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QUANTUM STIEFEL MANIFOLD AND DOUBLE COSETS OF QUANTUM UNITARY GROUP

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Dedicated to the memory of Professor Yuri L. Daletskii

We study the homogeneous space of the quantum group $U_q(n)$ related to the subgroup $U_q(n-m)$ ($m < n$), classify its irreducible representations and get a formula for its invariant integral. We also study the double cosets $U_q(n-m)\backslash U_q(n)/U_q(n-m)$ and the hypergroup structure associated with them.

1. Introduction.

If a group G acts on a set S transitively on the right, then one can view S as G/K, where K is a subgroup of G. Thus a function on S can be considered as a function on G, invariant with respect to the right shifts by elements of K. It is especially interesting to consider bi-invariant functions on G since they can be identified with the functions on the set of G-orbits in S. If G is a locally compact group and K is its compact subgroup with Haar measures μ_G and μ_K respectively, then the set $B \subset L_1(G, \mu_G)$ of all bi-invariant functions is an algebra with respect to the convolution and has a natural hypergroup structure related to generalized translation operators (see the survey [23] and the references given there):

$$R^h f(g) = \int_K f(gkh)d\mu_K(k) = \Delta(f)(g,h), \quad g,h \in G. \tag{1}$$

If the subalgebra B is commutative (or, equivalently, the coproduct Δ is cocommutative) then (G, K) forms a Gel'fand pair [6]. In many cases the characters of B are well known special functions of mathematical physics. This explains the importance of the notion of a Gel'fand pair.

The case of a noncommutative subalgebra B has been studied in a number of papers. In particular, a pair $(SO(n), SO(m))$ was considered in [8], [22]. It is known that the homogeneous space $SO(n)/SO(m)$ can be regarded as a Stiefel manifold $S^{n,n-m}$. The infinitesimal object for the corresponding hypergroup structure was investigated in [15], [16].

A similar situation arises while considering functions on compact quantum groups [29] (see also [20]). Here, for a pair of compact quantum groups

$(\mathcal{H}_1, \mathcal{H}_2)$ and a surjection $\pi : \mathcal{H}_1 \to \mathcal{H}_2$, we also consider an algebra H of bi-invariant elements and define a coalgebra structure on it [4], [24], [25], [26]. In this situation we say that H is endowed with a hypergroup structure. If the coalgebra is cocommutative, we call $(\mathcal{H}_1, \mathcal{H}_2)$ a Gel'fand pair [12], [24]. If, in addition to, the algebra is commutative, we call such a Gel'fand pair strict [24], [25].

When algebra and coalgebra of bi-invariant elements are noncommutative, a corresponding hypergroup structure is more complicated. One of the simplest examples of such a situation is given by the pair $\mathcal{H}_1 = U_q(n)$, $\mathcal{H}_2 = U_q(n-m)$ (quantum unitary groups, $m < n$). For this pair we study a structure of a homogeneous space $S_q^{n,m} = U_q(n)/U_q(n-m)$ which is a quantum analogue of a Stiefel manifold. We give its description in terms of generators and relations of commutation, obtain a classification of its irreducible representations and a formula for an invariant integral on it. These results generalize the results obtained in [14], [17], [21], [28] for $m = 1$. In this case $S_q^{n,1}$ are quantum spheres. We also investigate the double cosets $U_q(n-m)\backslash U_q(n)/U_q(n-m)$ and the corresponding hypergroup structure. In a special case $m = 1$ these and related questions were studied in [4], [7], [11], [12], [24], [25], [26].

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2. Preliminaries.

All modules, comodules, algebras, coalgebras, Hopf algebras [1], linear maps, homomophisms, tensor products are considered over the field C of complex numbers. Algebras (coalgebras) are associative (coassociative) unital (counital).

2.1. Quantum group $GL_q(n, C)$ [5], [14], [18], [19].

Let a Hopf algebra $\mathcal{H} := A(GL_q(n, C))$ be generated by letters t_{ij} $(i, j \in \{1, \ldots, n\})$, ${\det{}^{-1}}_q$ satisfying the following relations of commutation ($q \in C$):

$$t_{ik}t_{jk} = qt_{jk}t_{ik}, \quad t_{ki}t_{kj} = qt_{kj}t_{ki} \ (i < j), \tag{2}$$

$$t_{il}t_{jk} = t_{jk}t_{il} \ (i < j, k < l), \tag{3}$$

$$t_{ik}t_{jl} - qt_{il}t_{jk} = t_{jl}t_{ik} - q^{-1}t_{jk}t_{il} \ (i < j, k < l). \tag{4}$$

$$\det{}_q^{-1} t_{ij} = t_{ij} \det{}_q^{-1}. \tag{5}$$

The coproduct $\Delta : H \to H \otimes H$ and counit $\varepsilon : H \to C$ act in the following way:

$$\Delta(t_{ij}) = \sum_k t_{ik} \otimes t_{kj},\ \Delta(\det{}^{-1}{}_q) = \det{}_q^{-1} \otimes \det{}_q^{-1},$$

$$\varepsilon(t_{ij}) = \delta_{ij},\ \varepsilon(\det{}_q^{-1}) = 1.$$

The antipode S is a homomorphism $S : H \to H$ such that:

$$S(t_{ij}) = (-q)^{i-j} \xi_{\hat{i}}^{\hat{j}} \det{}_q^{-1},$$

$$S(\det{}_q^{-1}) = \det{}_q,$$

where

$$\xi_J^I = \sum_{\tau \in P_r} (-q)^{l(\tau)} t_{i_{\tau(1)} j_1} \cdots t_{i_{\tau(r)} j_r}$$

is the quantum minor determinant; $\hat{k} = (1, \cdots, k-1, k+1, \cdots, n)$, $\det_q = \xi_{1,\cdots,n}^{1,\cdots,n}$ $I = (i_1, \cdots, i_r)$; $J = (j_1, \cdots, j_r)$, P_r is the permutation group of the set $(1, \cdots, r)$; $l(\tau) = (\tau(1), \cdots, \tau(r))$ is the number of inversions in τ.

The commutation relations (2), (3), (4) can be rewritten as

$$\mathbf{R}(\mathbf{T} \otimes \mathbf{T}) = (\mathbf{T} \otimes \mathbf{T})\mathbf{R}, \tag{6}$$

where $\mathbf{T} := (t_{ij})_{i,j=1,\ldots,n} = \sum_{i,j} t_{ij} E_{ij} \in \mathrm{Mat}(n, A(GL_q(n, C)))$, $\mathbf{T} \otimes \mathbf{T} = \sum_{i,j,k,l} t_{ij} t_{kl} E_{ij} \otimes E_{kl}, \mathbf{R} \in \mathrm{Mat}(n, C) \otimes \mathrm{Mat}(n, C)$ is a so-called constant R-matrix of type A_{n-1} [**9**], [**18**]:

$$\mathbf{R} = q \sum_i E_{ii} \otimes E_{ii} + \sum_{i \neq j} E_{ji} \otimes E_{ij} + (q - q^{-1}) \sum_{i<j} E_{ii} \otimes E_{jj},$$

E_{ij} are matrix units.

Remark 1. The quantum minor determinants satisfy the following relations:

$$\Delta(\xi_J^I) = \sum_{\#K=r} \xi_K^I \otimes \xi_J^K, \quad \varepsilon(\xi_J^I) = \delta_{IJ}.$$

2.2. Quantum universal enveloping algebra $U_q(gl(n, C))$ [5], [14], [18].

Let L_n be a free Z-module of rank n with a canonical basis $(\varepsilon_1, \cdots, \varepsilon_n)$: $L_n = \oplus_{k=1}^n Z\varepsilon_k$. We fix a symmetric bilinear form $L_n \times L_n \to Z$ defined by $\langle \varepsilon_i, \varepsilon_j \rangle = \delta_{ij}$. An element of L_n will be called *an integral weight.*

The quantum universal enveloping algebra $U_q(gl(n, C))$ is a C-algebra defined by generators $e_k, f_k \quad (k \in \{1, \ldots, n\})$ and $q^\lambda \quad (\lambda \in L_n)$ and the following relations of commutation:

$$q^\lambda e_k q^{-\lambda} = q^{\langle \lambda, \varepsilon_k - \varepsilon_{k+1} \rangle} e_k,\ q^\lambda f_k q^{-\lambda} = q^{-\langle \lambda, \varepsilon_k - \varepsilon_{k+1} \rangle} f_k,\ \lambda \in L_n,\ k \in \{1, ..., n\}$$

$$e_i f_j - f_j e_i = \frac{\delta_{ij}(q^{\varepsilon_i - \varepsilon_{i+1}} - q^{-\varepsilon_i + \varepsilon_{i+1}})}{q - q^{-1}}, \quad i, j \in \{1, \ldots, n\}$$

$$e_i^2e_j-(q+q^{-1})e_ie_je_i+e_je_i^2=0,\ f_i^2f_j-(q+q^{-1})f_if_jf_i+f_jf_i^2=0,\ |i-j|=1$$
$$e_ie_j=e_je_i;\ f_if_j=f_jf_i\quad |i-j|>1$$
$$q^0=1,\ q^\lambda q^\mu=q^{\lambda+\mu}\quad \lambda,\mu\in L_n.$$

This algebra has also a structure of a Hopf algebra with the following coproduct Δ, counit ε, and antipode S:

$$\Delta(q^\lambda)=q^\lambda\otimes q^\lambda,\quad \varepsilon(q^\lambda)=1,\quad S(q^\lambda)=q^\lambda$$
$$\Delta(e_k)=e_k\otimes q^{-(\varepsilon_k-\varepsilon_{k+1})/2}+q^{(\varepsilon_k-\varepsilon_{k+1})/2}\otimes e_k,\quad \varepsilon(e_k)=0,S(e_k)=-q^{-1}e_k$$
$$\Delta(f_k)=f_k\otimes q^{-(\varepsilon_k-\varepsilon_{k+1})/2}+q^{(\varepsilon_k-\varepsilon_{k+1})/2}\otimes f_k,\quad \varepsilon(f_k)=0,S(f_k)=-qf_k.$$

Given two Hopf algebras A and H over C, we say that a C-bilinear form $\langle a,\phi\rangle : H\times A\to C$ is *a pairing* of Hopf algebras if it satisfies the following conditions:

$$\langle a,\phi\psi\rangle=\langle\Delta_H(a),\phi\otimes\psi\rangle,\quad \langle a,1\rangle=\varepsilon_H(a)$$
$$\langle ab,\phi\rangle=\langle a\otimes b,\Delta_A(\phi)\rangle,\quad \langle 1,\phi\rangle=\varepsilon_A(\phi),$$
$$\langle S_H(a),\phi\rangle=\langle a,S_A(\phi)\rangle,$$

for any $a,b\in H$ and $\phi,\psi\in A$.

The following proposition can be found in [**5**], [**19**]:

Proposition 1. *There exists a unique pairing of Hopf algebras $U_q(gl(n,C))$ and $A(GL_q(n,C))$ (briefly $GL_q(n,C)$), such that:*

$$\langle q^\lambda,t_{ij}\rangle=\delta_{ij}q^{\langle\lambda,\varepsilon_i\rangle}\quad \langle q^\lambda,\det\nolimits_q^m\rangle=q^{m\langle\lambda,\varepsilon_1+\cdots+\varepsilon_n\rangle}\quad (m\in Z)$$
$$\langle e_k,t_{ij}\rangle=\delta_{ik}\delta_{j,k+1};\quad \langle f_k,t_{ij}\rangle=\delta_{i,k+1}\delta_{jk}$$
$$\langle e_k,\det\nolimits_q^m\rangle=\langle f_k,\det\nolimits_q^m\rangle=0\ (m\in Z).$$

Let V be a right $GL_q(n,C)$-comodule (resp. left $GL_q(n,C)$-comodule) with a structure mapping $R_G:V\to V\otimes GL_q(n,C)$ (resp. $L_G:V\to GL_q(n,C)\otimes V$), then V has a left (resp. right) module structure over $U_q(gl(n,C))$ defined by

$$a\cdot v=(\mathrm{id}\otimes a)\circ R_G(v)\quad (\text{resp. } v\cdot a=(a\otimes\mathrm{id})\circ L_G(v))$$

$\forall a\in U_q(gl(n,C))$ and $v\in V$.

In particular, $GL_q(n,c)$ is a bimodule over $U_q(gl(n,C))$. The actions of the generators q^λ,e_k,f_k are given by

$$\begin{aligned} q^\lambda t_{ij}&=t_{ij}q^{\langle\lambda,\varepsilon_j\rangle}; & t_{ij}q^\lambda&=t_{ij}q^{\langle\lambda,\varepsilon_i\rangle};\\ e_kt_{ij}&=t_{i,j-1}\delta_{j,k+1}; & t_{ij}e_k&=\delta_{ik}t_{i+1,j};\\ f_kt_{ij}&=t_{i,j+1}\delta_{jk}; & t_{ij}f_k&=\delta_{i,k+1}t_{i-1j}.\end{aligned}$$

2.3. Quantum G-spaces and relative invariants [14].

Let G be a quantum group with a coordinate ring $A(G)$. Then a quantum space X is called a quantum left $A(G)$-space if the coordinate ring $A(X)$ of X has a structure of a left $A(G)$-comodule $L_G : A(X) \to A(G) \otimes A(X)$ such that L_G is a C-algebra homomorphism. An element χ of $A(G)$ is called *a linear character* of G if

$$\Delta(\chi) = \chi \otimes \chi, \quad \varepsilon(\chi) = 1.$$

For a given linear character χ of G, an element ϕ of $A(X)$ is called *a left relative invariant with character* χ if $L_G(\phi) = \chi \otimes \phi$. The subspace of all left relative G-invariants in $A(X)$ with a character χ is denoted by

$$(G\backslash X; \chi) = (\phi \in A(X) : L_G(\phi) = \chi \otimes \phi).$$

The notions of right G-space and right relative G-invariants are defined similarly. If $\chi = 1$, the subalgebras of all right- and left- invariants are called quantum homogeneous spaces and denoted by $X\backslash G$ and G/X respectively. In the similar way one can define a subalgebra of bi-invariants:

$$G\backslash X/G = \{\phi \in A(X) : R_G(\phi) = \phi \otimes 1, L_G(\phi) = 1 \otimes \phi\}.$$

2.4. Corepresentations of $GL_q(n, C)$ [14].

In what follows, we use an abbrevation $\xi_J = \xi_{j_1 \dots j_r}$ (resp. $\xi^J = \xi^{j_1 \dots j_r}$) to refer to a quantum r-minor determinant $\xi^{1 \dots r}_{j_1 \dots j_r}$ (resp. $\xi^{j_1 \dots j_r}_{1 \dots r}$) where $J = (j_1 < \cdots < j_r)$.

For each positive integer r with $r \in \{1, \cdots n\}$, we define the fundamental weight Λ_r by $\Lambda_r = \varepsilon_1 + \cdots + \varepsilon_r$. Let λ be an integral weight in L_n in the form $\lambda = \Lambda_{m_1} + \cdots + \Lambda_{m_l}, \;\; 0 \le m_l \le \cdots \le m_1 \le n$. This condition is equivalent to stating that λ is written as $\lambda = \lambda_1 \varepsilon_1 + \cdots + \lambda_n \varepsilon_n$ with $0 \le \lambda_n \le \cdots \le \lambda_1$.

For such λ, let $T = (T_{rs}; 1 \le r \le n, \; 1 \le s \le \lambda_r)$ be a family of elements in $(1, \dots, n)$. If T satisfies the conditions

$$T_{r-1,s} < T_{r,s} \;\text{ for }\; 1 \le s \le l, \;\; 2 \le r \le m_s, \tag{7}$$

$$T_{r,s-1} \le T_{r,s} \;\text{ for }\; 1 \le r \le n, \;\; 2 \le s \le \lambda_r, \tag{8}$$

then T is called *a semi-standard tableau of shape* λ *with labels in* $(1, \dots, n)$. We denote the totality of all semi-standard tableaus $T = (T_{r,s})$ by $SSTab_n(\lambda)$ and define the standard monomial ξ_T indexed by T as the product of quantum minor determinants

$$\xi_T = \xi_{J_1} \dots \xi_{J_l} \in GL_q(n, C),$$

where $J_s = (T_{1,s}, \dots, T_{m_s,s})$ for $s \in \{1, \dots l\}$.

Suppose now that q is not a root of unity.

Proposition 2. *If $0 \le \lambda_n \le \cdots \le \lambda_1$, then the standard monomials $\xi_T = \xi_{J_1} \ldots \xi_{J_l}$ indexed by the semi-standard tableaus T in $SSTab_n(\lambda)$ form a C-basis for a right $GL_q(n, C)$-comodule denoted by $V^R(\lambda)$.*

In a similar way one can construct a left irreducible $GL_q(n, C)$-comodule $V^L(\lambda)$. We say that an integral weight λ in L_n is *dominant* if $\lambda_n \le \cdots \le \lambda_1$.

Proposition 3. (1) *If λ is dominant, then the monomials $(\det_q)^{-m}\xi_T$ indexed by semi-standard tableaus T in $SSTab_n(\lambda + m(\varepsilon_1 + \cdots + \varepsilon_n))$ form a basis for the right $GL_q(n, C)$-comodule $V^R(\lambda)$ for any $m \in Z$ with $\lambda_n \ge -m$.*

(2) *Any finite-dimensional irreducible right (resp. left) $GL_q(n, C)$- comodule is isomorphic to $V^R(\lambda)$ for some dominant integral weight λ in L_n.*

(3) *Any finite-dimensional right and left $GL_q(n, C)$-comodule is completly reducible.*

Proposition 4. *The coordinate ring $A(CL_q(n, C))$ is decomposed into the direct sum of irreducible two-sided $GL_q(n, C)$-comodules:*

$$A(GL_q(n, C)) = \oplus_\lambda W(\lambda) \tag{9}$$

where the two-sided $GL_q(n, C)$-comodule $W(\lambda)$is isomorphic to the tensor product of the left and right irreducible $GL_q(n, C)$-comodules $V^L(\lambda)$ and $V^R(\lambda)$:

$$W(\lambda) \sim V^L(\lambda) \otimes V^R(\lambda),$$

and the summation runs over all dominant integral weights λ in L_n.

2.5. Invariant integral [14], [29], [21], [27].

Definition 1. A linear functional $\nu : GL_q(n, C) \to C$ is called right-invariant (resp. left-invariant) integral if

$$(\nu \otimes \mathrm{id}) \circ \Delta(\phi) = \nu(\phi) \cdot 1 \quad (\text{resp. } (\mathrm{id} \otimes \nu) \circ \Delta(\phi) = 1 \cdot \nu(\phi))$$

for all $\phi \in GL_q(n, C)$. A bi-invariant integral is called a Haar integral.

Proposition 5. *There exists a unique Haar integral h with $h(1) = 1$ and it is the projection $\nu : \oplus_\lambda W(\lambda) \to W(0)$.*

2.6. Quantum group $U_q(n)$ and quantum homogeneous space $U_q(n-1)\backslash U_q(n)$ [14], [28].

The definition of a Hopf *-algebra can be found in [20], [19]:

Definition 2. A Hopf algebra $\mathcal{H}$ is a Hopf *-algebra if it is equipped with a conjugate linear mapping $* : H \to H$, such that:

(1) $1^* = 1; \underline{(\phi\psi)^*} = \psi^*\phi^*$.

(2) $\varepsilon(\phi^*) = \overline{\varepsilon(\phi)}$; $\Delta \circ * = (* \otimes *) \circ \Delta$; $\qquad (\forall \phi, \psi \in H)$.

(3) $* \circ * = \mathrm{id}; * \circ S \circ * \circ S = \mathrm{id}$.

Now we define the "compact real form" $U_q(n)$ of $GL_q(n,C)$ by introducing an involution in the Hopf algebra $GL_q(n,C)$, if q is real, $q \notin \{-1,0,1\}$. The anti-homomorphism $*$ acts on $A(GL_q(n,C))$ in the following way:

$$t^*{}_{ij} = S(t_{ji}) \quad \forall i,j \in \{1,\dots,n\} \quad (\det_q^{-1})^* = \det_q.$$

One can check that $\mathbf{T}^*\mathbf{T} = \mathbf{T}\mathbf{T}^* = 1$, where $\mathbf{T}^* = ((t^*{}_{ji})_{i,j=1}^n)$, and that $A(U_q(n)) = A((GL_q(n,C)),*)$ is a Hopf $*$-algebra.

Definition 3. The above Hopf $*$-algebra is the algebra of polynomials on the quantum unitary group $U_q(n)$. Sometimes we denote it briefly $U_q(n)$. The algebra of polynomials on the quantum group $SU_q(n)$ is a Hopf $*$-algebra specified by the condition $\det_q = 1$ with the same $\Delta, \varepsilon, S, *$.

For $1 \le m < n$ we define an epimorphism $\gamma_m : A(U_q(n)) \to A(U_q(n-m))$ of Hopf $*$-algebras by:

$$\gamma_m(t_{ij}) = s_{ij} \quad (1 \le i,j \le n-m);$$

$$\gamma_m(t_{kl}) = \delta_{kl}1 \quad (k \textit{ or } l > n-m); \quad \gamma_m(\det_q^{-1}) = \det_q^{-1}. \tag{10}$$

Proposition 6. *The algebra $U_q(n-1)\backslash U_q(n)$ is generated by t_{nk} and t^*_{nk} $(1 \le k \le n)$, satisfying the following relations:*

$$t_{ni}t_{nj} = qt_{nj}t_{ni}, \ qt^*_{ni}t^*_{nj} = t^*_{nj}t^*_{ni} \ (1 \le i < j \le n),$$
$$t^*_{nj}t_{ni} = qt_{ni}t^*_{nj}, \ 1 \le i,j \le n$$
$$t^*_{nk}t_{nk} = t_{nk}t^*_{nk} + (1-q^2)\sum_{l<k} t_{nl}t^*_{nl} (1 \le k \le n)$$
$$\sum_{k=1}^n t_{nk}t^*_{nk} = 1.$$

The structure of the quantum homogeneous space $U_q(n)/U_q(n-1)$ is similar.

2.7. Double cosets of quantum groups [4], [24], [25].

Let $\mathcal{H}_i = (H_i, d_i, 1_i, \Delta_i, \varepsilon_i, S_i)$ $(i = 1,2)$ be two Hopf algebras, where d_i is a product, 1_i is a unit, Δ_i is a coproduct, ε_i is a counit, S_i is an antipode in H_i. Let $\gamma : \mathcal{H}_1 \to \mathcal{H}_2$ be an epimorphism of Hopf algebras, i.e. it is an algebra epimorphism such that $(\gamma\otimes\gamma)\circ\Delta_1 = \Delta_2\circ\gamma$, $\varepsilon_1 = \varepsilon_2\circ\gamma$, $\gamma\circ S_i = S_2\circ\gamma$.

With respect to the coactions

$$L_{H_2} = (\gamma \otimes \mathrm{id}) \circ \Delta_1, \quad L_{H_2} : H_1 \to H_2 \otimes H_1$$
$$(\text{resp., } R_{H_2} = (\mathrm{id} \otimes \gamma) \circ \Delta_1, \quad R_{H_2} : H_1 \to H_1 \otimes H_2)$$

H_1 is a left (resp., right) H_2-comodule. Define the sets $H_2\backslash H_1$, H_1/H_2, $H_2\backslash H_1/H_2$ of left-, right- and bi-invariants in H_1 with respect to H_2 exactly as in 2.3.

Remark 2. a) Since L_{H_2} and R_{H_2} are homomorphisms, the above sets are unital subalgebras of H_1.

b) A straightforward verification shows that:

$$\Delta_1(H_2\backslash H_1) \subset H_2\backslash H_1 \otimes H_1, \Delta_1(H_1/H_2) \subset H_1 \otimes H_1/H_2,$$
$$\Delta_1(H_2\backslash H_1/H_2) \subset H_2\backslash H_1 \otimes H_1/H_2.$$

c) From $\Delta_1 \circ S_1 = \sigma \circ (S_1 \otimes S_1) \circ \Delta_1$ (here $\sigma(a \otimes b) = b \otimes a$), one can deduce that

$$S_1(H_2\backslash H_1) \subset H_2\backslash H_1,\ S_1(H_1/H_2) \subset H_1/H_2,$$
$$S_1(H_2\backslash H_1/H_2) \subset H_2\backslash H_1/H_2.$$

Let ν_2 be an invariant integral on $\mathcal{H}_2$ (it exists if $\mathcal{H}_2$ is associated with a compact quantum group [**29**]). Introduce a new coproduct on $H_2\backslash H_1/H_2$:

$$\Delta := (\mathrm{id} \otimes \nu_2 \circ \gamma \otimes \mathrm{id}) \circ (\Delta_1 \otimes \mathrm{id}) \circ \Delta_1. \tag{11}$$

This is a generalization of (1) for Hopf algebras case.

We denote the restrictions of ε_1, S_1, to $H_2\backslash H_1/H_2$ by the same letters.

Proposition 7. *Let a mapping Δ be defined by (11). Then:*

a) *Δ maps $H_2\backslash H_1/H_2$ into $H_2\backslash H_1/H_2 \otimes H_2\backslash H_1/H_2$;*

b) *Δ is coassociative, i.e.*

$$(\mathrm{id} \otimes \Delta) \circ \Delta = (\Delta \otimes \mathrm{id}) \circ \Delta;$$

c) *ε_1 is a counit on $H_2\backslash H_1/H_2$ with respect to Δ:*

$$(\varepsilon_1 \otimes \mathrm{id}) \circ \Delta = (\mathrm{id} \otimes \varepsilon_1) \circ \Delta = \mathrm{id};$$

d) *if ν_1 is a left- (resp., right-) invariant integral on H_1, then its restriction to $H_2\backslash H_1/H_2$ is left- (resp., right-) invariant with respect to Δ:*

$$(\nu_1 \otimes \mathrm{id}) \circ \Delta(h) = \nu_1(h)1_1$$
$$(\textit{resp.},\ \ (\mathrm{id} \otimes \nu_1) \circ \Delta(h) = \nu_1(h)1_1);$$

e) *the following relation holds:*

$$\Delta \circ S_1 = \sigma \circ (S_1 \otimes S_1) \circ \Delta;$$

f) *if both $\mathcal{H}_1$, $\mathcal{H}_2$ are Hopf $*$-algebras and γ is their $*$-epimorphism, then $H_2\backslash H_1$, H_1/H_2, $H_2\backslash H_1/H_2$ are unital $*$-algebras, $\Delta \circ * = (* \otimes *) \circ \Delta$ and Δ maps the cone of positive elements of $H_2\backslash H_1/H_2$ to the cone of positive elements of its tensor product.*

Now recall that if $\mathcal{H}_1$ is a Hopf *-algebra associated with a compact quantum group, then [**29**] H_1 can be represented as

$$H = \sum_{\alpha} \sum_{i,j=1}^{d_\alpha} \mathrm{C} u^{\alpha}_{i,j}, \tag{12}$$

where $u^{\alpha}_{i,j}$ are matrix elements of d_α-dimensional unitary corepresentation of H_1 ($d_\alpha < \infty$ for all α running over some discrete set $\hat{Q}$) and there exists an invariant integral ν on H, which is a state and such that α-sum in (12) defines an orthogonal decomposition in the sense of the inner product given by $\langle f, g\rangle := \nu(f \cdot g^*)$ after a suitable choice of an orthonormal basis for each representation space. In this case, the comodules $H_2\backslash H_1,\ H_1/H_2$ as well as $H_2\backslash H_1/H_2$ may be given by

$$H_2\backslash H_1 = \sum_{\alpha}\sum_{i=1}^{d'_\alpha}\sum_{j=1}^{d_\alpha} \mathrm{C}u^{\alpha}_{i,j},\ \ H_1/H_2 = \sum_{\alpha}\sum_{i=1}^{d_\alpha}\sum_{j=1}^{d'_\alpha} \mathrm{C}u^{\alpha}_{i,j},$$

$$H_2\backslash H_1/H_2 = \sum_{\alpha}\sum_{i,j=1}^{d'_\alpha} \mathrm{C}u^{\alpha}_{i,j}$$

where $d'_\alpha \le d_\alpha$ for all α. One can see that

$$\Delta(u^{\alpha}_{i,j}) = \sum_{k=1}^{d'_\alpha} u^{\alpha}_{i,k} \otimes u^{\alpha}_{k,j}. \tag{13}$$

3. Quantum Stiefel manifold $U_q(n)/U_q(n-m)$.

3.1.

For studying the structure of the quantum space $U_q(n)/U_q(n-m)$, decompose $U_q(n)$ into a direct sum of irreducible $U_q(n-m)$-comodules.

Definition 4. A dominant integral weight $\mu \in L_i$ is said to be subordinated to a dominant integral weight $\nu \in L_j$ $(1 \le i < j \le n)$ if

$$\nu_{k+j-i} \le \mu_k \le \nu_k \qquad (\forall\ 1 \le k \le i). \tag{14}$$

In this case we write $\mu \prec \nu$.

Lemma 1. *Let λ be a dominant integral weight in L_n, $V^R(\lambda)$ be an irreducible right $U_q(n)$-comodule of the weight λ. Considering $V^R(\lambda)$ as a right $U_q(n-m)$-comodule $(1 \le m \le n)$ with a coaction $R_{U_q(n-m)} := (\mathrm{id} \otimes \gamma_m)R_{U_q(n)}$, we have a decomposition:*

$$V^R(\lambda) \sim \oplus_{\mu^{n-m}} K^{\lambda}_{\mu^{n-m}} V^R(\lambda, \mu^{n-m}), \tag{15}$$

where μ^{n-m} runs over the set of all dominant integral weights in L_{n-m} subordinated to λ, $V^R(\lambda, \mu^{n-m})$ is an irreducible $U_q(n-m)$-comodule of the weight μ^{n-m}, $K^{\lambda}_{\mu^{n-m}} \ge 1$ is its multiplicity, $\sim$ means an isomorphism.

Proof. For $m = 1$ the result has been obtained in [**14**]:

$$V^R(\lambda) = \oplus_{\mu^{n-1}} V^R(\lambda, \mu^{n-1}). \tag{16}$$

Every $U_q(n-1)$-subcomodule $V^R(\lambda, \mu^{n-1})$ of $V^R(\lambda)$ is isomorphic to an irreducible right $U_q(n-1)$-comodule $V^R(\mu^{n-1})$ with weight μ^{n-1}. By this isomorphism and a decomposition of $V^R(\mu^{n-1})$ similar to (16), on the second step we have

$$V^R(\lambda) = \oplus_{\mu^{n-1}} \oplus_{\mu^{n-2}} V^R(\lambda, \mu^{n-1}, \mu^{n-2}) \sim \oplus_{\{\mu\}^{n-2}} V^R(\lambda, \{\mu\}^{n-2}),$$

where $\{\mu\}^{n-2}$ is the collection of the weights $\{\mu^{n-1}, \mu^{n-2}\}$ with $\mu^{n-2} \prec \mu^{n-1}$. $V^R(\lambda, \{\mu\}^{n-2})$ is a $U_q(n-2)$-subcomodule of the right $U_q(n-2)$-comodule $V^R(\lambda)$, and $V^R(\lambda, \{\mu\}^{n-2})$ is isomorphic to an irreducible right $U_q(n-2)$-comodule of the weight μ^{n-2}. On the m-th step of this process one gets a decomposition:

$$V^R(\lambda) = \oplus_{\{\mu\}^{n-m}} V^R(\lambda, \{\mu\}^{n-m}),$$

where $\{\mu\}^{n-m}$ runs over the set of all collections of the dominant integral weights $\{\mu\}^{n-m} = \{\mu^{n-1}, \cdots, \mu^{n-m}\}$, $\mu^i \in L_i$, with $\mu^{i-1} \prec \mu^i$.

Here $V^R(\lambda, \{\mu\}^{n-m})$ is a $U_q(n-m)$-subcomodule of the right $U_q(n-m)$-comodule $V^R(\lambda)$ and it is isomorphic to an irreducible $U_q(n-m)$-comodule of the weight μ^{n-m}. There are several collections $\{\mu\}^{n-m}$ "leading" from λ to μ^{n-m}; the number of these collections defines the corresponding multiplicity $K^{\lambda}_{\mu^{n-m}}$ (the formula for computation of this number is quite complicated).

3.2.

In a usual way, one can give a definition of the highest vector of the weight μ^{n-i} in the irreducible $U_q(n-i)$-comodule $V^R(\lambda, \mu^{n-i})$. It can be constructed as follows. Remind that the basis in $V^R(\lambda)$ is formed by all possible standard monomials ξ_T indexed by the semi-standard tableaus T in $SSTab_n(\lambda)$.

(1) Let a tableau T be such that ξ_T is the highest vector of an irreducible $U_q(n-i)$-comodule. This is possible if and only if $e_k \cdot \xi_T = 0\ \forall k \in \{1, \dots, n-i-1\}$. Since $e_k \cdot \xi_{j_1 \cdots j_s} = \xi_{j_1 \cdots j_p - 1 \cdots j_s}$, if some $j_p = k+1$, and $e_k \cdot \xi_{j_1 \cdots j_s} = 0$, if $j_1, \cdots, j_s \neq k+1$ $(j_1 \leq \cdots \leq j_s \leq n)$, then one gets that

$$e_k \cdot \xi_{J_s} = 0,\ \forall k \in \{1, \cdots, n-i-1\},$$

if $J_s = (1, \cdots, p_s, j_{p_s+1}, \cdots, j_s)$, where $1 \leq p_s \leq n-i$, $j_{p_s+1}, \cdots j_s \geq n-i$. So the above structure of the lines of the tableau T is necessary. After that it remains to check that for such a tableau we also have:

$$q^{\varepsilon_k} \cdot \xi_T = q^{\mu_k^{n-i}} \xi_T, \varepsilon_k \in L_{n-i}, \forall k = \overline{1, n-i}.$$

(2) If a tableau T has the above property, one can get a collection of tableaus $T^{(n-i)} (1 \leq i \leq n)$ (each of them is obtained by removing from T its elements T_{rs} exceeding $n-i$). Every r-column of the tableau $T^{(n-i)}$ has, obviously, the height $\mu_r^{n-i} = \mathrm{card}\{1 \leq s \leq \lambda_r | T_{rs} \leq n-i\}$, and it is clear that $\mu^{n-i} = 0$ if $r \geq n-i$. So, $\mu^{n-i} = \sum_{r=1}^{n-i} \mu_r^{n-i} \varepsilon_r \in L_{n-i}$, and the

collection of the weights $\{\mu\}^{n-i}$ satisfies the condition of subordination. Let us show that the monomial $\xi_{T^{(n-i)}}$ is the highest vector of the weight μ^{n-i} in the irreducible $U_q(n-i)$-comodule.

In fact, according to the construction of $T^{(n-i)}$, one has $e_k \cdot \xi_{T^{(n-i)}} = 0$ $\forall k \in \{1, \cdots, n-i-1\}$. Now it is sufficient to show that $q^{\varepsilon_k} \cdot \xi_{T^{(n-i)}} = q^{\mu_k^{n-i}} \xi_{T^{(n-i)}}$ $\forall k \in \{1, \cdots, n-i\}$. Taking into consideration the formulae $q^{\varepsilon_k} \cdot t_{ij} = q^{\delta_{kj}} t_{ij}$, $q^{\varepsilon_k} \cdot \xi_J = q^{\langle \varepsilon_k, \Lambda_J \rangle} \xi_J$ and $q^{\varepsilon_k} \cdot \xi_{T^{(n-i)}} = q^{\langle \varepsilon_k, \Lambda_{J_1^{n-i}} + \cdots + \Lambda_{J_s^{n-i}} \rangle} \xi_{T^{(n-i)}}$ $= q^{\mu_k^{n-i}} \xi_{T^{(n-i)}}$. Considering a line J and the corresponding line $J^{(n-i)}$, we have $q^{\varepsilon_k} \cdot \xi_J = q^{\langle \varepsilon_k, \Lambda_J \rangle} \xi_J = q^{\langle \varepsilon_k, \Lambda_{J^{(n-i)}} \rangle} \xi_J$ $\forall k \in \{1, \cdots, n-i\}$. This means that $q^{\varepsilon_k} \cdot \xi_T = q^{\langle \varepsilon_k, \Lambda_{J_1^{n-i}} + \cdots + \Lambda_{J_s^{n-i}} \rangle} \xi_T = q^{\mu_k^{n-i}} \xi_T$.

So one gets that ξ_T (corresponding to the semi-standard tableau T with the above property) is the highest vector of the weight $\mu_r^{(n-i)}$ in the irreducible $U_q(n-i)$-comodule $V^R(\lambda)$. The epimophism $\gamma_i : U_q(n) \to U_q(n-i)$ maps ξ_T to $\xi_{T^{(n-i)}}$ and in this way generates an isomorphism of irreducible $U_q(n-i)$-comodules.

Remark 3. Lemma 1 and the mentioned construction of the highest vector can be applied also to left comodules.

3.3.

In what follows we consider the homogeneous space $U_q(n)/U_q(n-m)$, i.e., the subcomodule $V^R(\lambda)$ of comodule $U_q(n)$ of the weight $\mu^{n-m} = 0$. From inequalities $0 = \mu_1^{n-m} \geq \mu_2^{n-m-1} \geq \cdots \geq \mu_{n-m}^1 \geq \lambda_{n-m+1} \geq \cdots \geq \lambda_n$ we obtain $\lambda_n \leq 0$. The condition $\mu^{n-m} = 0$ means that the tableaus $T \in SSTab_n(\lambda)$ do not contain the numbers $1, \cdots, n-m$. Using Proposition 3, we can proceed to the case $\lambda_n \geq 0$. The new weights can be written as: $\tilde{\lambda} = \lambda - \lambda_m(\varepsilon_1 + \cdots + \varepsilon_n)$, $\tilde{\mu}^{n-m} = \mu^{n-m} - \lambda_n = -\lambda_n$. That is why the new tableaus $T \in SSTab_n(\lambda - \lambda_n(\varepsilon_1 + \cdots + \varepsilon_n))$ corresponding to the highest weights of this comodule will already contain the numbers $1, \cdots, n-m$. Moreover, each of them should contain a rectangle block beginning from the first line and the first column, every its line containing a completely ordered set $1, \cdots, n-m$, the number of its columns being equal to $-\lambda_n$.

Consider now the generators $v_T = \det_q^{\lambda_n} \xi_T$ of the above $U_q(n-m)$-comodule. Since the number of lines of T containing the numbers $1, \cdots, n-m$ equals to $-\lambda_n$, then these generators can be written in the form: $v_T = \det_q^{-1} \xi_{T_1} \cdots \det_q^{-1} \xi_{T_{-\lambda_n}} \xi_{T_{-\lambda_n+1}} \cdots \xi_{T_{\lambda_1 - \lambda_n}}$, where T_i are lines of T. By the definition of $*$ we have: $\det_q^{-1} \xi_{T_i} = sgn_q(T_i, T_i^c)(\xi_{T_i^c}^{I_i})^*$ and then $v_T = (\xi_{T_1^c}^{I_1})^* \cdots (\xi_{T_{-\lambda_n}^c}^{I_{-\lambda_n}})^* \cdot \xi_{T_{-\lambda_n+1}} \cdots \xi_{T_{\lambda_1-\lambda_n}}$. Here T_i^c is the complement of T_i to the set $(1, \cdots, n)$, I_i is the line $(p, \cdots, n)$, p is the length of the T_i, and $sgn_q(I, J) = (-q)^{l(I,J)}$, $l(I, J) = \#\{(i, j) : i \in I, j \in J, i > j\}$.

The lines $T_{-\lambda_n+1}, \cdots, T_{\lambda_1-\lambda_n}$ do not contain the numbers $1, \cdots, n-m$ by the construction.

The comodule generated by the vectors v_T is the $U_q(n-m)$-subcomodule of the weight $\mu^{n-m} = 0$ in $U_q(n-m)$-comodule $V^R(\lambda)$. We denote it by $V^R(\lambda)^0_{n-m}$. There is an isomorphism $W(\lambda)/U_q(n-m) \sim V^L(\lambda) \otimes V^R(\lambda)^0_{n-m}$ and $U_q(n)/U_q(n-m) \sim \oplus_\lambda V^L(\lambda) \otimes V^R(\lambda)^0_{n-m}$, where λ runs over the set of all weights such that $\mu^{n-m} = 0 \prec \lambda$. This isomorphism imposes a restriction on lower indices of the minors generating $W(\lambda)/U_q(n-m)$.

Thus, the algebraic generators of $U_q(n)/U_q(n-m)$ are the minors $\xi^{i_1,\cdots i_r}_{j_1,\cdots j_r}$ and $(\xi^{i_1,\cdots i_r}_{j_1,\cdots j_r})^*$ $(1 \le i_1 < \cdots < i_r \le n;\ n-m+1 \le j_1 < \cdots < j_r \le n;\ 1 \le r \le m)$. These minors are polynomials in $t_{ij}; t^*_{ij}$ $(1 \le i \le n,\ n-m+1 \le j \le n)$.

The result of our considerations can be summarized as follows:

Theorem 1. 1) *The quantum homogeneous space* $S^{n,m}_q = U_q(n)/U_q(n-m)$ *is the algebra generated by* t_{ij}, t^*_{ij} $(1 \le i \le n, n-m+1 \le j \le n)$ *satisfying the following relations:*

$$t_{ik}t_{jk} = qt_{jk}t_{ik}, \quad t_{ki}t_{kj} = qt_{kj}t_{ki} \ \ (i<j) \ \ t_{il}t_{jk} = t_{jk}t_{il} \ \ (i<j, k<l)$$

$$t_{ik}t_{jl} - qt_{il}t_{jk} = t_{jl}t_{ik} - q^{-1}t_{jk}t_{il} \ (i<j, k<l) \tag{17}$$

$$t^*_{ik}t^*_{jk} = q^{-1}t^*_{jk}t^*_{ik}, \quad t^*_{ki}t^*_{kj} = q^{-1}t^*_{kj}t^*_{ki} \ \ (i<j) \ \ t^*_{il}t^*_{jk} = t^*_{jk}t^*_{il} \ \ (i<j, k<l)$$

$$t^*_{ik}t^*_{jl} - q^{-1}t^*_{il}t^*_{jk} = t^*_{jl}t^*_{ik} - qt^*_{jk}t^*_{il} \ \ (i<j, k<l) \tag{18}$$

$$qt_{lp}t^*_{lp} + (q-q^{-1})\sum_{m>p} t_{lm}t^*_{lm} = qt^*_{lp}t_{lp} + (q-q^{-1})\sum_{r<l} t^*_{rp}t_{rp}$$

$$t_{ij}t^*_{is} = qt^*_{is}t_{ij} + (q-q^{-1})\sum_{p<i} t^*_{ps}t_{pj} \ \ (s \ne j)$$

$$qt_{lp}t^*_{jp} + (q-q^{-1})\sum_{m>p} t_{lm}t^*_{jm} = t^*_{jp}t_{lp} \ (l \ne j)$$

$$t_{kj}t^*_{ps} = t^*_{ps}t_{kj} \ (k \ne p,\ j \ne s) \tag{19}$$

$$\sum_{i=1}^{n} t_{il}t^*_{ij} = \delta_{lj}. \tag{20}$$

2) *The relations* (17), (18), (19), (20) *form the full system of relations between the generators* t_{ik}, t^*_{ik} $(1 \le i \le n,\ n-m+1 \le k \le n)$ *of the algebra* $S^{n,m}_q$.

Proof. 1) It was explained already that the algebra $S_q^{n,m}$ is generated by t_{ik}, t_{ik}^* $(1 \le i \le n,\ n-m+1 \le k \le n)$. We shall prove the formulae (17)-(20) using the matrices $\mathbf{T} = (t_{ij})_{i,j=1}^n$ and $\mathbf{T}^* = (t_{ji}^*)_{i,j=1}^n$.

The formula (20) follows from the equation $\mathbf{T}\mathbf{T}^* = \mathbf{T}^*\mathbf{T} = I$ (see 2.6). The formula (17) follows from the Equation (6): $(\mathbf{T} \otimes \mathbf{T})\mathbf{R} = \mathbf{R}(\mathbf{T} \otimes \mathbf{T})$, where $\mathbf{T} \otimes \mathbf{T} = (\mathbf{T} \otimes I)(I \otimes \mathbf{T})$. Multiplying both sides by $(\mathbf{T}^* \otimes I)$ on the left, one gets $(I \otimes \mathbf{T})\mathbf{R} = (\mathbf{T}^* \otimes I)\mathbf{R}(\mathbf{T} \otimes I)(I \otimes \mathbf{T})$; multiplying both sides by $I \otimes \mathbf{T}^*$ on the right, one gets $(I \otimes \mathbf{T})\mathbf{R}(I \otimes \mathbf{T}^*) = (\mathbf{T}^* \otimes I)\mathbf{R}(\mathbf{T} \otimes I)$.

The coordinate form of this formula gives us (19). And, finally, multiplying the last line by the $I \otimes \mathbf{T}^*$ on the right and by the $\mathbf{T}^* \otimes I$ on the left, one gets $\mathbf{R}(I \otimes \mathbf{T}^*)(\mathbf{T}^* \otimes I) = (I \otimes \mathbf{T}^*)(\mathbf{T}^* \otimes I)\mathbf{R}$ or $\mathbf{R}(\mathbf{T}^* \otimes \mathbf{T}^*) = (\mathbf{T}^* \otimes \mathbf{T}^*)\mathbf{R}$. From this formula we obtain (18).

2) It is known that monomials in t_{ik}, t_{ik}^* $(1 \le i \le n,\ 1 \le k \le n)$ having the lexicographic order form a basis in $U_q(n)$ [**14**], [**18**]. Similarly one can show, using the relations (17),(18), (19),(20), that monomials in t_{ik}, t_{ik}^* $(1 \le i \le n,\ n-m+1 \le k \le n)$ having the lexicographic order form a basis in $S_q^{n,m}$. This allows us to identify the algebra generated by the above letters and relations with certain subalgebra of $U_q(n)$. After that it is clear that there is no nontrivial polynomial in t_{ik}, t_{ik}^* $(1 \le i \le n,\ n-m+1 \le k \le n)$ equal to 0.

Remark 4. The similar facts are valid for $U_q(n-m)\backslash U_q(n)$. In the classical case $q = 1$ the homogeneous space above is called a Stiefel manifold. That is why in what follows we call $S_q^{n,m} = U_q(n-m)/U_q(n)$ a quantum Stiefel manifold.

4. Irreducible representations of $S_q^{n,m}$.

4.1.

Recall some basic facts referring to representations of the algebra of polynomials on the quantum group $SU_q(n)$ [**20**], [**19**]. The algebra of polynomials on $SU_q(2)$ has the following irreducible representations: one-dimensional $\chi_\phi(s_{11}) = e^{i\phi},\ \chi_\phi(s_{22}) = e^{-i\phi},\ \chi_\phi(s_{12}) = \chi_\phi(s_{21}) = 0$ $(\phi \in [0, 2\pi))$ and an infinite-dimensional ρ_0 in $l_2(\mathbf{Z}_+) : \rho_0(s_{11})e_0 = 0, \rho_0(s_{11})e_k = (1-q^{2k})^{1/2}e_{k-1}$ $k \ge 1$; $\rho_0(s_{21})e_k = -q^k e_k$; $\rho_0(s_{22}) = \rho_0(s_{11})^*$; $\rho_0(s_{12}) = -q\rho_0(s_{21})$ (from here on we consider only $q \in (0, 1)$).

The representations of $SU_q(n)$ can be constructed using the above representations of $SU_q(2)$. Let $\psi_i : U_q(sl(2)) \to U_q(sl(n))$ be the inclusion of the quantum enveloping algebras such that $\psi_i(e) = e_i$; $\psi_i(f) = f_i$; $\psi_i(q^\varepsilon) = q^{\varepsilon_i}$ $1 \le i \le n-1$. Then $\psi_i^* : SU_q(n) \to SU_q(2)$ and $\pi_i = \rho_0\psi_i^*$ are the irreducible $*$-representations of $SU_q(n)$. By this construction, $\pi_i(t_{ii})$ and $\pi_i(t_{i+1,i+1})$ contain the shift operators, $\pi_i(t_{i,i+1})$ and $\pi_i(t_{i+1,i})$ are diagonal and $\pi_i(t_{k,l}) = \delta_{kl}1$ for all other generators.

Let $S_n \ni \omega = \tau_{i_1} \cdots \tau_{i_k}$ be a decomposition of some element ω of the permutation group, which has the least possible number of transpositions $\tau_j = (j, j+1)$. The representation $\pi_\omega = \pi_{i_1} \otimes \cdots \otimes \pi_{i_k}$ corresponds to the element ω (recall that if $\mathcal{H}$ is a Hopf algebra, ρ_1 is a representation of its algebra H, A is a right comodule algebra over $\mathcal{H}$ with respect to the coaction $R_A : H \to H \otimes A$, ρ_2 is a representation of the algebra A, then one can construct a new representation of the algebra H: $\rho_1 \otimes \rho_2 := (\rho_1 \otimes \rho_2) \circ R_A$).

Proposition 8. *The $*$-representations π_ω are irreducible and any irreducible $*$-representation of $SU_q(n)$ is equivalent to some π_ω up to a one-dimensional tensor factor.*

Remark 5. A similar statement is also valid for $U_q(n)$. The only difference is that the set of one-dimensional representations of the group $SU_q(n)$ is $(n-1)-$parametric and of the group $U_q(n)$ is $n-$parametric.

4.2.

One can construct an irreducible $*$-representation of the quantum Stiefel manifold $S_q^{n,m} = U_q(n)/U_q(n-m) = SU_q(n)/SU_q(n-m)$ using these facts. Let us consider cosets S_n/S_{n-m} of the permutation group S_n with respect to the subgroup S_{n-m}. Chose from every coset an element having the least possible length $\omega = \tau_{j_1} \cdots \tau_{j_l}$.

Theorem 2. *Representations π_ω of $S_q^{n,m}$ corresponding to elements ω of the least possible length, among the representatives of the class from S_n/S_{n-m}, are irreducible.*

Proof. Let us consider the structure of elements of S_n (resp., S_n/S_{n-m}) and the structure of the corresponding representations of $U_q(n)$ (resp., $S-q^{n,m}$). An arbitrary element of S_n can be written as follows:

$$\omega = \tau_{i_1} \cdots \tau_{n-1} \tau_{i_2} \cdots \tau_{n-2} \cdots \tau_{i_k} \cdots \tau_{n-k},$$

$$(1 \le k \le n-1,\ i_1 \le \cdots \le n-1, \cdots,\ i_k \le \cdots \le n-k),$$

(in particular, the greatest element $\omega_0 \in S_n$ has the form:

$$\omega_0 = \tau_1 \tau_2 \cdots \tau_{n-1} \tau_1 \cdots \tau_{n-2} \cdots \tau_1 \tau_2 \tau_1).$$

The corresponding series of representations of $U_q(n)$

$$\pi_{\omega,\bar\phi} = \pi_{i_1} \otimes \cdots \otimes \pi_{n-1} \otimes \pi_{i_2} \otimes \cdots \otimes \pi_{n-2} \otimes \cdots \otimes \pi_{i_k} \otimes \cdots \otimes \pi_{n-k} \otimes \kappa_{\bar\phi},$$

is parametrized by one-dimensional representations $\kappa_{\bar\phi}$. Here one-dimensional representations $\kappa_{\bar\phi}$ correspond to the greatest torus in $U_q(n)$.

Respectively, an arbitrary representative of a class from S_n/S_{n-m} having the least possible length can be written as follows:

$$\omega = \tau_{i_1} \cdots \tau_{n-1} \tau_{i_2} \cdots \tau_{n-2} \cdots \tau_{i_k} \cdots \tau_{n-k} \quad (1 \le k \le m),$$

(in particular, the greatest element from S_n/S_{n-m}, the representative of the class $\omega_0 S_{n-m}$ having the least possible length, is

$$\omega_m = \tau_1\tau_2\cdots\tau_{n-1}\tau_1\cdots\tau_{n-2}\cdots\tau_1\cdots\tau_{n-m}).$$

The corresponding series of representations of $S_q^{n,m}$

$$\pi_{\omega,\bar\phi} = \pi_{i_1}\otimes\cdots\otimes\pi_{n-1}\otimes\pi_{i_2}\otimes\cdots\otimes\pi_{n-2}\otimes\cdots\otimes\pi_{i_k}\otimes\cdots\otimes\pi_{n-k}\otimes\kappa_{\bar\phi},$$

is parametrized by one-dimensional representations corresponding to the greatest torus in $S_q^{n,m}$.

Now let us show that every representation π_ω is irreducible. The irreducibility of this representation follows from the fact that for any tensor component of $\pi_\omega(\cdot)$ one can find such a generator t of $S_q^{n,m}$ for which this tensor component of $\pi_\omega(t)$ containes the shift operator. More precisely, one can see that for any tensor component of $\pi_\omega(\cdot)$ with a number from the group $\{i_l,\dots,n-l\}$ $(l=1,\dots,k)$ it is possible to find such a generator t from the group $\{t_{1,n-l+1},\dots,t_{n,n-l+1}\}$ for which this tensor component of $\pi_\omega(t)$ containes the shift operator. Let us show the typical consideration supporting this statement. Let us consider, for example, the greatest representation π_{ω_m}. Then

$$\pi_{\omega_m}(t_{in}) = \pi_1(t_{ii})\otimes\cdots\otimes\pi_{i-1}(t_{ii})\otimes\pi_i(t_{i,i+1})\otimes\cdots\otimes\pi_{n-1}(t_{n-1,n})\otimes\pi_1(t_{nn})\otimes\cdots$$
$$\otimes\pi_{n-2}(t_{nn})\otimes\cdots\otimes\pi_1(t_{nn})\otimes\pi_{n-m}(t_{nn})\quad(1\le i\le n).$$

The operator $\pi_{i-1}(t_{ii})$ containes the shift operator by the construction. The operators $\pi_j(t_{j,j+1})$ $(i\le j\le n-1)$ are diagonal and all the others are unit operators. Hence, the above statement is true for the tensor components of $\pi_{\omega_m}(\cdot)$ with numbers from the group $\{1,\dots,n-1\}$. Similarly one can consider all other cases. Thus, the proof is completed.

Theorem 3. *Each irreducible $*$-representation ρ of $S_q^{n,m}$ is equivalent* (*up to a one-dimensional tensor factor*) *to one of the representations π_ω, where ω has the least possible length among the representatives of the class from S_n/S_{n-m}.*

Proof. Continue ρ up to the representation of the quantum group $SU_q(n)$ in the following way: $\rho(t_{ij})=\rho(t_{ij}^*)=\delta_{ij}1$ $(1\le j\le n-m,\ 1\le i\le n)$. It is irreducible because it is irreducible on the subalgebra. Then, according to Proposition 8, it is equivalent to the representation $\pi_{i_1}\otimes\cdots\otimes\pi_{i_k}\otimes\kappa_\phi$ (κ_ϕ is one-dimensional representation).

Let us consider the permutation $\omega=\tau_{i_1}\cdots\tau_{i_k}$ corresponding to this representation. The representation ρ and, consequently, π_ω are irreducible on $S_q^{n,m}$. Then let us show that π_ω is irreducible only if ω has the least possible length. Consider a transposition $\tau_s\in S_{n-m}$, a representation $\pi_\omega\otimes\pi_s=\pi_{i_1}\otimes\cdots\otimes\pi_{i_k}\otimes\pi_s$ and its action on the generators t_{ik} $(1\le i\le n,\ n-m+1\le k\le n)$ of $S_q^{n,m}$: $\pi_\omega\otimes\pi_s(t_{ik})=\sum_{r=1}^n\pi_\omega(t_{ir})\otimes\pi_s(t_{rk})$. But $\pi_s(t_{rk})=\delta_{rk}1$ $(1\le r\le n,\ n-m+1\le k\le n)$ by the construction of the representations π_s $(s \le n-m-1)$. So $\pi_\omega \otimes \pi_s(t_{ik}) = \pi_\omega(t_{ik}) \otimes 1$ and this representation is not irreducible. Thus we have proved that the length of ω should be the least possible.

4.3.

Let us consider now the set of representations $\pi_{\omega,\bar\phi} = \pi_\omega \otimes \kappa_{\bar\phi}$, where $\kappa_{\bar\phi}$ are one-dimensional representations of $S_q^{n,m}$ and $\omega \in S_n/S_{n-m}$.

Theorem 4. *For the representations of $S_q^{n,m}$ we have:*

$$\cap_{\omega\in S_n/S_{n-m},\bar\phi} \mathrm{Ker}\pi_{\omega,\bar\phi} = 0.$$

Proof. Let $\cap_{\omega\in S_n/S_{n-m},\bar\phi} \mathrm{Ker}\pi_{\omega,\bar\phi} = L$. It was shown in the proof of Theorem 3 that $(\pi_{\omega,\bar\phi} \otimes \pi_\sigma)(x) = (\pi_{\omega,\bar\phi} \otimes 1)(x)$ $\forall\sigma \in S_{n-m}$ $\forall x \in S_q^{n,m}$. So for $x \in L$ we have $(\pi_{\omega,\bar\phi} \otimes \pi_\sigma)(x) = 0$. Taking all $\sigma \in S_{n-m}$, one can obtain all the representations of $U_q(n)$. Thus, $L = \cap_{\gamma\in S_n,\bar\phi} \mathrm{Ker}\pi_{\gamma,\bar\phi} = 0$, because $\pi_{\gamma,\bar\phi}$ $(\gamma \in S_n)$ give all the representations of $U_q(n)$, and it is known [**28**] that the intersection of their kernels is 0.

Now one can construct a C^*-algebra $\mathbf{C}(S_q^{n,m})$ of the quantum Stiefel manifold $S_q^{n,m}$, considering the completion of $S_q^{n,m}$ with respect to the C^*-norm

$$||\cdot|| = \sup_{\omega,\bar\phi} ||\pi_{\omega,\bar\phi}(\cdot)||.$$

Remark 6. The C^*-algebra $\mathbf{C}(U_q(n))$ of the quantum group $U_q(n)$ was introduced in a number of papers, for example, in [**18**], [**29**]. The C^*-algebra $\mathbf{C}(S_q^{n,1})$ was introduced in [**17**], [**18**], [**28**].

5. Invariant integral.

5.1.

The existence and the uniqueness of the invariant integral ν_n on $U_q(n)$ are known (2.5, 2.6). Our aim is to obtain a formula for its calculation as well as a similar formula for an invariant integral $\nu_{n,n-m}$ on $S_q^{n,m}$ (i.e., a linear functional on $S_q^{n,m}$ such that $(\mathrm{id} \otimes \nu_{n,n-m}) \circ \Delta(f) = \nu_{n,n-m}(f)1$ for every $f \in S_q^{n,m}$). We shall do it by induction in n. For this we use the formula for the invariant integral $\nu_{n,n-1}$ on $S_q^{n,1} = SU_q(n)/SU_q(n-1) = U_q(n)/U_q(n-1)$ which has been obtained in [**28**]:

$$\nu_{n,n-1}(f) = (2\pi)^{-1} \int_0^{2\pi} \mathrm{tr}(\pi_{\omega_1,\phi}(f)Q)d\phi,$$

where ω_1 is the greatest element in S_n/S_{n-1}, $Q : l_2(\mathbf{Z}_+)^{\otimes(n-1)} \to l_2(\mathbf{Z}_+)^{\otimes(n-1)}$ is a linear operator

$$Q(e_{m_1} \otimes \cdots \otimes e_{m_{n-1}}) = Q(m_1, \ldots, m_{n-1}) e_{m_1} \otimes \cdots \otimes e_{m_{n-1}},$$

where $Q(m_1,\dots,m_{n-1}) = \prod_{j=1}^{n-1}(1-q^{2(n-j)})^{-1}q^{2\sum_{j=1}^{n-1}m_j(n-j)}$. One can see that the operator Q is equal to the operator

$$\pi_1(t_{12}^*t_{12})(1-q^2)^{-1}\otimes\pi_2((t_{23}^*t_{23})^2)(1-q^4)^{-1}\otimes\cdots$$
$$\cdots\otimes\pi_{n-1}((t_{n-1,n}^*t_{n-1,n})^{n-1})(1-q^{2(n-1)})^{-1}$$

and its trace equals to 1.

Lemma 2. *The formula for the invariant integral on $U_q(n)$ is:*

$$\nu_n(f) = (2\pi)^{-n}\int_{T_0}\mathrm{tr}[\pi_{\omega_0,\bar{\phi}}(f)(\otimes_{k=1}^{n-1}Q_k)]d\bar{\phi},$$

where $Q_k = \otimes_{i=1}^{n-k}\pi_i(t_{i,i+1}^*t_{i,i+1})^i(1-q^{2i})^{-1}$, T_0 is the greatest torus in $U_q(n)$.

The formula for the invariant integral on $S_q^{n,m}$ is:

$$\nu_{n,n-m}(f) = (2\pi)^{-m}\int_{T_m}\mathrm{tr}[\pi_{\omega_m,\bar{\phi}}(f)(\otimes_{k=1}^{m-1}Q_k)]d\bar{\phi},$$

where Q_k are the same as above, T_m is the greatest torus in $S_q^{n,m}$.

Proof. The induction step is given by the following equality:

$$\nu_n = (\nu_{n,n-1}\otimes\nu_{n-1})(\mathrm{id}\otimes\gamma_1)\Delta,$$

where $\gamma_1 : U_q(n)\to U_q(n-1)$ is an epimorphism, a map $P_r = (\mathrm{id}\otimes\nu_{n-1})(\mathrm{id}\otimes\gamma_1)\Delta$ is exactly a projector from $U_q(n)$ on $S_q^{n,1}$ such that $(\mathrm{id}\otimes P_r)\Delta = \Delta\circ P_r$ [**4**], [**24**], [**25**], [**26**]. In fact, by the properties of P_r and $\nu_{n,n-1}$, one can see that the right-hand side of the above equality is exactly a right-invariant integral on $U_q(n)$. Since such an integral is unique (see 2.5), the above equality is true.

The base of the induction is the following expression for an invariant integral on $U_q(1) : \nu_1(t) = 0,\ \nu_1(1) = 1$. Now, using the formula for $\nu_{n,n-1}$, we obtain the statements of the lemma.

Corollary 1. *The invariant integrals ν_n and $\nu_{n,n-m}$ are the faithful states on the corresponding C^*-algebras $\mathbf{C}(U_q(n))$ and $\mathbf{C}(S_q^{n,m})$.*

Remark 7. The statement of the above corollary was obtained in [**13**] by different considerations.

5.2.

The construction of the invariant integral allows us to prove the following:

Theorem 5. *For the series of the greatest representations $\pi_{\omega_m,\bar{\phi}}$ of $S_q^{n,m}$ the following statement holds:*

$$\cap_{\bar{\phi}}\mathrm{Ker}\pi_{\omega_m,\bar{\phi}} = 0.$$

Proof. Let $\cap_{\bar\phi}\mathrm{Ker}\pi_{\omega_m,\bar\phi} = L \ni v$. Since L is a $*$-ideal, then $vv^* \in L$, and we get $\pi_{\omega_m,\bar\phi}(vv^*) = 0$. But at the same time, vv^* is a strictly positive element, hence $\nu_{n,n-m}(vv^*) > 0$. This contradiction proves the theorem.

The above theorem allows us to state that the representation

$$R_n(f) = \oplus \int_{T_m} \pi_{\omega_m,\bar\phi}(f) d\bar\phi$$

is the faithful representation of C^* algebra $\mathbf{C}(S_q^{n,m})$.

This result generalizes the construction of the faithful representation for $S_q^{n,1}$ [**28**].

6. Double cosets $U_q(n-m)\backslash U_q(n)/U_q(n-m)$.

6.1.

The definition of $U_q(n-m)\backslash U_q(n)/U_q(n-m)$ as the intersection of $U_q(n-m)\backslash U_q(n)$ and $U_q(n)/U_q(n-m)$ along with the results of Section 3 allow to describe the hypergroup structure on it more precisely than in Section 2.7. First, a straightforward corollary of Subsection 3.3 and Theorem 1 is given by the following:

Lemma 3. *The algebra $U_q(n-m)\backslash U_q(n)/U_q(n-m)$ is generated by the generators t_{ij}, t_{ij}^* $(n-m+1 \le i,j \le n)$ for which the relations of commutation* (17), (18), (19) *hold.*

Moreover, this algebra has the following comodule decomposition:

$$U_q(n-m)\backslash U_q(n)/U_q(n-m) \sim \oplus_\lambda V^L(\lambda)^0_{n-m} \otimes V^R(\lambda)^0_{n-m},$$

where λ is an integral dominant weight such that $\mu^{n-m} = 0 \prec \lambda$.

Remark 8. We can also consider an algebra $U_q(n-m_1)\backslash U_q(n)/U_q(n-m_2)$. This algebra is generated by generators t_{ij}, t_{ij}^* $(n-m_1+1 \le i \le n,\ n-m_2+1 \le j \le n)$ for which the relations of commutation (17), (18), (19) hold.

6.2.

One can introduce a new coproduct on the algebra $U_q(n-m)\backslash U_q(n)/U_q(n-m)$: $\tilde\Delta := (\mathrm{id}\otimes\nu_{n-m}\otimes\mathrm{id})(\mathrm{id}\otimes\gamma_m\otimes\mathrm{id})(\Delta\otimes\mathrm{id})\Delta$, where ν_{n-m} is the invariant integral on $U_q(n-m)$, γ_m is the epimorphism from $U_q(n)$ to $U_q(n-m)$ and Δ is the coproduct on $U_q(n)$.

Lemma 4. *The coproduct $\tilde\Delta$ "respects" the comodule structure of $U_q(n-m)\backslash U_q(n)/U_q(n-m) = \oplus_\lambda U_q(n-m)\backslash W(\lambda)/U_q(n-m)$:*

$$\tilde\Delta : U_q(n-m)\backslash W(\lambda)/U_q(n-m) \to$$

$$U_q(n-m)\backslash W(\lambda)/U_q(n-m) \otimes U_q(n-m)\backslash W(\lambda)/U_q(n-m).$$

Proof. The coproduct Δ acts on $W(\lambda)/U_q(n-m)$ in the following way: $\Delta : W(\lambda)/U_q(n-m) \to W(\lambda) \otimes W(\lambda)/U_q(n-m)$ and hence $(\Delta \otimes \mathrm{id})\Delta : U_q(n-m)\backslash W(\lambda)/U_q(n-m) \to U_q(n-m)\backslash W(\lambda) \otimes W(\lambda) \otimes W(\lambda)/U_q(n-m)$. It was shown earlier that $W(\lambda)$ can be decomposed into the direct sum as the right $U_q(n-m)-$comodule: $W(\lambda) = \oplus_{\mu^{n-m} \prec \lambda} W(\lambda, \mu^{n-m})$.

In the similar way $W(\lambda)$ can be decomposed as the left $U_q(n-m)-$comodule: $W(\lambda) = \oplus_{\mu^{n-m} \prec \lambda} W(\mu^{n-m}, \lambda)$.

Then one has: $(\mathrm{id} \otimes \gamma_m \otimes \mathrm{id})(\Delta \otimes \mathrm{id})\Delta : U_q(n-m)\backslash W(\lambda)/U_q(n-m) \to \oplus_{\mu^{n-m} \prec \lambda} U_q(n-m)\backslash W(\lambda, \mu^{n-m}) \otimes W(\mu^{n-m}) \otimes W(\mu^{n-m}, \lambda)/U_q(n-m)$. One uses the fact that $\nu_{n-m}(W(\mu^{n-m})) = 0$ if $\mu^{n-m} \neq 0$ and $W(0) = \mathbf{C}$; $W(\lambda)/U_q(n-m) = W(\lambda, 0)$; $U_q(n-m)\backslash W(\lambda) = W(0, \lambda)$.

After that $(\mathrm{id} \otimes \nu_{n-m}\gamma_m \otimes \mathrm{id})(\Delta \otimes \mathrm{id})\Delta : U_q(n-m)\backslash W(\lambda)/U_q(n-m) \to U_q(n-m)\backslash W(\lambda, 0) \otimes W(0) \otimes W(0, \lambda)/\, U_q(n-m) = U_q(n-m)\backslash W(\lambda)/U_q(n-m) \otimes U_q(n-m)\backslash W(\lambda)/U_q(n-m)$.

6.3.

Theorem 2 allows to get the following statement referring to representations of double cosets:

Lemma 5. (i) *Representations $\pi_{j_1} \otimes \cdots \otimes \pi_{j_l}$ corresponding to the elements ω of the least possible length (among the representatives of the class from $S_{n-m}\backslash S_n/S_{n-m}$) are irreducible representations of the double cosets $U_q(n-m)\backslash U_q(n)/U_q(n-m)$.*

(ii) *The series of greatest representations of $U_q(n-m)\backslash U_q(n)/U_q(n-m)$*

$$\pi_{{}_m\omega_m, \bar{\phi}} = \pi_{n-m} \otimes \pi_{n-1} \otimes \pi_{n-m-1} \otimes \cdots \otimes \pi_{n-2} \otimes \cdots \otimes \pi_{n-2m} \otimes \cdots \otimes \pi_{n-m} \otimes \kappa_{\bar{\phi}}$$

is parametrized by the greatest torus in $U_q(n-m)\backslash U_q(n)/U_q(n-m)$. Here ${}_m\omega_m$ are the greatest elements of double cosets $S_{n-m}\backslash S_n/S_{n-m}$.

(iii) *The representation $R_n(f) = \oplus \int_{T_m} \pi_{\omega_m, \bar{\phi}}(f) d\bar{\phi}$ is the faithful representation of C^* algebra generated by $U_q(n-m)\backslash U_q(n)/U_q(n-m)$.*

Finally, Lemma 2 gives a formula for the invariant integral on $U_q(n-m)\backslash U_q(n)/U_q(n-m)$ which is the restriction of $\nu_{n,n-m}$ on this algebra. The mentioned formula shows that the invariant integral is the faithful state on the C^*-algebra generated by the double cosets.

References

[1] E. Abe, *Hopf algebras*, Cambridge Tracts in Math., Vol. 74, Cambridge University Press, 1980.

[2] N. Bourbaki, *Elements de mathématique, Algèbre*, Chapitre 2, Hermann, Paris, 1962.

[3] K. Bragiel, *On the spherical and zonal spherical functions on a compact quantum group*, Letters in Math. Phys., **22** (1991), 195-201.

[4] Y. Chapovsky and L. Vainerman, *Hypergroup structures, associated with a pair of quantum groups* $(SU_q(n), U_q(n-1))$, in 'Methods of functional analysis in problems of mathematical physics', pages 47-69, Inst. Math. Acad. Sci. Ukraine, Kiev, 1992.

[5] V.G. Drinfeld, *Quantum Groups*, in 'Proceedings of the International Congress of Mathematicians', Berkeley, California, USA, (1986), 798-820.

[6] J. Faraut, *Analyse harmonique sur les paires de Guelfand et les espaces hyperboliques*, in 'Analyse Harmonique', pages 315-446, CIMPA, Nice, 1982.

[7] P. Floris, *A Non-Commutative Discrete Hypergroup Associated with q-Disk Polynomials*, preprint, 1994.

[8] S. Gelbart, *A theory of Stiefel harmonics*, Trans. Amer. Math. Soc., **192** (1974), 29-50.

[9] M. Jimbo, *A q-difference analogue of* $u(g)$ *and the Yang-Baxter equation*, Lett. Math. Phys., **10** (1985), 63-69.

[10] T. Koornwinder, *Orthogonal polynomials in connection with quantum groups*, in 'Orthogonal Polynomials: Theory and Practice' (P. Nevai, editor), Vol. 294, (1990), 257-292, Norwell, MA, NATO-ASI Series C, Kluwer.

[11] ———, *Positive convolution structures associated with quantum groups*, in 'Probability Measures on Groups', X, pages 249-268, Plenum, 1991.

[12] ———, *Discrete hypergroups associated with compact quantum Gel'fand pairs*, Contemporary Math., **183** (1995), 213-235.

[13] G. Nagy, *A Framework for Deformation Quantization*, Ph. D. Thesis, Berkeley, 1992.

[14] M. Noumi, H. Yamada and K. Mimachi, *Finite dimensional representations of the quantum group* $GL_q(n;C)$ *and the zonal spherical functions on* $U_q(n-1)\backslash U_q(n)$, Japan. J. Math., **19**(**1**) (1993), 31-80.

[15] G. Podkolzin, *Infinitesimal algebra for hypercomplex system with a basis* $SO(4)//SO(2)$, Ukrainian Math. Journal, **42**(**3**) (1990), 427-429; English transl. in Ukrainian Math. J., **3** (1990), 98-101.

[16] ———, *Infinitesimal object for hypercomplex system generated by double cosets, and nonlinear differential equations*, Funktsional. Anali. Prilozhen., **25**(**1**) (1991), 91-93; English transl. in Funct. Anal. Appl., **25** (1991), 78-80.

[17] P. Podles, *Quantum spheres*, Lett. Math. Phys., **14** (1987), 193-202.

[18] N. Reshetikhin, L. Takhtadjan and L. Faddeev, *Quantization of Lie groups and Lie algebras*, Algebra i Analiz, **1**(**1**) (1989), 178-206. English transl. in Leningrad Math. J., **1** (1990), 193-225.

[19] Y. Soibelman, *Irreducible representations of the algebra of functions on the Quantum group* $SU(n)$ *and Shubert cells*, Dokl. Acad. Nauk SSSR, **307** (1989), 41-45; English Transl. in Soviet Math. Dokl., **40** (1990).

[20] ———, *The algebra of functions on a compact Quantum group and its representations*, Algebra i Analiz, **2**(**1**) (1990), 190-211; English Transl. in Leningrad Math. J., **2**(**1**) (1991), 161-178.

[21] Y. Soibelman and L. Vaksman, *On some problems in the Theory of Quantum groups*, Adv. in Soviet Math., **9** (1992), 3-55.

[22] R. Strichartz, *The explicit fourier decomposition of* $L^2(SO(n)/SO(n-m))$, Canadian Math. Journal, **27** (1975), 294-310.

[23] L. Vainerman, *Duality for algebras with involution and generalized shift operators*, Journal of Soviet Math., **42**(**6**) (1988), 52-59.

[24] ———, *Gel'fand pairs of quantum groups, hypergroups and q-special functions*, Contemporary Mathematics, **183** (1995), 373-394.

[25] ———, *Hypergroup structures associated with Gel'fand pairs of compact quantum group*, Asterisque, **232** (1995), 231-243.

[26] L. Vainerman and Y. Weiss, *Hypergroup associated with double cosets of the quantum group $SU_q(2)$*, in 'Applications of the functional analysis methods in math. physics', pages 52-59, Inst. Math. Acad. Sci. Ukraine, Kiev, 1991, Russian; to be transl. in Selecta Math. Sov.

[27] L. Vaksman and Y. Soibelman, *Algebra of functions on the quantum group $SU(2)$*, Funct. Analyz i ego Pril., **22**(**3**) (1988), 1-14; English transl. Funct. Anal. Appl., **22** (1988).

[28] ———, *Algebra of functions on quantum group $SU(n+1)$ and odd dimensional spheres*, Algebra i Analyz, **2**(**5**) (1990), 101-120; English transl. in Leningrad Math. J., **2**(**5**) (1991), 1023-1042.

[29] S. Woronowicz, *Compact matrix pseudogroups*, Commun. Math. Phys., **111** (1987), 613-665.

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