interpolating sequence and if $\{a_n\} \in \ell^\infty$, then there is an $f \in H^\infty$ “interpolating” the values a_n at z_n . In other words, $f(z_n) = a_n$ for all n . The formulation of interpolating sequences in terms of B' is important because the quantity $(1 - |z|^2)|B'(z)|$ is the conformally invariant derivative of B , in the following sense: Let $z = \tau(w)$ and $C(w) = B(\tau(w))$, where τ is a conformal map of \mathbb{D} onto \mathbb{D} . Then $(1 - |z|^2)|B'(z)| = (1 - |w|^2)|C'(w)|$.

Carleson’s proof of his interpolation theorem gave an important geometric characterization of interpolating sequences. A measure μ defined on \mathbb{D} is called a *Carleson measure* if there is a constant $K < \infty$ so that for all approximate squares

$$Q = \{re^{i\theta} : 1 - a < r < 1, \theta_0 \leq \theta \leq \theta_0 + a\}$$

the following estimate holds:

$$\mu(Q) \leq Ka.$$

A sequence $\{z_n\} \subset \mathbb{D}$ is interpolating if and only if there is an $\varepsilon > 0$ such that

$$(3) \quad \rho(z_n, z_m) \geq \varepsilon \text{ for all } n \neq m \text{ and}$$

$$(4) \quad \sum_n (1 - |z_n|^2) \delta_{z_n} \text{ is a Carleson measure,}$$

where δ_z is unit point mass at z . Carleson measures and interpolating sequences also proved to be of great importance in the proof of the Corona theorem, by Carleson [Ca], and in finding L^∞ solutions to $\bar{\partial}$ -problems by Jones [J2], for example. Carleson measures, were also fundamental in the development of BMO by C. Fefferman [Fe], Jones [J3] and others. Carleson measures can be obtained as weak-* limits of measures associated with interpolating sequences [J3].

The problem of characterizing the linear span of the interpolating Blaschke products arose in 1978 [J1]. See also [G, p. 430]. Ziskind [Z] earlier proved that interpolating Blaschke products characterize the Shilov boundary of H^∞ , the natural analog of Newman’s theorem. Another indication that the interpolating Blaschke products span a “large” subset of H^∞ is the Chang-Marshall theorem [Ch, M2]: Every closed subalgebra of L^∞ containing H^∞ is generated by H^∞ and a collection of complex conjugates of interpolating Blaschke products. The analog of Frostman’s theorem for interpolating Blaschke products is not true, however, since Kahane [K], Piranian [P], and Shapiro [S] proved there are inner functions I , which are not finite Blaschke products, such that

$$(5) \quad \lim_{|z| \rightarrow 1} (1 - |z|^2) |I'(z)| = 0.$$

Since $(I - \lambda)/(1 - \bar{\lambda}I)$ must have this same property, it is never an interpolating Blaschke product. In [J1] Jones proved the analogs of the Douglas-Rudin and Hoffman theorems mentioned above: Every unimodular L^∞ function can be uniformly approximated by a quotient of interpolating Blaschke products and the interpolating Blaschke products separate the points of the maximal ideal space of H^∞ .

Let \mathcal{IBP} denote the uniform closure of finite linear combinations of interpolating Blaschke products:

$$\mathcal{IBP} = cl \left\{ \sum_{n=1}^m \alpha_n B_n : B_n \text{ is interpolating and } \alpha_n \in \mathbb{C} \right\}.$$

While we cannot solve Jones’ problem of characterizing \mathcal{IBP} , we can prove that an H^∞ function is in \mathcal{IBP} if it is not too “pathological”. Let \mathcal{RI} denote

the bounded analytic functions on D which extend to be continuous on $\mathbb{D} \setminus E$ for some $E \subset \partial\mathbb{D}$ with $|E| \equiv \text{length}(E) = 0$. These are precisely the bounded analytic functions with Riemann integrable boundary values, and hence the name \mathcal{RI} . By Corollary 1.3 of [DGG], \mathcal{RI} is the uniform closure of the set of bounded analytic functions that extend to be continuous on $\mathbb{D} \setminus E$ for some closed set $E \subset \partial\mathbb{D}$ with $|E| \equiv \text{length}(E) = 0$.

Theorem. $\mathcal{RI} \subset \mathcal{IBP}$.

With different techniques, and at the same time, A. Nicolau proved that a Blaschke product which is continuous on $\mathbb{D} \setminus E$ for some closed set $E \subset \partial\mathbb{D}$ with $|E| \equiv \text{length}(E) = 0$ is in the algebra generated by the interpolating Blaschke products. We remark that Wolff [unpublished] has shown that there are Blaschke products in \mathcal{RI} satisfying (5) above.

Lemma 1. \mathcal{IBP} is an algebra.

Proof of Lemma 1. It suffices to show that the product of two interpolating Blaschke products is in \mathcal{IBP} . Suppose B_1 and B_2 are interpolating Blaschke products with zero sets $\{a_n\}$ and $\{b_n\}$ respectively, then $\{z_n\} = \{a_n\} \cup \{b_n\}$ satisfies (4). It is possible to move the $\{b_n\}$ slightly so as to satisfy (3) and still have a Blaschke product close to B_1B_2 . The $\{b_n\}$ cannot just be arbitrarily moved, for example if $a_n = b_n = 1 - 2^{-n}$ and if B_3 is the Blaschke product with zeros $\zeta_n = z_n + i\delta 2^{-n}$, where $\delta > 0$, then $\|B_1B_2 - B_1B_3\| = 2$.

By (3), we may suppose that the disks $D_k = \{z : \rho(z, a_k) \leq \delta\}$ are disjoint and each D_k contains at most one b_n . By Earl’s proof of Carleson’s interpolation theorem [E], if δ is sufficiently small, there is a constant K , $0 < K < 1$ so that

$$(6) \quad |B_1(z)| \geq K\delta$$

on $\Omega_\delta = \mathbb{D} \setminus \cup_k D_k$. Moreover, we can move some of the zeros of B_1 to B_2 so that we can assume that for each zero a_k of B_1 , there is a zero d_k of B_2 with $\rho(a_k, d_k) < K\delta/4$. By Frostman’s theorem, we can choose ε , with $K\delta/4 < \varepsilon < K\delta/3$ so that $C_3 = (B_1 - \varepsilon)/(1 - \varepsilon B_1)$ is a Blaschke product. Then C_3 has exactly one zero in each “annular” region $A_k = \{z : \varepsilon < \rho(z, a_k) < \delta\}$ and no other zeros. To see this, first note that

$$(7) \quad \|C_3 - B_1\|_\infty \leq \frac{2\varepsilon}{1 - \varepsilon} < K\delta.$$

So if $C_3(z) = 0$, then $|B_1(z)| < K\delta$ and hence by (6) $z \notin \Omega_\delta$. Also if $C_3(z) = 0$, then

$$\varepsilon = |B_1(z)| = \rho(B_1(z), B_1(a_n)) < \rho(z, a_n)$$

by Schwarz’s lemma. On ∂D_k , by (6) and (7),

$$|C_3(z) - B_1(z)| < |B_1(z)|,$$

so Rouché’s theorem, C_3 has exactly one zero in each D_k and hence exactly one in each A_k . Thus ρ -distance between the zeros of C_3 and the zeros of B_2 is at least $\rho(\varepsilon, K\delta/4) > 0$. It is not hard to verify, using (3) and (4), that $B \equiv C_3 B_2$ is interpolating and

$$\|B_1 B_2 - B\|_\infty < \frac{2\varepsilon}{1 - \varepsilon}.$$

Since δ , and hence ε , can be chosen arbitrarily small, this proves the lemma. □

We remark that since $z \in IBP$

$$1 = -2 \left(\frac{z - \frac{1}{2}}{1 - \frac{1}{2}z} \right) + 3 \sum_{n=1}^\infty \frac{z^n}{2^n} \in IBP,$$

by Lemma 1.

The estimate in the next Lemma is the key to the proof of the theorem. We transfer the problem from the disk to the upper half plane \mathbb{H} . In this context, the pseudo-hyperbolic distance is given by

$$\rho(z, w) = \left| \frac{z - w}{z - \bar{w}} \right|,$$

$z, w \in \mathbb{H}$, and a Blaschke product is given by

$$B(z) = \prod_{n=1}^\infty c_n \frac{z - z_n}{z - \bar{z}_n}$$

where $\text{Im } z_n > 0$, and $\{c_n\}$ are constants of absolute value 1 chosen so that the product converges. The conformally invariant derivative of B on the upper half plane is $\text{Im } z |B'(z)|$, and B is interpolating if and only if

$$(8) \quad \inf_n 2 \text{Im } z_n |B'(z_n)| = \inf_m \prod_{n \neq m} \rho(z_n, z_m) > 0.$$

Lemma 2. *Suppose B is a Blaschke product on \mathbb{H} with zeros $\{z_n\} \subset \mathbb{H} \cap \{z : \text{Re } z \leq 0\}$. If $\delta > 0$ and $0 < \arg z < \frac{\pi}{2} - \delta$, then*

$$\text{Im } z |B'(z)| \geq C |B(z)| \log \frac{1}{|B(z)|},$$

where C is a constant that depends only on δ

Proof of Lemma 2. Note that for some $\varepsilon = \varepsilon(\delta) > 0$, $\rho(z, z_n) \geq \varepsilon$. Thus

$$(9) \quad \log \frac{1}{|B(z)|^2} \leq C_1 \sum_{n=1}^{\infty} (1 - \rho^2(z, z_n)) = 4C_1 \sum_{n=1}^{\infty} \frac{\text{Im } z \text{ Im } z_n}{|z - \bar{z}_n|^2},$$

and

$$(10) \quad (\text{Im } z) \frac{B'(z)}{B(z)} = \sum_{n=1}^{\infty} \frac{\text{Im } z \text{ Im } z_n}{|z - \bar{z}_n|^2} \frac{2i(\bar{z} - z_n)}{z - z_n}.$$

By our assumptions on z and z_n , (see Figure 1)

$$0 \leq \theta = \arg \left(\frac{z - z_n}{\bar{z} - z_n} \right) \leq \pi - 2\delta.$$

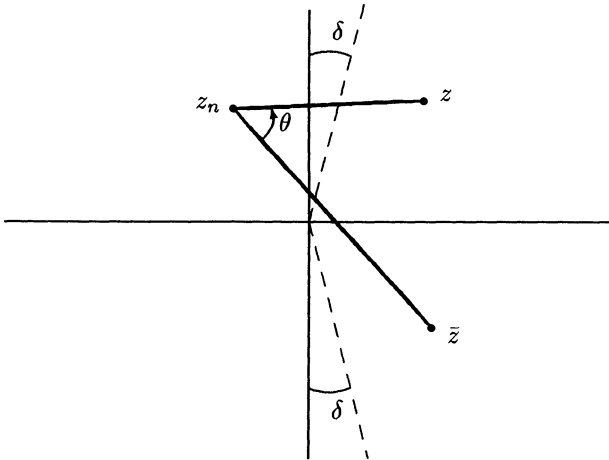


Figure 1.

Thus if $\zeta = e^{-i\delta}$,

$$\left| \arg \left(\frac{2i(\bar{z} - z_n)}{z - z_n} \zeta \right) \right| \leq \frac{\pi}{2} - \delta,$$

and since $\left| \frac{\bar{z} - z_n}{z - z_n} \right| \geq 1$,

$$(11) \quad \text{Re} \frac{2i(\bar{z} - z_n)}{z - z_n} \zeta \geq C_2 \equiv 2 \cos \left(\frac{\pi}{2} - \delta \right) > 0.$$

By (9), (10) and (11)

$$\text{Im } z \left| \frac{B'(z)}{B(z)} \right| \geq \text{Im } z \text{ Re} \left(\frac{B'(z)\zeta}{B(z)} \right) \geq C_2 \sum_{n=1}^{\infty} \frac{\text{Im } z \text{ Im } z_n}{|z - \bar{z}_n|^2} \geq \frac{C_2}{4C_1} \log \frac{1}{|B(z)|^2}. \square$$

Lemma 3. *If B is a Blaschke product which is continuous on $\partial\mathbb{D} \setminus E$, where E is a closed set of length 0, then $B \in \mathcal{IBP}$.*

Proof of Lemma 3. Transfer the problem to the upper-half plane \mathbb{H} and without loss of generality, suppose the zeros $\{z_n\}$ of

$$B(z) = \prod_{n=1}^{\infty} c_n \frac{z - z_n}{z - \bar{z}_n}$$

are contained in the unit square $Q_0 = \{x + iy : 0 < x < 1, 0 < y < 1\}$. A dyadic square Q is a square sitting on \mathbb{R} of the form $Q = \{x + iy : \frac{k}{2^n} \leq x \leq \frac{k+1}{2^n}, 0 < y \leq 2^{-n}\}$ where k and n are integers. Let $\{Q_j\}$ be the set of dyadic squares which do not contain any zero of B . The region $Q_0 \setminus \cup Q_j$ contains all zeros of B and by our assumption on B , the curve $\Gamma = \partial(Q_0 \setminus \cup Q_j)$ satisfies

$$(12) \quad \text{length}(\Gamma \cap \mathbb{R}) = 0.$$

Note that arc-length on Γ is a Carleson measure. Given $\varepsilon > 0$, sprinkle points $\{\zeta_n\}$ on Γ such that

$$(13) \quad \rho(\zeta_n, \zeta_m) \geq \varepsilon \text{ for all } n \neq m \text{ and}$$

$$(14) \quad \text{if } \zeta \in \Gamma \cap \mathbb{H} \text{ then } \rho(\zeta, \zeta_n) \leq 2\varepsilon, \text{ for some } n.$$

As in Ziskind [Z] (see also [G]), the Blaschke product B_I with zeros $\{\zeta_n\}$ is interpolating. Moreover

$$|BB_I| \leq 2\varepsilon \text{ on } Q_0 \setminus \cup Q_j,$$

by (12), (14) and the maximum principle.

There is a $c_1, 0 < c_1 < 1$, so that if $\rho(z, Q_0 \setminus \cup Q_j) > c_1$ and $z \in Q_0$, then z lies in a disk D centered on \mathbb{R} with $D \subset \cup Q_j$. Thus there is a $\sigma > 2\varepsilon$ and $\delta > 0$, so that if $|B(z)B_I(z)| > \sigma$ then there is a linear fractional transformation φ of \mathbb{H} onto \mathbb{H} so that $\varphi(Q_0 \setminus \cup Q_j) \subset \{z : \text{Re } z \leq 0\}$ and $0 < \arg \varphi(z) < \frac{\pi}{2} - \delta$. A similar argument works for $z \notin Q_0$.

By (8), Lemma 2 and Frostman's theorem

$$D = \frac{BB_I - \lambda}{1 - \lambda BB_I}$$

is an interpolating Blaschke product for some $\lambda > \sigma$. By Lemma 1 and the comment immediately after Lemma 1,

$$BB_I = \lambda + (1 - \lambda^2) \sum_{n=1}^{\infty} (-\lambda)^{n-1} D^n \in \mathcal{IBP}.$$

By the arguments used in the proof of Lemma 1 and Frostman's theorem,

$$\frac{B_I - \eta}{1 - \bar{\eta}B_I}$$

is an interpolating Blaschke product for some arbitrarily small $|\eta|$. In the proof above that D is interpolating, we really only needed that B_I is small on Γ and that the zeros of B_I are hyperbolically close to Γ . Thus

$$C = B \left(\frac{B_I - \eta}{1 - \bar{\eta}B_I} \right) \in \mathcal{IBP}.$$

and

$$B = \frac{1}{\eta}(BB_I - C(1 - \bar{\eta}B_I)) \in \mathcal{IBP}.$$

□

Proof of the Theorem. By the comment after the definition of \mathcal{RI} and since $1 \in \mathcal{IBP}$, it suffices to approximate only $f \in H^\infty$ such that f extends to be continuous on $\bar{\mathbb{D}} \setminus E$, where E is closed and $|E| = 0$, and such that $\|f - \frac{1}{2}\|_\infty < \frac{1}{2}$. Let C_E denote the closed subalgebra of L^∞ generated by quotients of Blaschke products which extend to be analytic on $\partial\mathbb{D} \setminus E$. In other words, C_E is the set of uniform limits of finite linear combinations of quotients of Blaschke products which are analytic on $\partial\mathbb{D} \setminus E$. Since each point of $\{z : |z - \frac{1}{2}| < \frac{1}{2}\}$ is the average of two unique points on $\partial\mathbb{D}$, we can write $f = (u_1 + u_2)/2$ where u_1 and u_2 are unimodular functions on $\partial\mathbb{D}$ which are continuous on $\partial\mathbb{D} \setminus E$. By Theorem 12.1 [DDG], we have $f \in C_E$. Let \mathcal{A} be the uniformly closed algebra generated by H^∞ and C_E . It is shown in [MS] that the inner functions which are invertible in \mathcal{A} belong to C_E . By Theorem 4.1 [CM], f can be uniformly approximated by convex combinations of Blaschke products in $H^\infty \cap C_E$. The Theorem now follows from Lemma 1 and Lemma 3. □

We remark that \mathcal{RI} is a uniformly closed subalgebra of H^∞ with inner-outer factorization. See [MS].

References

- [Ca] L. Carleson, *Interpolations by bounded analytic functions and the corona problem*, Annals of Math., **76** (1962), 547-559.
- [Ch] S.Y. Chang, *A characterization of Douglas subalgebras*, Acta Math., **137** (1976), 81-89.
- [CM] S.Y. Chang and D.E. Marshall, *Some algebras of bounded analytic functions containing the disk algebra*, Banach Spaces of Analytic Functions, Springer Lecture Notes, **604** (1977), 12-20.

- [DGG] A.M. Davie, T.W. Gamelin, and J. Garnett, *Distance estimates and pointwise bounded density*, Trans. Amer. Math. Soc., **175** (1973), 37-68.
- [E] J.P. Earl, *On the interpolation of bounded sequences by bounded functions*, J. London. Math. Soc., **2** (1970), 544-548.
- [Fe] C. Fefferman and E.M. Stein, *H^p spaces of several variables*, Acta Math., **129** (1972), 137-193.
- [Fr] O. Frostman, *Potential d'équilibre et capacité des ensembles avec quelques applications à la théorie des fonctions*, Medd. Lund. Univ. Math. Sem., **3** (3) (1935), 1-118.
- [G] J.B. Garnett, *Bounded Analytic Functions*, Academic Press, New York, 1981.
- [H] K. Hoffman, *Banach Spaces of Analytic Functions*, Prentiss-Hall, Inc., Englewood Cliffs, 1962.
- [J1] P.W. Jones, *Constructions with Functions of Bounded Mean Oscillation*, Thesis, UCLA, 1978. See also P.W. Jones, *Ratios of interpolating Blaschke products*, Pac. J. Math., **95** (1981), 311-321.
- [J2] ———, *L^∞ estimates and the ∂ -problem*, Acta Math., **150** (1983), 137-152.
- [J3] ———, *Carleson measures and the Fefferman-Stein decomposition of $BMO(\mathbb{R})$* , Annals of Math., **111** (1980), 197-208.
- [K] J.P. Kahane, *Trois notes sur les ensembles parfaits linéaires*, Enseign. Math., **15** (1969), 185-192.
- [M1] D.E. Marshall, *Blaschke products generate H^∞* , Bull. Amer. Math. Soc., **82** (1976), 494-496.
- [M2] ———, *Subalgebras of L^∞ containing H^∞* , Acta Math., **137** (1976), 91-98.
- [MS] D.E. Marshall and K. Stephenson, *Inner divisors and composition operators*, J. Funct. Anal., **46** (1982), 131-148.
- [P] G. Piranian, *Two monotonic singular, uniformly almost smooth functions*, Duke Math J., **33** (1966), 255-262.
- [S] H.S. Shapiro, *Monotonic singular functions of high smoothness*, Michigan Math. J., **15** (1968), 265-275.
- [Z] S. Ziskind, *Interpolating sequences and the Shilov boundary of $H^\infty(\Delta)$* , J. Funct. Anal., **21** (1976), 380-388.

Received August 10, 1993. The first author is supported in part by NSF grant DMS-9302823.

UNIVERSITY OF WASHINGTON
 SEATTLE, WA 98195
E-mail address: marshall@math.washington.edu

AND

UNIVERSITETET I BERGEN
 MATEMATISK INSTITUTT
 5014 BERGEN-UNIVERSITETET
 NORWAY

Peng Lin and Richard Rochberg , Trace ideal criteria for Toeplitz and Hankel operators on the weighted Bergman spaces with exponential type weights	127
Donald E. Marshall and Arne Stray , Interpolating Blaschke products ..	491
Kathy D. Merrill and Lynne H. Walling , On quadratic reciprocity over function fields	147
Takahiko Nakazi and Masahiro Yamada , (A_2) -conditions and Carleson inequalities in Bergman spaces	151
C. Ott , A note on a paper of E. Boasso and A. Larotonda	173
Victor Patrangenaru , Classifying 3 and 4 dimensional homogeneous Riemannian manifolds by Cartan triples	511
Carlo Pensavalle and Tim Steger , Tensor products with anisotropic principal series representations of free groups	181
Ying Shen , On Ricci deformation of a Riemannian metric on manifold with boundary	203
Albert Jeu-Liang Sheu , The Weyl quantization of Poisson $SU(2)$	223
Alexandra Shlapentokh , Polynomials with a given discriminant over fields of algebraic functions of positive characteristic	533
Eric Stade and D.I. Wallace , Weyl's law for $SL(3, \mathbb{Z}) \backslash SL(3, \mathbb{R}) / SO(3, \mathbb{R})$	241
Christopher W. Stark , Resolutions modeled on ternary trees	557
Per Tomter , Minimal hyperspheres in two-point homogeneous spaces	263
Jun Tomiyama , Topological Full groups and structure of normalizers in transformation group C^* -algebras	571
Nik Weaver , Subalgebras of little Lipschitz algebras	283

PACIFIC JOURNAL OF MATHEMATICS

Volume 173 No. 2 April 1996

A mean value inequality with applications to Bergman space operators	295
PATRICK ROBERT AHERN and ZELJKO CUCKOVIC	
H^p -estimates of holomorphic division formulas	307
MATS ANDERSSON and HASSE CARLSSON	
Group structure and maximal division for cubic recursions with a double root	337
CHRISTIAN JEAN-CLAUDE BALLOT	
The Weil representation and Gauss sums	357
ANTONIA WILSON BLUHER	
Duality for the quantum $E(2)$ group	375
ALFONS VAN DAELE and S. L. WORONOWICZ	
Cohomology complex projective space with degree one codimension-two fixed submanifolds	387
KARL HEINZ DOVERMANN and ROBERT D. LITTLE	
On the mapping intersection problem	403
ALEXANDER DRANISHNIKOV	
From the L^1 norms of the complex heat kernels to a Hörmander multiplier theorem for sub-Laplacians on nilpotent Lie groups	413
XUAN THINH DUONG	
Isoperimetric inequalities for automorphism groups of free groups	425
ALLEN E. HATCHER and KAREN VOGTMANN	
Approximation by normal elements with finite spectra in C^* -algebras of real rank zero	443
HUAXIN LIN	
Interpolating Blaschke products	491
DONALD EDDY MARSHALL and ARNE STRAY	
Interpolating Blaschke products generate H^∞	501
JOHN BRADY GARNETT and ARTUR NICOLAU	
Classifying 3- and 4-dimensional homogeneous Riemannian manifolds by Cartan triples	511
VICTOR PATRANGENARU	
Polynomials with a given discriminant over fields of algebraic functions of positive characteristic	533
ALEXANDRA SHLAPENTOKH	
Resolutions modeled on ternary trees	557
CHRISTOPHER W. STARK	
Topological full groups and structure of normalizers in transformation group C^* -algebras	571
JUN TOMIYAMA	