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In this paper, we proved a unicity theorem for meromorphic functions with one sharing pair and a condition on deficiency. An example shows that the condition on deficiency is best possible. This result gives a general answer to the problem due to C.C. Yang (1977).

1. Introduction.

In this paper, by meromorphic function we always mean a function which meromorphic in the plane. Let $f(z)$ be meromorphic. We shall use the following standard notations in Nevanlinna theory:

$$T(r, f), \quad m(r, f), \quad N(r, f), \dots$$

(see Gross [5]). We denote by $S(r, f)$ any function satisfying

$$S(r, f) = o\{T(r, f)\}$$

as $r \rightarrow +\infty$, possibly outside a set of finite Lebesgue measure. A meromorphic function $a(z)$ is said to be a small function of f if

$$T(r, a) = S(r, f).$$

In this case, we define

$$\delta(a, f) = 1 - \lim_{r \rightarrow \infty} \frac{N(r, \frac{1}{f-a})}{T(r, f)},$$

and $a(z)$ is said to be a deficient function of f if $\delta(a, f) > 0$.

Let $g(z)$, $a_1(z)$ and $a_2(z)$ be meromorphic functions. If the two functions $f(z) - a_1(z)$ and $g(z) - a_2(z)$ assume the same zeros with the same multiplicities, then we call that f and g share the pair (a_1, a_2) CM. In particular, if $a_1 = a_2 = a$, then the word "the pair" is replaced by "the value" or "the function" provided that a is a constant or a is a function respectively (cf. Frank-Ohlenroth [4], Gundersen [6], etc.). In addition, if

$$N(r, (f = a_1)\Delta(g = a_2)) = \min\{S(r, f), \quad S(r, g)\},$$

then we say that f and g almost share the pair (a_1, a_2) CM. Here, $N(r, (f = a_1)\Delta(g = a_2))$ is the counting function of those points which satisfy one of the following three cases: (i) $f = a_1$ but $g \neq a_2$; (ii) $f \neq a_1$ but $g = a_2$; (iii) $f = a_1$ and $g = a_2$ but the multiplicities are not the same.

In 1977, Yang [9] proved the following result.

Theorem A. *Suppose that F is a family of the functions which are of the form $\alpha_1(z)e^{\mu(z)} + \alpha_2(z)$, where $\mu(z)$ is an entire functions with finite order, $\alpha_j(z)$ ($j = 1, 2$) are meromorphic functions of finite order, $\alpha_1 \not\equiv 0$, $\alpha_2 \not\equiv \text{const.}$, the order of α_j ($j = 1, 2$) is less than the order of μ . Let c_1 and c_2 be two distinct constants, and let $f \in F$, $g \in F$. If f and g share the two values c_1 and c_2 CM, then $f \equiv g$ or*

$$\left(f - \frac{c_2 - c_1\lambda(z)}{1 - \lambda(z)}\right) \left(g + \frac{c_2 - c_1\lambda(z)}{1 - \lambda(z)}\right) = -\frac{(c_2 - c_1)^2\lambda(z)}{(1 - \lambda(z))^2},$$

where $\lambda(z)$ is a nonconstant meromorphic function.

Based on this result, Yang [9] proposed the following problem.

Yang's problem. *Whether can we omit the restrictions on the order in the family F ?*

It is easy to see from the hypotheses of Theorem A that, if $f = \alpha_1(z)e^{\mu(z)} + \alpha_2(z) \in F$ and $g = \alpha_3(z)e^{\nu(z)} + \alpha_4(z) \in F$, then $N(r, \frac{1}{f-\alpha_2}) = o\{T(r, f)\}$ and $N(r, \frac{1}{g-\alpha_4}) = o\{T(r, g)\}$. Thus

$$(1) \quad \delta(\alpha_2, f) = \delta(\alpha_4, g) = 1,$$

$$(2) \quad \delta(\infty, f) = \delta(\infty, g) = 1.$$

These observations lead to our main result.

Theorem 1. *Let $f(z)$, $g(z)$, $a(z)$, $b(z)$, $\alpha(z)$ and $\beta(z)$ be meromorphic functions in the plane, where $a(z)$ and $\alpha(z)$ are small functions of f , $b(z)$ and $\beta(z)$ are small functions of $g(z)$, $a(z) \not\equiv \alpha(z)$, $b(z) \not\equiv \beta(z)$. Suppose that f and g share the pair (a, b) CM and*

$$(3) \quad \delta = \delta(\alpha, f) + \delta(\beta, g) + \delta(\infty, f) + \delta(\infty, g) > 3.$$

Then either

$$(4) \quad \frac{f - \alpha}{a - \alpha} = \frac{g - \beta}{b - \beta}$$

or

$$(5) \quad \frac{f - \alpha g - \beta}{a - \alpha b - \beta} = 1.$$

Remark 1. The number 3 in the inequality (3) is sharp.

For example, let P and Q be two nonzero polynomials, $f = e^{2z} - Qe^z$, $g = \frac{e^{2z}}{e^z + \frac{P}{Q}}$. Then one can check that f and g share the pair (P, Q) CM and

$$\delta(0, f) + \delta(0, g) + \delta(\infty, f) + \delta(\infty, g) = \frac{1}{2} + 1 + 1 + \frac{1}{2} = 3.$$

However, $\frac{f}{P} \neq \frac{g}{Q}$ and $\frac{f}{P} \frac{g}{Q} \neq 1$.

Remark 2. Note that Theorem A needs two shared values. However, in our theorem 1, we only need one shared pair.

Remark 3. The topic on unicity theorem concerning deficiency were studied by Ozawa [7], Ueda [8] etc. The case that f and g are entire functions and $a(z) = b(z) = 1$ was considered by Yi [10].

Remark 4. From the proof of Theorem 1 we see that the word “share” can be replaced by “almost share”.

As an application, we obtain the following

Corollary. *The answer to Yang’s problem is affirmative.*

2. Some Symbols.

For the sake of convenience, we shall use some symbols introduced by Chuang [1] and Chuang-Hua [2].

For meromorphic function $f(z)$ and a point z , according as z is a pole of f or not, we denote by $\omega(f, z)$ the multiplicity of z or 0 and by $\bar{\omega}(f, z)$ the value 1 or 0. For three meromorphic functions f , g and h , we divide the set of the poles of f and g on $\{|z| \leq r\}$ into five pairwise disjoint subsets as follows:

$$\begin{aligned} V_1 &=: \{z : f(z) \neq \infty, g(z) = \infty\}, \\ V_2 &=: \{z : f(z) = \infty, g(z) \neq \infty\}, \\ V_3 &=: \{z : f(z) = \infty, g(z) = \infty, h(z) = \infty\}, \\ V_4 &=: \{z : f(z) = \infty, g(z) = \infty, h(z) \neq 0, \infty\}, \\ V_5 &=: \{z : f(z) = \infty, g(z) = \infty, h(z) = 0\}. \end{aligned}$$

Furthermore, for each $j \in \{1, \dots, 5\}$, we denote by $n_j(f)$ and $n_j(g)$ the number of the poles of f and g in the set V_j respectively, with due count of multiplicity. The corresponding counting functions are denoted by $N_j(f)$ and $N_j(g)$ respectively. Obviously,

$$(6) \quad N(r, f) = N_2(f) + N_3(f) + N_4(f) + N_5(f),$$

$$(7) \quad N(r, g) = N_1(g) + N_3(g) + N_4(g) + N_5(g).$$

3. One Basic Lemma.

For the proof of our results, we need the following lemma which can be found in Chuang-Yang [3, p. 39] or Gross [5, pp. 70-73].

Lemma 1. *Let f_j ($j = 1, \dots, n \geq 2$) be n linearly independent meromorphic functions. If $f_1 + \dots + f_n \equiv 1$, then we have*

$$\begin{aligned} T(r, f_1) \leq & \sum_{j=1}^n N\left(r, \frac{1}{f_j}\right) + N(r, W) - N\left(r, \frac{1}{W}\right) \\ & - \sum_{j=2}^n N(r, f_j) + S(r, f_1) + \dots + S(r, f_n), \end{aligned}$$

where $W = W(z)$ is the Wronskian of f_1, \dots, f_n .

4. Proof of Theorem 1.

Let

$$\begin{aligned} F_1 & =: \{z : a(z) = \infty\} \cup \{z : b(z) = \infty\} \cup \{z : \alpha(z) = \infty\} \cup \{z : \beta(z) = \infty\}, \\ F_2 & =: \{z : a(z) = \alpha(z)\} \cup \{z : b(z) = \beta(z)\}. \end{aligned}$$

Set

$$F =: F_1 \cup F_2,$$

the corresponding counting function is denoted by $N_F(r)$. Put

$$(8) \quad h(z) =: \frac{f(z) - a(z)}{g(z) - b(z)}.$$

Since $a(z)$ and $b(z)$ are small functions of f and g respectively, we know that $h(z) \not\equiv 0, \infty$. Let z_o be a pole of h with $z_o \notin F$. Since f and g share the

pair (a, b) CM, we have $z_o \in V_2 \cup V_3$. If $z_o \in V_2$, then $\omega(h, z_o) = \omega(f, z_o)$; If $z_o \in V_3$, then $\omega(h, z_o) = \omega(f, z_o) - \omega(g, z_o)$. Thus

$$(9) \quad N_2(f) + N_3(f) - N_3(g) - N_F(r) \leq N(r, h) \leq N_2(f) + N_3(f) - N_3(g) + N_F(r).$$

Similarly we have

$$(10) \quad N\left(r, \frac{1}{h}\right) \leq N_1(g) + N_5(g) - N_5(f) + N_F(r).$$

Let

$$f_1 =: \frac{f - \alpha}{a - \alpha}, \quad f_2 =: -\frac{g - \beta}{a - \alpha}h, \quad f_3 =: \frac{b - \beta}{a - \alpha}h.$$

Then

$$(11) \quad N\left(r, \frac{1}{f_1}\right) \leq N\left(r, \frac{1}{f - \alpha}\right) + N_F(r),$$

$$(12) \quad N\left(r, \frac{1}{f_3}\right) \leq N\left(r, \frac{1}{h}\right) + N_F(r),$$

$$(13) \quad N(r, h) - N_F(r) \leq N(r, f_3) \leq N(r, h) + N_F(r).$$

From (8) it is easy to see that any zero of h which is not in the set F is not a zero of f_2 . Thus

$$(14) \quad N\left(r, \frac{1}{f_2}\right) \leq N\left(r, \frac{1}{g - \beta}\right) + N_F(r).$$

Now for any pole z_o of f_2 with $z_o \notin F$, we know that z_o is a pole of g or h . If $z_o \in V_1$, then $\omega(g, z_o) = \omega(\frac{1}{h}, z_o)$, and so, $\omega(f_2, z_o) = 0$; If $z_o \in V_2$, then $\omega(f_2, z_o) = \omega(h, z_o) = \omega(f, z_o)$; If $z_o \in V_3$, then $\omega(h, z_o) = \omega(f, z_o) - \omega(g, z_o)$, and so, $\omega(f_2, z_o) = \omega(g, z_o) + \omega(h, z_o) = \omega(f, z_o)$; If $z_o \in V_4$, then $h(z_o) \neq 0, \infty$ and $\omega(f_2, z_o) = \omega(g, z_o)$; If $z_o \in V_5$, then $\omega(\frac{1}{h}, z_o) = \omega(g, z_o) - \omega(f, z_o)$ and $\omega(f_2, z_o) = \omega(g, z_o) - \omega(\frac{1}{h}, z_o) = \omega(f, z_o)$. Combining all these facts we get

$$(15) \quad \begin{aligned} N_2(f) + N_3(f) + N_4(g) + N_5(f) - N_F(r) &\leq N(r, f_2) \\ &\leq N_2(f) + N_3(f) + N_4(g) + N_5(f) + N_F(r). \end{aligned}$$

Next we rewrite (8) in the form

$$(16) \quad f_1 + f_2 + f_3 = 1.$$

Without loss of generality, we suppose that there exists a set I with infinite measure such that

$$(17) \quad T(r, g) \leq T(r, f), \quad r \in I.$$

(Otherwise, we only need to consider $T(r, g)$ instead of $T(r, f)$ in the following discussions.) Thus $N_F(r) = S(r, f)$, $r \in I$.

In the sequel, we always let $r \in I$. Now we prove the following lemma.

Lemma 2. f_1, f_2 and f_3 are linearly dependent.

Proof. Suppose on the contrary that the f 's are linearly independent. By lemma 1,

$$\begin{aligned} T(r, f_1) &\leq \sum_{j=1}^3 N\left(r, \frac{1}{f_j}\right) + N(r, W) - N\left(r, \frac{1}{W}\right) - N(r, f_2) - N(r, f_3) \\ &\quad + S(r, f_1) + S(r, f_2) + S(r, f_3) \end{aligned}$$

where W is the Wronskian of f_1, f_2, f_3 , i.e.,

$$W = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = - \begin{vmatrix} f_1' & f_3' \\ f_1'' & f_3'' \end{vmatrix}$$

by (16). Now by (10), (11), (12) and (14),

$$\begin{aligned} \sum_{j=1}^3 N\left(r, \frac{1}{f_j}\right) &\leq N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{g - \beta}\right) \\ &\quad + N_1(g) + N_5(g) - N_5(f) + 4N_F(r). \end{aligned}$$

In addition, by the inequalities on the left hand sides of (9), (13) and (15),

$$N(r, f_2) + N(r, f_3) \geq 2N_2(f) + 2N_3(f) + N_4(f) + N_5(f) - N_3(g) - 3N_F(r).$$

Combining the three inequalities above we get

$$\begin{aligned} T(r, f_1) &\leq N(r, W) - N\left(r, \frac{1}{W}\right) + N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{g - \beta}\right) \\ &\quad + N_1(g) + N_3(g) + N_5(g) \\ &\quad - 2N_2(f) - 2N_3(f) - N_4(f) - 2N_5(f) \\ &\quad + 7N_F(r) + S(r, f_1) + S(r, f_2) + S(r, f_3) \end{aligned}$$

$$\begin{aligned}
&= N(r, W) - N\left(r, \frac{1}{W}\right) + N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{g - \beta}\right) \\
&\quad + N_1(g) + N_3(g) + N_4(g) + N_5(g) \\
&\quad - 2N_2(f) - 2N_3(f) - 2N_4(f) - 2N_5(f) \\
&\quad - N_4(g) + N_4(f) \\
&\quad + 7N_F(r) + S(r, f_1) + S(r, f_2) + S(r, f_3).
\end{aligned}$$

Substituting (6) and (7) into the above inequality and using the facts that

$$\begin{aligned}
N_4(f) &= N_4(g), \quad T(r, f) = T(r, f_1) + S(r, f), \\
T(r, a), \quad T(r, b), \quad T(r, \alpha), \quad T(r, \beta) &= S(r, f), \\
N_F(r), \quad S(r, f_j) &= S(r, f), \quad (j = 1, \dots, 3)
\end{aligned}$$

we obtain

$$\begin{aligned}
(18) \quad T(r, f) &\leq N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{g - \beta}\right) + N(r, W) - N\left(r, \frac{1}{W}\right) \\
&\quad + N(r, g) - 2N(r, f) + S(r, f).
\end{aligned}$$

Next we estimate the term $N(r, W) - N\left(r, \frac{1}{W}\right)$. Since

$$(19) \quad W = -(f_1' f_3'' - f_1'' f_3'),$$

from the expressions of f_1 and f_3 we see that the poles of W only occur at the poles of f and the points in F . Let z_0 be a pole of f with $z_0 \notin F$.

If $z_0 \in V_2$, then near $z = z_0$,

$$f_1 = \frac{1}{(z - z_0)^{\omega(f, z_0)}} \{x + \mathcal{O}(z - z_0)\}, \quad f_3 = \frac{1}{(z - z_0)^{\omega(f, z_0)}} \{y + \mathcal{O}(z - z_0)\},$$

where x and y are nonzero constants. If $\omega(f, z_0) \geq 2$, then

$$f_1' f_3'' = \frac{1}{(z - z_0)^{2\omega(f, z_0)+3}} \{-\omega(f, z_0)^2 (\omega(f, z_0) + 1) xy + \mathcal{O}(z - z_0)\},$$

$$f_1'' f_3' = \frac{1}{(z - z_0)^{2\omega(f, z_0)+3}} \{-\omega(f, z_0)^2 (\omega(f, z_0) + 1) xy + \mathcal{O}(z - z_0)\},$$

and so,

$$f_1' f_3'' - f_1'' f_3' = \mathcal{O}\left\{\frac{1}{(z - z_0)^{2\omega(f, z_0)+2}}\right\}.$$

If $\omega(f, z_0) = 1$, then

$$f_1' f_3'' = \frac{-2xy}{(z - z_0)^5} + \frac{\mathcal{O}(1)}{(z - z_0)^3} + \dots,$$

$$f_1'' f_3' = \frac{-2xy}{(z - z_0)^5} + \frac{\mathcal{O}(1)}{(z - z_0)^3} + \dots,$$

and so,

$$f_1' f_3'' - f_1'' f_3' = \mathcal{O} \left\{ \frac{1}{(z - z_0)^3} \right\}.$$

Thus

$$\begin{aligned} \omega(W, z_0) &\leq \begin{cases} 2\omega(f, z_0) + 2, & \text{if } \omega(f, z_0) \geq 2 \\ 3, & \text{if } \omega(f, z_0) = 1 \end{cases} \\ &\leq 3\omega(f, z_0). \end{aligned}$$

If $z_0 \in V_3$, then $\omega(g, z_0) \geq 1$ and $\omega(f, z_0) \geq 2$. Thus, by (19),

$$\begin{aligned} \omega(W, z_0) &\leq 2\omega(f, z_0) + 3 - \omega(g, z_0) \\ &\leq 2\omega(f, z_0) + 2 \leq 3\omega(f, z_0). \end{aligned}$$

If $z_0 \in V_4$, then $\omega(f, z_0) = \omega(g, z_0)$, and so, $\omega(f_3, z_0) = 0$. By (19), we get

$$\omega(W, z_0) \leq \omega(f, z_0) + 2 \leq 3\omega(f, z_0).$$

If $z_0 \in V_5$, and if z_0 is a pole of W , then by (19),

$$\begin{aligned} \omega(W, z_0) &\leq \omega(f, z_0) + 2 \\ &\leq 3\omega(f, z_0). \end{aligned}$$

Combining all the cases above and noting (6), we deduce that

$$N(r, W) \leq 3N(r, f) + N_F(r).$$

This and (18) give

$$(20) \quad T(r, f) \leq N \left(r, \frac{1}{f - \alpha} \right) + N \left(r, \frac{1}{g - \beta} \right) + N(r, f) + N(r, g) + S(r, f).$$

Now by the definition of deficiency, for $\epsilon = \frac{\delta - 3}{8} > 0$, where δ is the sum in (3), there exists $r_o > 0$ such that

$$N \left(r, \frac{1}{f - \alpha} \right) \leq (1 - \delta(\alpha, f) + \epsilon)T(r, f),$$

$$N\left(r, \frac{1}{g-\beta}\right) \leq (1 - \delta(\beta, g) + \epsilon)T(r, g),$$

$$N(r, f) \leq (1 - \delta(\infty, f) + \epsilon)T(r, f)$$

and

$$N(r, g) \leq (1 - \delta(\infty, g) + \epsilon)T(r, g)$$

hold for $r \in I$ and $r > r_0$. Substituting all these inequality into (20) and noting (17), we get $\delta \leq 3$, which contradicts our hypothesis. This completes the proof of the lemma.

Now by Lemma 2, there exist three constants c_1, c_2 and c_3 with

$$(21) \quad |c_1| + |c_2| + |c_3| \neq 0$$

and

$$(22) \quad c_1 f_1 + c_2 f_2 + c_3 f_3 = 0.$$

If $c_1 = 0$, then $c_2 c_3 \neq 0$ and $f_2 = -\frac{c_3}{c_2} f_3$. This leads to $g = \frac{c_3}{c_2} b(z) + \left(1 - \frac{c_3}{c_2}\right) \beta(z)$, which contradicts the assumptions that $b(z)$ and $\beta(z)$ are small functions of g . Thus, $c_1 \neq 0$. We may suppose $c_1 = -1$, and (22) reads $f_1 = c_2 f_2 + c_3 f_3$. Combining this and (16) we obtain

$$(23) \quad (1 + c_2) f_2 + (1 + c_3) f_3 = 1.$$

Next we consider two cases.

(i) $1 + c_2 = 0$. Then $1 + c_3 \neq 0$ and $(1 + c_3) f_3 = 1$. It follows from (8) and the definition of f_3 between (10) and (11) that

$$(24) \quad f - \frac{c_3 a + \alpha}{1 + c_3} = f - a + a - \frac{c_3 a + \alpha}{1 + c_3}$$

$$= \left(\frac{1}{1 + c_3}\right) \frac{a - \alpha}{b - \beta} (g - b) + a - \frac{c_3 a + \alpha}{1 + c_3}$$

$$= \left(\frac{1}{1 + c_3}\right) \frac{a - \alpha}{b - \beta} (g - \beta).$$

If $c_3 \neq 0$, then $\frac{c_3 a + \alpha}{1 + c_3} \neq \alpha$. By the Nevanlinna “three-functions theorem” we deduce that

$$T(r, f) \leq N(r, f) + N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{f - \frac{c_3 a + \alpha}{1 + c_3}}\right) + S(r, f)$$

$$= N(r, f) + N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{g - \beta}\right) + S(r, f).$$

This is impossible by the same reasoning as in the proof of Lemma 2. Therefore $c_3 = 0$ and (24) reads

$$\frac{f - \alpha}{g - \beta} = \frac{a - \alpha}{b - \beta}.$$

This is what we need.

(ii) $1 + c_2 \neq 0$. It follows from (8), (23) and the definitions of f_2 and f_3 between (10) and (11) that $-(1 + c_2)\frac{g-\beta}{a-\alpha} + (1 + c_3)\frac{b-\beta}{a-\alpha} = \frac{g-b}{f-a}$, which can be written as

$$(25) \quad f - \frac{c_2 a + \alpha}{1 + c_2} = \left(\frac{c_2 - c_3}{(1 + c_2)^2} \right) \frac{(a - \alpha)(b - \beta)}{g - \frac{1+c_3}{1+c_2}b - \frac{c_2-c_3}{1+c_2}\beta}.$$

If $c_2 \neq 0$, then $\frac{c_2 a + \alpha}{1 + c_2} \neq \alpha$. By the “three-functions theorem”, we have

$$\begin{aligned} T(r, f) &\leq N(r, f) + N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{f - \frac{c_2 a + \alpha}{1 + c_2}}\right) + S(r, f) \\ &\leq N(r, f) + N\left(r, \frac{1}{f - \alpha}\right) + N(r, g) + S(r, f). \end{aligned}$$

By the same reasoning as in the proof of Lemma 2, we can get a contradiction. Thus $c_2 = 0$, and (25) reads

$$(26) \quad f - \alpha = -c_3 \frac{(a - \alpha)(b - \beta)}{g - (1 + c_3)b + c_3\beta}.$$

If $c_3 = -1$, then

$$(f - \alpha)(g - \beta) = (a - \alpha)(b - \beta),$$

as asserted. If $c_3 \neq -1$, then $\frac{\alpha + c_3 a}{1 + c_3} \neq \alpha$ and (26) can be written as

$$f - \frac{\alpha + c_3 a}{1 + c_3} = -\left(\frac{c_3}{1 + c_3} \right) \frac{(a - \alpha)(g - \beta)}{g - (1 + c_3)b + c_3\beta}.$$

Thus, the “three-functions theorem” gives

$$\begin{aligned} T(r, f) &\leq N(r, f) + N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{f - \frac{\alpha + c_3 a}{1 + c_3}}\right) + S(r, f) \\ &\leq N(r, f) + N\left(r, \frac{1}{f - \alpha}\right) + N\left(r, \frac{1}{g - \beta}\right) + S(r, f). \end{aligned}$$

By the same reasoning as in the proof of Lemma 2 we obtain a contradiction.

This completes the proof of the theorem. \square

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