Pacific Journal of Mathematics

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Volume 202 No. 2 February 2002

PACIFIC JOURNAL OF MATHEMATICS
Vol. 202, No. 2, 2002

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We construct examples having remarkable properties of cohomological dimension.

1. Introduction.

It is well-known that $\dim X \le n$ if and only if every map of a closed subspace of X into the n-dimensional sphere $\mathbb{S}^n$ can be extended over X. It is also well-known that for the cohomological dimension $\dim_G X$ of X with respect to an abelian coefficient group G, $\dim_G X \le n$ if and only if every map of a closed subspace of X into the Eilenberg-Mac Lane complex $K(G, n)$ extends over X. These properties give rise to the notion of extensional dimension [3]. Let K be a CW complex. The extensional dimension of X does not exceed K, written e-dim$X \le K$, if every map of a closed subset of X into K extends over X. Here e-dim $X > K$ means that e-dim $X \le K$ does not hold. We write e-dim$X > n$ if e-dim$X > K$ for every CW-complex K which is not n-connected. Thus e-dim$> n$ implies both dim$> n$ and $\dim_G > n$ for every group $G \neq 0$.

Below are listed some remarkable examples in cohomological dimension.

Theorem 1.1 (Dranishnikov [1])**.** *There is a locally compact separable metric space X such that $\dim_{\mathbb{Z}} X \le 4$ and $\dim_{\mathbb{Z}} \beta X = \infty$ where βX is the Stone-Čech compactification of X.*

Theorem 1.2 (Dydak [5], cf. [6])**.** *For each abelian group G there is a separable metric space X such that $\dim_G X \le 3$ and every Hausdorff compactification of X is of $\dim_G > 3$.*

Theorem 1.3 (Dranishnikov-Repovš [4], cf. [11])**.** *There is a compactum X such that $\dim_{\mathbb{Z}_2} X \le 1$ and e-dim $X > \mathbb{R}P^m$ for all integers $m > 0$.*

The goal of this note is to improve these results with a simpler construction. Namely we will prove the following theorems.

Theorem 1.4. *There is a locally compact separable metric space X such that for every abelian group G and every non-contractible* CW*-complex P, $\dim_G X \le 2$ and e-dim $\beta X > P$.*

Theorem 1.5. *There is a separable metric space X such that for every abelian group G and every Hausdorff compactification X' of X, $\dim_G X \leq 2$ and e-$\dim X' > 2$.*

Theorem 1.6. *There is a compactum X such that for every cyclic finite* CW*-complex P and every abelian group G, e-$\dim X > P$, $\dim_G X \leq 2$ and $\dim_G X \leq 1$ if G is finite.*

A space is called cyclic if at least one of its (reduced integral) homology groups does not vanish. We call a map homologically essential if it induces a nontrivial homomorphism of at least one of the homology groups.

The main tool for constructing our examples is the following theorem which was proved in [8]. We will formulate this theorem without using notations of truncated cohomology (note that no algebraic properties of truncated cohomology were used in [8]).

For a CW-complex K and a space L, $[K, L]$ denotes the set of pointed homotopy classes of maps from K to L. Let $\text{map}\,(K, L)$ stand for the space of pointed maps from K to L. By $\text{map}\,(K, L) \cong 0$ we mean that $\text{map}\,(K, L)$ is weakly homotopy equivalent to a point, that is $\pi_n(\text{map}\,(K, L)) = [\Sigma^n K, L] = [K, \Omega^n L] = 0$ for every $n \geq 0$. Clearly $\text{map}\,(K, L) \cong 0$ implies both $\text{map}\,(\Sigma^n K, L) \cong 0$ and $\text{map}\,(K, \Omega^n L) \cong 0$ for all $n \geq 0$. A space L and CW-complexes in Theorems 1.7-1.10 are assumed to be pointed. Maps between pointed spaces are also assumed to be pointed.

Theorem 1.7 ([8])**.** *Let K and P be countable* CW*-complexes and let a space L have finite homotopy groups. If $\text{map}\,(K, L) \cong 0$ and $[P, L] \neq 0$ then there exists a compactum X such that $P < e - \dim X \leq K$.*

Theorem 1.7 was formulated in [8] in a slightly different form. First, it was assumed in [8] that K and P are countable simplicial complexes. Since each countable CW-complex is homotopy equivalent to a countable simplicial complex we can replace simplicial complexes by CW-complexes. Secondly, it was assumed in [8] that $\text{map}\,(K, L) \cong 0$ and $|\pi_i(L)| < \infty, i = 0, 1, \ldots$ for any base point in L. This restriction can be omitted. Indeed, this is obvious if L is path connected. Let a pointed map $f : P \longrightarrow L$ be essential. If P is mapped by f into the path component of the base point of L then replace L by the path component of the base point and we are done. So the only case one needs to consider is when both P and L are not path connected. Then the condition $[K, L] = 0$ (derived from $\text{map}(K, L) \cong 0$) implies that K is connected. In this case one can define $X = [0, 1]$ which obviously satisfies $P <$e-$\dim X \leq K$.

We will need a more precise version of Theorem 1.7 (which was actually proved in [8]).

Theorem 1.8. *Let K, $P = P_0$ be countable* CW*-complexes, let a space L have finite homotopy groups and let* $\operatorname{map}(K, L) \cong 0$*. Let $P_1, \dots, P_n$ be* CW*-complexes and let maps $f_i : P_i \longrightarrow P_{i+1}, i = 0, 1, \dots, n-1$ and $f_n : P_n \longrightarrow L$ be such that $f = f_n \circ \cdots \circ f_0 : P \longrightarrow L$ is essential. Then there are a compactum X, a closed subset X' of X and a map $g : X' \longrightarrow P$ such that $e\text{-}\dim X \le K$ and the maps $g_0 = g$ and $g_i = f_{i-1} \circ \cdots \circ f_0 \circ g : X' \longrightarrow P_i$, $i = 1, 2, \dots, n$ do not extend over X. In particular $e\text{-}\dim X > P_i$ for every $i = 0, 1, \dots, n$.*

Theorem 1.8 was proved in [**8**] for the case $n = 0$, see the proof of Theorem 1.2(b) in [**8**]. The general case can easily be derived from the case $n = 0$. We recall that X and X' were constructed in [**8**] as the inverse limit $(X, X') = \varprojlim((M_j, N_j), p^j_{j-1})$ of a sequence of pairs of finite complexes (M_j, N_j), $j = 0, 1, \dots$ with bonding maps $p^j_{j-1} : (M_j, N_j) \longrightarrow (M_{j-1}, N_{j-1})$ such that N_0 is a finite subcomplex of P, $N_j = (p^j_{j-1})^{-1}(N_{j-1})$ and the map $p^j_0 = p^1_0 \circ \cdots \circ p^{j-1}_{j-2} \circ p^j_{j-1} : (M_j, N_j) \longrightarrow (M_0, N_0)$ has the property that $f \circ p^j_0|_{N_j} : N_j \longrightarrow L$ does not extend over M_j where $p^j_0|_{N_j} : N_j \longrightarrow N_0$ is considered as a map to P. Let $p : (X, X') \longrightarrow (M_0, N_0)$ be the projection. Consider $g_0 = p|_{X'} : X' \longrightarrow N_0$ as a map to P and let $g_i = f_{i-1} \circ \cdots \circ f_0 \circ g_0 : X' \longrightarrow P_i$, $i = 1, 2, \dots, n$. Then for every i the map g_i does not extend over X since otherwise for a sufficiently large j the map $(f_{i-1} \circ \cdots \circ f_0) \circ p^j_0|_{N_j} : N_j \longrightarrow P_i$ if $i \ge 1$ or the map $p^j_0|_{N_j} : N_j \longrightarrow P$ if $i = 0$ would extend over M_j and this would imply that $f \circ p^j_0|_{N_j} : N_j \longrightarrow L$ also extends over M_j. This contradiction proves Theorem 1.8.

Note that if L is a CW-complex (or a space homotopy equivalent to a CW-complex) then we can assume $P_{n+1} = L$ and get that $g_{n+1} = f_n \circ g_n : X' \longrightarrow L$ does not extend over X and hence $\text{e-}\dim X > L$ (cf. the remark at the end of [**8**]).

The following two theorems provide us with a very important class of CW-complexes to which Theorems 1.7 and 1.8 apply.

Theorem 1.9 (Miller's theorem (the Sullivan conjecture) [**10**])**.** *Let G be a finite group and L a finite* CW*-complex. Then* $\operatorname{map}(K(G, 1), L) \cong 0$.

Theorem 1.10 (Dydak-Walsh [**7**])**.** *Let L have finite homotopy groups. Then* $\operatorname{map}(K(\mathbb{Q}, 1), L) \cong 0$.

The Dranishnikov-Repovš example (Theorem 1.3) can be obtained as an application of Theorem 1.7 and Miller's theorem. Indeed, fix $m > 0$ and let $k \ge m$ be even. The homology groups of $\mathbb{R}P^k$ are finite and hence so are the homology groups of $\Sigma\mathbb{R}P^k$. Since $\Sigma\mathbb{R}P^k$ is simply connected the Hurewicz isomorphism theorem modulo the class of finite abelian groups ([**12**], Sec. 9.6) implies that the homotopy groups of $\Sigma\mathbb{R}P^k$ are finite and hence so are the homotopy groups of $\Omega\Sigma\mathbb{R}P^k$. The inclusion $i : \mathbb{R}P^m \longrightarrow \mathbb{R}P^k$ induces the nontrivial homomorphism $i_* : H_1(\mathbb{R}P^m) \longrightarrow H_1(\mathbb{R}P^k)$ and hence $\Sigma i : \Sigma\mathbb{R}P^m \longrightarrow \Sigma\mathbb{R}P^k$ is essential. Thus $[\mathbb{R}P^m, \Omega\Sigma\mathbb{R}P^k] = [\Sigma\mathbb{R}P^m, \Sigma\mathbb{R}P^k] \neq 0$. By the Sullivan conjecture (Theorem 1.9) $\mathrm{map}\,(K(\mathbb{Z}_2,1),\ \Sigma\mathbb{R}P^k) \cong 0$ and hence $\mathrm{map}\,(K(\mathbb{Z}_2,1),\ \Omega\Sigma\mathbb{R}P^k) \cong 0$. Then Theorem 1.7 applied to $K = K(\mathbb{Z}_2,1), P = \mathbb{R}P^m$ and $L = \Omega\Sigma\mathbb{R}P^k$ produces a compactum X_m with $\dim_{\mathbb{Z}_2} X_m \leq 1$ and e-dim$X_m > \mathbb{R}P^m$. Let X be the one point compactification of the disjoint union of X_m, $m = 1, 2, \ldots$ and we have constructed the Dranishnikov-Repovš example (Theorem 1.3).

More or less the same strategy is applied for proving Theorems 1.4, 1.5 and 1.6 but this time instead of a specific structure of the real projective spaces $\mathbb{R}P^m$ we need Lemma 2.1 which plays a key role in our proofs.

2. Proofs.

Lemma 2.1. *Let $m \geq 2$ and let A be a finite* CW*-complex with $H_m(A) \neq 0$. Then there exists a finite* CW*-complex B with finite homotopy groups such that $[A, B] \neq 0$. Moreover, if $0 \neq \alpha \in H_m(A)$ then B can be constructed such that there is a map $\phi : A \longrightarrow B$ with $\phi_*(\alpha) \neq 0$.*

Proof. By adjoining to A finitely many cells of dim$\leq m$ we can kill the homotopy groups $\pi_i(A)$ for $i = 0, 1, \ldots, m-1$. Clearly α will remain nonzero in this enlarged complex and hence without loss of generality we may assume that $\pi_i(A) = 0$ for $i = 0, 1, \ldots, m-1$.

Assume that α is of infinite order. By Hurewicz's isomorphism theorem we can adjoin an $(m+1)$-cell to A to kill the element 2α, leaving $\alpha \neq 0$. Thus we may assume that α is of finite order.

Let $z_1, \ldots, z_k \in H_m(A)$ be a maximal collection of elements of infinite order such that $t_1 z_1 + \cdots + t_k z_k$, $t_i \in \mathbb{Z}$ is of finite order if and only if $t_i = 0$ for all $1 \leq i \leq k$. By Hurewicz's isomorphism theorem attach to A k cells of dim $= m+1$ to kill z_i, $1 \leq i \leq k$. Then the elements of $H_m(A)$ of finite order, and α in particular, will remain untouched and $H_m(A)$ will become a finite group. Thus we may assume that $H_m(A)$ is finite.

Let $n = \dim A > m$. If $m+1 < n$ adjoin to A finitely many cells of $m+2 \leq$dim$\leq n$ to kill the homotopy groups $\pi_i(A)$ for $m+1 \leq i \leq n-1$. Then $H_m(A)$ remains unchanged and by the Hurewicz isomorphism theorem modulo the class of finite abelian groups we may assume that $H_i(A)$ are finite for $i < n$.

Now once again take a maximal collection $z_1, \ldots, z_k \in H_n(A)$ of elements of infinite order such that $t_1 z_1 + \cdots + t_k z_k$, $t_i \in \mathbb{Z}$ is of finite order only if $t_i = 0$ for all $1 \leq i \leq k$. Let $\psi : \pi_n(A) \longrightarrow H_n(A)$ be the Hurewicz homomorphism. By the Hurewicz isomorphism theorem modulo the class of finite abelian groups, cokerψ is finite and hence there are $t_i \neq 0$, $1 \leq i \leq k$ such that $t_i z_i \in \psi(\pi_n(A))$. Attach to A cells $C_1, \ldots, C_k$ of dim $= n+1$ to kill $t_1 z_1, \ldots, t_k z_k$ respectively. Then $H_n(A)$ will become a finite group.

Let $C = n_1C_1 + \cdots + n_kC_k$, $n_i \in \mathbb{Z}$ be an $(n+1)$-dimensional chain. Then $\partial C = t_1n_1z_1 + \cdots + t_kn_kz_k$ if z_i's are considered as cycles. Therefore $\partial C = 0$ only if $C = 0$ and hence $H_{n+1}(A) = 0$.

Thus after all the enlargements of A we get a simply connected finite CW-complex B with finite homology groups such that for the inclusion $\phi : A \longrightarrow B$, $\phi_*(\alpha) \neq 0$. Then the homotopy groups of B are finite and the lemma follows. □

Proof of Theorem 1.6. Fix a cyclic finite CW-complex P. Then $\Sigma^2 P$ is simply connected and cyclic and hence by Lemma 2.1 there exists a finite CW-complex B with finite homotopy groups such that $[\Sigma^2 P, B] \neq 0$. Then $[P, \Omega^2 B] = [\Sigma^2 P, B] \neq 0$ and $\Omega^2 B$ also has finite homotopy groups. Let $K = K(\mathbb{Q}, 1) \bigvee (\bigvee \{K(G, 1) : G \text{ is finite}\})$ be the wedge of $K(\mathbb{Q}, 1)$ and $K(G, 1)$'s over all possible (up to isomorphism) finite abelian groups G. Since there are only countably many non-isomorphic finite groups, K is a countable CW-complex. By the Sullivan conjecture and Theorem 1.10 map $(K, \Omega^2 B) \cong 0$. Apply Theorem 1.7 to K, P and $L = \Omega^2 B$ and construct a compactum X_P such that e-dim $X_P > P$, $\dim_{\mathbb{Q}} X_P \leq 1$ and $\dim_G X_P \leq 1$ for every finite abelian group G. e-dim$\leq K$ implies $\dim_{\mathbb{Q}} \leq 1$ and $\dim_G \leq 1$ for every finite abelian group G. Hence by Bockstein's theorem and inequalities $\dim_G X_P \leq 2$ for every abelian group G. Since there are only countably many finite CW-complexes of different homotopy types define X as the one point compactification of the disjoint union of X_P's over all possible (up to homotopy equivalence) cyclic finite CW-complexes P. Then X is the desired compactum. □

Proof of Theorem 1.5. By a couple (f, F) we mean a finite CW-complex F and a map $f : \mathbb{S}^2 \longrightarrow F$ such that $f_* : H_2(\mathbb{S}^2) \longrightarrow H_2(F)$ is nontrivial. Two couples (f, F) and (f', F') are said to be of the same homotopy type or homotopy equivalent if there is a homotopy equivalence $h : F \longrightarrow F'$ such that $h \circ f \cong f'$.

Let $K = K(\mathbb{Q}, 1) \bigvee (\bigvee \{K(\mathbb{Z}_p, 1) : p \text{ prime }\})$ and let $T = (f, F)$ be a couple. By Lemma 2.1 and Theorems 1.8, 1.9, 1.10 there are a compactum X_1^T, a closed subset X_s^T of X_1^T and a map $g_s^T : X_s^T \longrightarrow \mathbb{S}^2$ such that e-dim$X_1^T \leq K$ and $g_1^T = f \circ g_s^T : X_s^T \longrightarrow F$ does not extend over X_1^T. Let X^T be the quotient space of X_1^T obtained by replacing X_s^T by $\mathbb{S}^2$ according to the map g_s^T. Then $\mathbb{S}^2$ can be considered as a subspace $\mathbb{S}^2 \subset X^T$ of X^T such that the map $f : \mathbb{S}^2 \longrightarrow F$ does not extend over X^T. e-dim$\leq K$ implies $\dim_{\mathbb{Q}} \leq 1$ and $\dim_{\mathbb{Z}_p} \leq 1$ for every prime p. Hence by Bockstein's theorem and inequalities $\dim_G X_1^T \leq 2$ for every G and clearly the latter property also holds for X^T.

Let $\mathcal{T}$ be a countable family of couples which includes all possible homotopy types of couples. Let X be the set obtained from the disjoint union of $X^T, T \in \mathcal{T}$ by identifying all the spheres $\mathbb{S}^2$, that is $X = \cup\{X^T : T \in \mathcal{T}\}$

with $\mathbb{S}^2 = \cap\{X^T : T \in \mathcal{T}\}$. Endow X with a separable metric topology which agrees with the topology of X^T for each X^T. Then $\dim_G X \leq 2$ for every G.

We are going to show that X has the required properties. Let X' be a Hausdorff compactification of X. Since $\mathbb{S}^2 \subset X \subset X'$, e-dim$X' > P$ for every CW-complex P which is not simply connected. Now assume that P is simply connected but not 2-connected. Take $f' : \mathbb{S}^2 \longrightarrow P$ such that $f'_*(H_2(\mathbb{S}^2)) \neq 0$. Assume that f' extends to $f'' : X' \longrightarrow P$ and take a finite subcomplex F' of P such that $f''(X') \subset F'$. Consider f' and f'' as maps to F'. Then $T' = (f', F')$ is a couple and hence there is a couple $T = (f, F) \in \mathcal{T}$ which is homotopy equivalent to T', that is there is a homotopy equivalence $h : F' \longrightarrow F$ such that $f \cong h \circ f'$. By our construction f does not extend over X^T and hence neither does $h \circ f'$. On the other hand $h \circ f''|_{X^T}$ is an extension of $h \circ f'$. This contradiction proves the theorem. $\square$

Proof of Theorem 1.4. By a pair $T = (P_1, P_0)$ we mean a pair of finite CW-complexes $P_0 \subset P_1$ such that such that the inclusion $f_0 : P_0 \longrightarrow P_1$ is homologically essential. By Lemma 2.1 there are a finite CW-complex B with finite homotopy groups and a map $\phi : \Sigma^2 P_1 \longrightarrow B$ such that $\phi \circ (\Sigma^2 f_0) : \Sigma^2 P_0 \longrightarrow B$ is essential. Let $f_1 : P_1 \longrightarrow \Omega^2 B$ be the adjoint of ϕ. Then $f = f_1 \circ f_0 : P_0 \longrightarrow \Omega^2 B$ is the adjoint of $\phi \circ (\Sigma^2 f_0)$ and hence f is also essential. Define $K = K(\mathbb{Q}, 1) \bigvee (\bigvee \{K(\mathbb{Z}_p, 1) : p \text{ prime }\})$, $L = \Omega^2 B$ and apply Theorems 1.8, 1.9 and 1.10 to construct a compactum X^T, a closed subset X_0^T of X^T and a map $g^T : X_0^T \longrightarrow P_0$ such that e-dim$X^T \leq K$ and g^T does not extend over X^T as a map to P_1.

Let $T = (P_1, P_0)$ be a pair. One can find a countable collection $\mathcal{Q}^T$ of maps from P_0 to P_0 such that each map from P_0 to P_0 is homotopic to some element of $\mathcal{Q}^T$. Consider $\mathcal{Q}^T$ as a discrete space. Let $\mathcal{T}$ be a countable collection of pairs which includes all possible homotopy types of pairs and define X as the disjoint union of $X^T \times \mathcal{Q}^T$, $T \in \mathcal{T}$. Clearly X is separable metrizable and locally compact and e-dim$X \leq K$. By the Bockstein theorem and inequalities $\dim_G X \leq 2$ for every abelian G.

Let us show that e-dim$\beta X > P$ for every non-contractible simply connected CW-complex P. Take a finite subcomplex P' of P supporting a nontrivial homology cycle in P. Then for any finite subcomplex P'' of P containing P' the inclusion of P' into P'' is homologically essential. Let $\mathcal{T}' = \{T : T = (P_0, P_1) \in \mathcal{T}$ such that $P' \cong P_0\}$ and let $X' = \cup\{X_0^T \times \mathcal{Q}^T : T \in \mathcal{T}'\}$. Then X' is a closed subset of X. For each $T = (P_1, P_0) \in \mathcal{T}'$ fix a homotopy equivalence $q^T : P_0 \longrightarrow P'$.

Define $f' : X' \longrightarrow P'$ by $f'(x, q) = q^T(q(g^T(x)))$ for $(x, q) \in X_0^T \times \mathcal{Q}^T, T \in \mathcal{T}'$.

Consider $\beta X'$ as a closed subset of βX and let $\beta f' : \beta X' \longrightarrow P'$ be the extension of f'. Let us show that $\beta f'$ considered as a map to P does not extend over βX. Assume that there is an extension $h : \beta X \longrightarrow P$ of $\beta f'$ and let P'' be a finite subcomplex of P containing both $h(\beta X)$ and P'. Take $T = (P_1, P_0) \in \mathcal{T}'$ such that $(P_1, P_0) \cong (P'', P')$ and let $q'' : (P'', P') \longrightarrow (P_1, P_0)$ be a homotopy equivalence. Let $q \in \mathcal{Q}^T$ be a homotopy inverse of $q'' \circ q^T : P_0 \longrightarrow P_0$, that is $q'' \circ q^T \circ q : P_0 \longrightarrow P_0$ is homotopic to the identity map.

From now we identify $X_0^T \times \{q\}$ and $X^T \times \{q\}$ with X_0^T and X^T respectively. Then the map $r = q'' \circ f'|_{X_0^T} = q'' \circ q^T \circ q \circ g^T : X_0^T \longrightarrow P_0$ is homotopic to g^T and hence by our construction r does not extend over X^T as a map to P_1. On the other hand $q'' \circ h|_{X^T} : X^T \longrightarrow P_1$ is an extension of r where h is considered as a map to P''. This contradiction shows that e-dim$\beta X > P$.

Now, by adding a 2-dimensional disk to X, we get that e-dim$\beta X > K$ for every non-simply connected CW-complex K. Clearly all the cohomological dimensions of X remain ≤ 2 and the theorem follows. □

Remarks. An interesting property of Theorems 1.4 and 1.5 is that the CW-complexes are not required to be countable and fixed in advance. This was achieved by using the so-called Rubin-Schapiro trick [**9**]. Another interesting point is that the space X constructed in the proof of Theorem 1.6 has the property $\dim_G X^n \leq n+1$ for every G and n. And finally let us note that it would be interesting to find out if Theorem 1.6 holds for non-contractible (not necessarily cyclic) finite complexes P.

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Received March 8, 2000 and revised January 25, 2001. The author was supported in part by a JSPS postdoctoral fellowship for foreign researchers.

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