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Journal of
Mathematics*

**A UNIQUENESS THEOREM FOR THE MINIMAL SURFACE
EQUATION**

JENN-FANG HWANG

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In 1991, Collin and Krust proved that if u satisfies the minimal surface equation in a strip with linear Dirichlet data on two sides, then u must be a helicoid. In this paper, we give a simpler proof of this result and generalize it.

1. Introduction.

Let $\Omega_\alpha \subset \mathbb{R}^2$ be a sector domain with angle $0 < \alpha < \pi$. Consider the minimal surface equation

$$(1) \quad \operatorname{div} Tu = 0$$

where $Tu = \frac{\nabla u}{\sqrt{1 + |\nabla u|^2}}$ and ∇u is the gradient of u . In 1965, Nitsche [7] announced the following results:

- (1) Given a continuous function f on $\partial\Omega_\alpha$, there always exists a solution u which satisfies the minimal surface equation in Ω_α with Dirichlet data f on $\partial\Omega_\alpha$;
- (2) If u satisfies the minimal surface equation with vanishing boundary value in Ω_α , then $u \equiv 0$.

Nitsche thus raised the following question: Let $\Omega \subset \Omega_\alpha$ and let f be an arbitrary continuous function on $\partial\Omega$. If the Dirichlet problem

$$\begin{cases} \operatorname{div} Tu = 0 & \text{in } \Omega, \\ u = f & \text{on } \partial\Omega \end{cases}$$

has a solution, is it unique?

We notice that similar questions for higher dimensions are raised in [6]. Results in this direction were obtained by Miklyukov [5] and Hwang [4] independently, in which the following result was established:

Theorem 1. *Let $\Omega \subset \mathbb{R}^2$ be an unbounded domain and let $u, v \in C^2(\Omega) \cap C^0(\bar{\Omega})$. For every $R > 0$, set $B_R = \{x \in \mathbb{R}^2 \mid |x| < R\}$ and $\Gamma_R = \partial(\Omega \cap B_R) \cap$*

∂B_R . Denote $|\Gamma_R|$ as the length of Γ_R . And suppose that

$$\begin{cases} \text{(i)} & \operatorname{div} Tu = \operatorname{div} Tv & \text{in } \Omega, \\ \text{(ii)} & u = v & \text{on } \partial\Omega, \\ \text{(iii)} & \max_{\Omega \cap B_R} |u - v| = O\left(\sqrt{\int_{R_0}^R \frac{1}{|\Gamma_r|} dr}\right) & \text{as } R \rightarrow \infty, \text{ for some} \\ & & \text{positive constant } R_0. \end{cases}$$

Then $u \equiv v$ in Ω .

A stronger version of Theorem 1 was discovered by Collin and Krust [2] independently, which is the following:

Theorem 1*. Let $\Omega, u, v, B_R, \Gamma_r$ and $|\Gamma_r|$ as in Theorem 1. And suppose that

$$\begin{cases} \text{(i)} & \operatorname{div} Tu = \operatorname{div} Tv & \text{in } \Omega, \\ \text{(ii)} & u = v & \text{on } \partial\Omega, \\ \text{(iii)} & \max_{\Omega \cap B_R} |u - v| = o\left(\int_{R_0}^R \frac{1}{|\Gamma_r|} dr\right) & \text{as } R \rightarrow \infty, \text{ for some} \\ & & \text{positive constant } R_0. \end{cases}$$

Then $u \equiv v$ in Ω .

In fact, for any unbounded domain Ω , we have $|\Gamma_R| = O(R)$, and condition (iii) in Theorem 1* becomes

$$\max_{\Omega \cap B_R} |u - v| = o(\log R) \quad \text{as } R \rightarrow \infty.$$

In the special case when Ω is a strip, then $|\Gamma_R| \leq \text{constant}$, and condition (iii) becomes $\max_{\Omega \cap B_R} |u - v| = o(R)$.

On the other hand, in a strip domain Ω , Collin [1] showed that there exist two different solutions for the minimal surface equation such that $u = v$ on $\partial\Omega$ and $\max_{\Omega \cap B_R} |u - v| = O(R)$ as $R \rightarrow \infty$. So condition (iii) is necessary.

This counterexample also answers Nitsche’s question in the negative.

In contrast, the following result is also given in [2].

Theorem 2. Let $\Omega = (0, 1) \times \mathbb{R}$ be a strip. Suppose that

$$\begin{cases} \operatorname{div} Tu = 0 & \text{in } \Omega, \\ u(0, y) = ay + b, \\ u(1, y) = cy + d \end{cases}$$

where a, b, c, d are constant. Then u must be a helicoid.

The following inequality was discovered by Miklyukov [5, p. 265], Hwang [4, p. 342] and Collin and Krust [2, p. 452]:

$$(Tu - Tv) \cdot (\nabla u - \nabla v) \geq \frac{\sqrt{1 + |\nabla u|^2} + \sqrt{1 + |\nabla v|^2}}{2} |Tu - Tv|^2$$

$$(2) \qquad \qquad \qquad \geq |Tu - Tv|^2.$$

Using this inequality, Miklyukov [5] and Hwang [4] proved Theorem 1 independently, and Collin and Krust [2] proved Theorem 1* also based on this inequality.

It seems that the method of proof of Theorem 1* can not be used to prove Theorem 2, and so Collin and Krust [2] resorted to the theory of Gauss maps instead.

In this paper, we will point out that the method of proof of Theorem 1 and Theorem 1* could be use to give a simpler proof of Theorem 2. Moreover, we shall generalize Theorem 1* and Theorem 2 to get the more general results as stated in Theorem 3 and Theorem 4. And we will make a remark after Theorem 3 to point out why Collin and Krust [2] could get a better result then Miklyukov [5] and Hwang [4].

2. A new proof for Theorem 2 and its generalization.

Without loss of generality, we may rephrase Theorem 2 in the following form:

Theorem 2*. *Let $\Omega = (b, a) \times \mathbb{R}$ be a strip domain in \mathbb{R}^2 where a, b are two constants with $-\frac{\pi}{2} < b < a < \frac{\pi}{2}$, and let $u \in C^0(\bar{\Omega}) \cap C^2(\Omega)$. Suppose that*

$$\begin{cases} \operatorname{div} Tu = 0 & \text{in } \Omega, \\ u = y \tan x & \text{on } \partial\Omega. \end{cases}$$

Then $u \equiv y \tan x$ in Ω ; in other words, u must be a helicoid.

Proof. For any $y > 0$, let

$$\begin{aligned} \Omega_y &= (b, a) \times (-y, y), \\ \Gamma_y &= \{(b, a) \times \{y\}\} \cup \{(b, a) \times \{-y\}\} \end{aligned}$$

and, set

$$\begin{aligned} g(y) &= \int_{\Gamma} (u - v)(Tu - Tv) \cdot \nu \, ds \\ &= \oint_{\partial\Omega_y} (u - v)(Tu - Tv) \cdot \nu \, ds \\ &= \int \int_{\Omega_y} (\nabla u - \nabla v) \cdot (Tu - Tv) \end{aligned}$$

where $v \equiv y \tan x$ and ν is the unit outward normal of Γ_y and $\partial\Omega_y$. Since $(\nabla u - \nabla v) \cdot (Tu - Tv) \geq 0$, Fubini's Theorem yields that the derivative $g'(y)$

exists for almost all $y > 0$ and

$$g'(y) = \int_{\Gamma_y} (\nabla u - \nabla v) \cdot (Tu - Tv)$$

whenever $g'(y)$ exists. Thus, in view of (2), for these y ,

$$\begin{aligned} g'(y) &\geq \int_{\Gamma_y} \frac{\sqrt{1 + |\nabla u|^2} + \sqrt{1 + |\nabla v|^2}}{2} |Tu - Tv|^2 \\ &\geq \left(\min_{\Gamma_y} \frac{\sqrt{1 + |\nabla v|^2}}{2} \right) \int_{\Gamma_y} |Tu - Tv|^2, \end{aligned}$$

in which, as $v_x = y \sec^2 x$, we have

$$\frac{\sqrt{1 + |\nabla v|^2}}{2} \geq \frac{y \sec^2 x}{2} \geq \frac{y}{2}.$$

Furthermore, by means of Schwarz's inequality,

$$|\Gamma_y| \int_{\Gamma_y} |Tu - Tv|^2 \geq \left(\int_{\Gamma_y} |Tu - Tv| \right)^2,$$

and $|\Gamma_y| = 2(a - b)$ (in virtue of the special geometry of Ω), thus

$$\int_{\Gamma_y} |Tu - Tv|^2 \geq \frac{1}{2(a - b)} \left(\int_{\Gamma_y} |Tu - Tv| \right)^2.$$

Hence, for any y where $g'(y)$ exists,

$$\begin{aligned} (3) \quad g'(y) &\geq \frac{y}{4(a - b)} \left(\int_{\Gamma_y} |Tu - Tv| \right)^2 \\ &\geq \frac{y}{4(a - b)} \left(\frac{1}{\pi} \int_{\Gamma_y} \tan^{-1}(u - v)(Tu - Tv) \cdot \nu \right)^2. \end{aligned}$$

Now, for all $y > 0$, set

$$\begin{aligned} h(y) &= \int_{\Gamma_y} \tan^{-1}(u - v)(Tu - Tv) \cdot \nu \\ &= \int \int_{\Omega_y} \frac{(\nabla u - \nabla v) \cdot (Tu - Tv)}{1 + (u - v)^2}. \end{aligned}$$

We note that $h \geq 0$ and $h(y)$ increases as y increases. Thus, if $h \equiv 0$, it is easy to see that Theorem 2* holds. Hence we may assume that $h \not\equiv 0$ and

that there exist two positive constants y_1 and c_1 such that $h(y) \geq c_1$ for all $y \geq y_1$.

Substituting this into (3), we obtain $g'(y) \geq \frac{c_1^2}{4(a-b)\pi^2}y$ for almost all $y \geq y_1$, which yields $g(y) - g(y_1) \geq \frac{c_1^2}{4(a-b)\pi^2}(y - y_1)^2$. Since $|u| = O(|y|)$ on $\partial\Omega$ as $|y| \rightarrow \infty$, by [7, p. 256], we have $|u| = O(|y|)$ in Ω as $|y| \rightarrow \infty$. Since for all $y > 0$, $g(y) = \int_{\Gamma_y} (u - v)(Tu - Tv) \cdot \nu$ and $|Tu - Tv| \leq 2$, we have $g(y) = O(y)$ as $y \rightarrow \infty$, which gives a contradiction and completes our proof. \square

By modifying the proof of Theorem 2*, we can derive the following

Theorem 3. *Let $\Omega \subseteq \mathbb{R}^2$ be an unbounded domain and let $u, v \in C^2(\Omega) \cap C^0(\bar{\Omega})$. Let B_R, Γ_R and $|\Gamma_R|$ be as in Theorem 1. Suppose that*

$$\begin{cases} \text{(i)} & \operatorname{div} Tu = \operatorname{div} Tv & \text{in } \Omega, \\ \text{(ii)} & u = v & \text{on } \partial\Omega, \\ \text{(iii)} & \max_{\Omega \cap B_R} |u - v| = o\left(\int_{R_0}^R \frac{1}{|\Gamma_R|} \min_{\Gamma_R} \sqrt{1 + |\nabla v|^2} dR\right) & \text{as } R \rightarrow \infty, \end{cases}$$

where R_0 is a positive constant. Then we have $u \equiv v$ in Ω .

Remark.

- (a) Notice that condition (iii) depends on $|\nabla v|$ only, without assuming any condition on $|\nabla u|$.
- (b) In Theorem 2*, since $\operatorname{div} Tu = 0$ in Ω and $u = y \tan x$ on $\partial\Omega$, by [7, p. 256], we have $u = O(|y|)$ in Ω as $|y| \rightarrow \infty$. And so, condition (iii) of Theorem 3 holds.

Proof of Theorem 3. The proof is similar to that of Theorem 2*. For every $R > 0$, let

$$\begin{aligned} M(R) &= \max_{\Omega \cap B_R} |u - v| = \max_{\Gamma_R} |u - v|, \\ Q(R) &= \min_{\Gamma_R} \frac{\sqrt{1 + |\nabla v|^2}}{2}, \\ g(R) &= \int_{\Gamma_R} (u - v)(Tu - Tv) \cdot \nu = \int \int_{\Omega_R} (\nabla u - \nabla v) \cdot (Tu - Tv) \end{aligned}$$

and

$$h(R) = \int_{\Gamma_R} \tan^{-1}(u - v)(Tu - Tv) \cdot \nu.$$

As in the proof of Theorem 2*, we may assume that $h \not\equiv 0$ and that there exist two positive constants R_1 and C_1 such that $R_1 > R_0$ and

$$(4) \quad h(R) \geq C_1 \quad \text{for all } R \geq R_1.$$

For almost all $R > 0$, we have

$$\begin{aligned}
 (5) \quad g'(R) &= \int_{\Gamma_R} (\nabla u - \nabla v) \cdot (Tu - Tv) \\
 &\geq \int_{\Gamma_R} Q(R) |Tu - Tv|^2 \\
 &\geq Q(R) |\Gamma_R|^{-1} \left(\int_{\Gamma_R} |Tu - Tv| \right)^2.
 \end{aligned}$$

Thus $g'(R) \geq (\frac{\pi}{2})^2 C_1^2 |\Gamma_R|^{-1} Q(R)$, for almost all $R > R_1$. Hence, for every R and R_2 such that $R > R_2 \geq R_1$, we have

$$(6) \quad g(R) - g(R_2) \geq \left(\frac{2C_1}{\pi} \right)^2 \int_{R_2}^R \frac{Q(r)}{|\Gamma_r|} dr.$$

By (4), we have $M(R) > 0$ for all $R \geq R_1$, hence (5) yields, for almost all $R \geq R_1$,

$$\begin{aligned}
 g'(R) &\geq Q(R) |\Gamma_R|^{-1} \int |Tu - Tv|^2 \\
 &\geq \frac{g^2(R) Q(R)}{M^2(R) |\Gamma_R|},
 \end{aligned}$$

and so, for every R and R_2 such that $R > R_2 \geq R_1$,

$$-\frac{1}{g} \Big|_{R_2}^R \geq \int_{R_2}^R \frac{g'}{g^2} \geq \int_{R_2}^R \frac{Q(r)}{M^2(r) |\Gamma_r|} dr \geq \frac{1}{M^2(R)} \int_{R_2}^R \frac{Q(r)}{|\Gamma_r|} dr,$$

and then

$$(7) \quad \frac{1}{g(R_2)} \geq \frac{1}{M^2(R)} \int_{R_2}^R \frac{Q(r)}{|\Gamma_r|} dr.$$

Now, since $M(R) > 0$ for all $R \geq R_1$, $M(R)$ is an increasing function of R and, in view of condition (iii),

$$(M(R))^{-1} \int_{R_1}^R \frac{Q(r)}{|\Gamma_r|} dr \rightarrow \infty \quad \text{as } R \rightarrow \infty,$$

and also

$$\int_{R_1}^R \frac{Q(r)}{|\Gamma_r|} dr \rightarrow \infty \quad \text{as } R \rightarrow \infty;$$

hence we can choose a constant $R_3 > R_1$ such that

$$(M(R))^{-1} \int_{R_1}^R \frac{Q(r)}{|\Gamma_r|} \geq \sqrt{2\pi} C_1^{-1}, \quad \text{for every } R \geq R_3,$$

and a constant $R_4, R_4 > R_3$, which depends on R_3 , such that

$$\int_{R_1}^{R_4} \frac{Q(r)}{|\Gamma_r|} dr = 2 \int_{R_1}^{R_3} \frac{Q(r)}{|\Gamma_r|} dr.$$

With this choice of R_3 and R_4 , we have

$$\begin{aligned} 1 &\geq \frac{g(R_3) - g(R_1)}{g(R_3)} \\ &\geq \left[\left(\frac{2C_1}{\pi} \right)^2 \int_{R_1}^{R_3} \frac{Q(r)}{|\Gamma_r|} \right] \left[(M^2(R_4))^{-1} \int_{R_3}^{R_4} \frac{Q(r)}{|\Gamma_r|} \right] \quad (\text{by (6), (7)}) \\ &= \left[\left(\frac{2C_1}{\pi} \right)^2 (M^2(R_4))^{-1} \right] \frac{1}{4} \left(\int_{R_1}^{R_4} \frac{Q(r)}{|\Gamma_r|} \right)^2 \quad (\text{by the choice of } R_3, R_4) \\ &\geq \frac{C_1^2}{\pi^2} (2\pi^2) C_1^{-2} \quad (\text{again by the choice of } R_3 \text{ and } R_4) \\ &\geq 2, \end{aligned}$$

which is desired contradiction. \square

Remark. The above proof is to show (6), which is the lower bound of $g(R)$, and (7), which is the upper bound of $g(R)$. And from (6) and (7), we get contradiction and so prove the theorem. Miklyukov [5] and Hwang [4] only observed the upper bound of $g(R)$, and so could not derive the better result as in Collin and Krust [2].

Let Ω be a domain in \mathbb{R}^2 . Consider the following equation in divergence form

$$\operatorname{div} A(x, u, \nabla u) = f(x, u, \nabla u),$$

where

$$\begin{aligned} A &= (A_1, A_2), \quad A_i : \Omega \times \mathbb{R} \times \mathbb{R}^2 \rightarrow \mathbb{R}, \quad i = 1, 2, \\ f &: \Omega \times \mathbb{R} \times \mathbb{R}^2 \rightarrow \mathbb{R}, \end{aligned}$$

and

$$A_i \in C^0(\bar{\Omega} \times \mathbb{R} \times \mathbb{R}^2) \cap C^1(\Omega \times \mathbb{R} \times \mathbb{R}^2), \quad i = 1, 2, \quad f \in C^0(\Omega \times \mathbb{R} \times \mathbb{R}^2).$$

We rewrite $A(x, u, \nabla u)$ briefly as Au .

Suppose that Au satisfies the following structural condition:

$$(8) \left\{ \begin{array}{l} (Au - Av) \cdot (\nabla u - \nabla v) \geq |Au - Av|^2 Q(R), \\ \quad \text{where } R = \sqrt{x^2 + y^2} \text{ and } Q(R) \text{ is a positive function,} \\ (\nabla u - \nabla v) \cdot (Au - Av) = 0, \quad \text{iff } \nabla u = \nabla v. \end{array} \right.$$

Now we have the following result:

Theorem 4. *Let $\partial\Omega = \Sigma^\alpha + \Sigma^\beta$ be a decomposition of $\partial\Omega$ such that $\Sigma^\beta \in C^1$. Let $u, v \in C^2(\Omega) \cap C^1(\Omega \cup \Sigma^\beta) \cap C^0(\bar{\Omega})$ and let $M(R) = \max_{\Omega \cap B_R}(u - v, 0)$. Suppose that*

$$\left\{ \begin{array}{ll} \text{(i)} & A \text{ satisfies the structural condition (8)} \\ \text{(ii)} & \operatorname{div} Au \geq \operatorname{div} Av \quad \text{in } \Omega \\ \text{(iii)} & u \leq v \quad \text{on } \Sigma^\alpha \\ \text{(iv)} & Au \cdot \nu \leq Av \cdot \nu \quad \text{on } \Sigma^\beta \\ \text{(v)} & M(R) = o\left(\int_{R_0}^R \frac{|Q_r|}{|\Gamma_r|} dr\right) \text{ as } R \rightarrow \infty, \text{ where } R_0 \text{ is} \\ & \text{a positive constant.} \end{array} \right.$$

Then, if $\partial\Omega = \Sigma^\beta$, we have either $u(x) \equiv v(x) + a$ positive constant or else $u(x) \leq v(x)$. Otherwise, $u(x) \equiv v(x)$.

The proof of Theorem 4 is exactly the same as that of Theorem 3. The interested readers may consult [4].

Acknowledgements. The author would like to thank the referee for many helpful comments and suggestions.

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Received November 4, 1994 and revised April 20, 1995. This author was partially supported by Grant NSC83-0208-M-001-072.

PACIFIC JOURNAL OF MATHEMATICS

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The *Pacific Journal of Mathematics* (ISSN 0030-8730) is published monthly except for July and August. Regular subscription rate: \$245.00 a year (10 issues). Special rate: \$123.00 a year to individual members of supporting institutions.

Subscriptions, back issues published within the last three years and changes of subscribers address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. Prior back issues are obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

The Pacific Journal of Mathematics at the University of California, c/o Department of Mathematics, 981 Evans Hall, Berkeley, CA 94720 (ISSN 0030-8730) is published monthly except for July and August. Second-class postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 6143, Berkeley, CA 94704-0163.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS at University of California,
Berkeley, CA 94720, A NON-PROFIT CORPORATION

This publication was typeset using AMS-LATEX,
the American Mathematical Society's TEX macro system.
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Volume 176 No. 2 December 1996

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