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# A LIOUVILLE-TYPE THEOREM FOR SEMILINEAR ELLIPTIC SYSTEMS VIA MOVING SPHERES

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**In this paper, we consider second order semilinear elliptic systems of the form $-\Delta u = a(x)v^p$ and $-\Delta v = b(x)u^q$ in $\mathbb{R}^N$ for $N \geq 3$, where $p, q > 0$ and $a, b \in C(\mathbb{R}^N)$. We prove a new Liouville-type theorem for the system under appropriate conditions on the nonlinearity.**

## 1. Introduction

We consider second order semilinear elliptic systems of the form

$$\left.\begin{aligned} -\Delta u &= a(x)v^p \\ -\Delta v &= b(x)u^q \end{aligned}\right\} \quad \text{in } \mathbb{R}^N \tag{1-1}$$

for $N \geq 3$, where $p, q > 0$ and $a, b \in C(\mathbb{R}^N)$.

The problem of existence and nonexistence of positive solutions of scalar elliptic equation

$$-\Delta u = K(x)u^p \quad \text{in } \mathbb{R}^N \tag{1-2}$$

has been investigated by many authors, see [Ding and Ni 1985; Gidas and Spruck 1981; Kusano and Naito 1987; Lin 1998; Ni 1982]. For $p = (N+2)/(N-2)$, we note that (1-2) has a geometric root. Given a smooth positive function $K$ defined on a Riemannian manifold $(M, g_0)$ of dimension $N \geq 3$, we ask whether there exists a metric $g$ conformal to $g_0$ such that $K$ is the scalar curvature of the new metric $g$. Let $g = u^{4/(N-2)}g_0$ for some positive function $u$; then the problem reduces to find solutions of the equation

$$\tfrac{4(N-1)}{N-2}\Delta_{g_0}u - k_0 u + K u^{(N+2)/(N-2)} = 0 \quad \text{in } M, \tag{1-3}$$

where $\Delta_{g_0}$ is the Laplace–Beltrami operator on $M$ and $k_0$ is the scalar curvature of $(M, g_0)$. In the special case that $M = R^N$ and $g_0$ is the standard metric of $R^N$, we have $k_0 \equiv 0$ and Equation (1-3) reduces to (1-2) with $p = (N+2)/(N-2)$. For

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more background material and other related problems, we refer to [Ni 1982; Lin 1998] and the references therein.

When $a(x)$ and $b(x)$ are positive constants, the system (1-1) becomes the Lane–Emden system

$$\left.\begin{aligned} -\Delta u &= v^p \\ -\Delta v &= u^q \end{aligned}\right\} \quad \text{in } \mathbb{R}^N. \tag{1-4}$$

There are some nonexistence results for positive solutions of the system (1-4); see [Busca and Manásevich 2002; de Figueiredo and Felmer 1994; Mitidieri 1993; 1996; Serrin and Zou 1994; 1996].

Next we turn our attention to system (1-1). As far as the author knows, there are no results that contain nonexistence criteria of positive solutions to (1-1) except for the following result of Mitideri [1996]. Let $a(x) = a(|x|)$ and $b(x) = b(|x|)$.

**Theorem.** *Suppose that $N \geq 3$ and $p, q > 1$ and let $a, b \in [0, +\infty) \to [0, +\infty)$ be functions such that*

(i) $a, b \in C[0, +\infty) \bigcap C^1(0, +\infty)$ *and* $a(r), b(r) > 0$ *for* $r > 0$;

(ii) $(a(r)r^\delta)', \ (b(r)r^\delta)' \geq 0$ *and* $r > 0$ *with*

$$\delta = \frac{2(p+1)(q+1) - N(pq-1)}{p+q+2};$$

(iii) $\lim_{r\to\infty} a(r)r^\delta = \lim_{r\to\infty} a(r)r^\delta = +\infty$.

*Then the problem (1-1) has no positive radial solutions.*

In this work, we consider nonradial solutions of (1-1), that is, $a(x)$ and $b(x)$ need not be radially symmetric. We generalize Mitidieri's result partially here. Our result is the following.

**Theorem 1.1.** *Suppose that $N \geq 3$ and $p, q > 1$ and that the nonnegative functions $a, b$ satisfy*

(i) $|x|^{\delta_1} a(x)$ *and* $|x|^{\delta_2} b(x)$ *are nondecreasing along each ray* $\{t\xi : t \geq 0\}$ *for any unit vector* $\xi$ *in* $\mathbb{R}^N$, *where*

$$\delta_1 = \tfrac{1}{2}(N + 2 - p(N-2)) \quad \text{and} \quad \delta_2 = \tfrac{1}{2}(N + 2 - q(N-2)),$$

(ii) $\lim_{|x|\to\infty} |x|^{\delta_1} a(x) = \lim_{|x|\to\infty} |x|^{\delta_2} b(x) = +\infty$.

*Then the system (1-1) has no positive classical solutions.*

We can easily obtain a corollary of Theorem 1.1.

**Corollary 1.2.** *If $1 < p, q < (N+2)/(N-2)$, then the system (1-4) has no positive classical solutions.*

We note that Figueiredo and Felmer [1994] proved Corollary 1.2, among other things, by using the moving plane method.

In the proof of Theorem 1.1, we use the method of moving spheres, a variant of the method of moving planes. Roughly speaking, we make reflection with respect to spheres instead of planes. The method of moving spheres was used in [Chou and Chu 1993; Padilla 1994; Chen and Li 1995; Li and Zhu 1995; Li and Zhang 2003]. Li and Zhang made significant simplifications and proved some Liouville type theorems for a single equation. The proof of our theorem is along the lines of the works cited above.

The method of moving planes was first introduced by Alexandrov [1958] and then used by several authors: Serrin [1971]; Gidas, Ni and Nirenberg [1979, 1981]; and Berestycki and Nirenberg [1988, 1991]. This method has become a powerful tool in the study of nonlinear partial differential equations.

The paper is organized as follows. In Section 2, we give some lemmas that are used in the proof of Theorem 1.1, which is then proved in Section 3.

## 2. Preliminary lemmas

For $\lambda > 0$, consider the Kelvin transformation of $u$ and $v$ for $x \in \mathbb{R}^N \setminus \{0\}$:

$$u_\lambda(x) = \frac{\lambda^{N-2}}{|x|^{N-2}} u\Big(\frac{\lambda^2}{|x|^2}x\Big) \quad \text{and} \quad v_\lambda(x) = \frac{\lambda^{N-2}}{|x|^{N-2}} v\Big(\frac{\lambda^2}{|x|^2}x\Big).$$

Our first lemma says that we can initiate the method of moving spheres.

**Lemma 2.1.** *There exists a $\lambda_0 > 0$ such that $u_\lambda(x) \le u(x)$ and $v_\lambda(x) \le v(x)$ for all $0 < \lambda < \lambda_0$ and $|x| \ge \lambda$.*

*Proof.* Clearly, there exists an $r_0 > 0$ such that

$$\frac{d}{dr}\big(r^{(N-2)/2} u(r,\theta)\big) > 0 \quad \text{for all } 0 < r < r_0 \text{ and } \theta \in S^{N-1}.$$

Consequently,

$$u_\lambda(x) < u(x) \quad \text{for all } 0 < \lambda < |x| < r_0.$$

By the superharmonicity of $u$ and the maximum principle,

$$u(x) \ge (\min_{\partial B_{r_0}} u) r_0^{N-2} |x|^{2-N} \quad \text{for all } |x| \ge r_0.$$

Let

$$\lambda_1 = r_0 \Big(\frac{\min_{\partial B_{r_0}} u}{\max_{\overline{B}_{r_0}} u}\Big)^{1/(N-2)} \le r_0.$$

Then for every $0 < \lambda < \lambda_1$ and $|x| \ge r_0$, we have

$$u_\lambda(x) \le \frac{\lambda_1^{N-2}}{|x|^{N-2}} \max_{\overline{B}_{r_0}} u \le \frac{r_0^{N-2} \min_{\partial B_{r_0}} u}{|x|^{N-2}} \le u(x).$$

Similarly, there exists $\lambda_2 > 0$ such that for every $0 < \lambda < \lambda_2$, we have

$$v_\lambda(x) \le v(x) \quad \text{for } |x| \ge \lambda.$$

We choose $\lambda_0 = \min\{\lambda_1, \lambda_2\}$. Thus, for every $0 < \lambda < \lambda_0$,

$$u_\lambda(x) \le u(x) \quad \text{and} \quad v_\lambda(x) \le v(x) \quad \text{for } |x| \ge \lambda. \qquad \square$$

Set

$$\bar{\lambda} = \sup\{\mu > 0 : u_\lambda(x) \le u(x) \text{ and } v_\lambda(x) \le v(x) \text{ for all } |x| \ge \lambda \text{ and } 0 < \lambda \le \mu\}.$$

By Lemma 2.1, $\bar{\lambda}$ is well defined, and $0 < \bar{\lambda} \le +\infty$.

**Lemma 2.2.** *If $\bar{\lambda} < +\infty$, then $u_{\bar{\lambda}}(x) \equiv u(x)$ and $v_{\bar{\lambda}}(x) \equiv v(x)$ in $\mathbb{R}^N \setminus \{0\}$.*

*Proof.* Let $\Sigma_\lambda = \{x : |x| > \lambda\}$. Clearly it suffices to show

$$u_{\bar{\lambda}} \equiv u \quad \text{and} \quad v_{\bar{\lambda}} \equiv v \quad \text{in } \Sigma_{\bar{\lambda}}.$$

We first prove $u_{\bar{\lambda}} \equiv u$. We prove it by contradiction. Supposing $u_{\bar{\lambda}} \not\equiv u$ in $\Sigma_{\bar{\lambda}}$, we know from the definition of $\bar{\lambda}$ that

$$u_{\bar{\lambda}} \le u \quad \text{and} \quad v_{\bar{\lambda}} \le v \quad \text{in } \Sigma_{\bar{\lambda}}.$$

From (1-1), a direct calculation yields

$$-\Delta u_\lambda = \left(\frac{\lambda}{|x|}\right)^{N+2-p(N-2)} a\left(\frac{\lambda^2}{|x|^2}x\right) v_\lambda^p.$$

Thus, by condition (i) of Theorem 1.1, we have

$$\begin{aligned}
-\Delta(u - u_{\bar{\lambda}}) &= a(x)v^p - \left(\frac{\bar{\lambda}}{|x|}\right)^{N+2-p(N-2)} a\left(\frac{\bar{\lambda}^2}{|x|^2}x\right) v_{\bar{\lambda}}^p \\
&= |x|^{-\delta_1}\left(|x|^{\delta_1} a(x)v^p - |x|^{\delta_1}\left(\frac{\bar{\lambda}}{|x|}\right)^{N+2-p(N-2)} a\left(\frac{\bar{\lambda}^2}{|x|^2}x\right) v_{\bar{\lambda}}^p\right) \\
&= |x|^{-\delta_1}\left(|x|^{\delta_1} a(x)v^p - \left(\frac{\bar{\lambda}^2}{|x|}\right)^{\delta_1} a\left(\frac{\bar{\lambda}^2}{|x|^2}x\right) v_{\bar{\lambda}}^p\right) \\
&\ge 0 \quad \text{in } \Sigma_{\bar{\lambda}}.
\end{aligned}$$

By the maximum principle, $u - u_{\bar{\lambda}} > 0$ in $\Sigma_{\bar{\lambda}}$. Thus, by the Hopf lemma and the compactness of $\partial B_{\bar{\lambda}}$, there exists a positive constant $b$ such that

$$\frac{d}{dr}(u - u_{\bar{\lambda}})\Big|_{\partial B_{\bar{\lambda}}} > b > 0.$$

By the continuity of $\nabla u$, there exists $R > \bar{\lambda}$ such that

$$\frac{d}{dr}(u - u_\lambda) > \frac{b}{2} > 0 \quad \text{for } \bar{\lambda} \le \lambda \le R \text{ and } \lambda \le r \le R.$$

Consequently, since $u - u_\lambda \equiv 0$ on $\partial B_\lambda$,

$$u(x) > u_\lambda(x) \quad \text{for } \bar\lambda \le \lambda \le R \text{ and } \lambda \le |x| \le R. \tag{2-1}$$

Set $c = \min_{\partial B_R}(u - u_{\bar\lambda}) > 0$. It follows from the superharmonicity of $u - u_{\bar\lambda}$ that $u - u_{\bar\lambda} \ge cR^{N-2}/|x|^{N-2}$ for $|x| \ge R$. Therefore

$$u - u_\lambda \ge \frac{cR^{N-2}}{|x|^{N-2}} - (u_\lambda - u_{\bar\lambda}) \quad \text{for } |x| \ge R. \tag{2-2}$$

By the uniform continuity of $u$ on $\bar B_R$, there exists an $\varepsilon \in (0, R - \bar\lambda)$ such that for all $\bar\lambda \le \lambda \le \bar\lambda + \varepsilon$,

$$\left|\lambda^{N-2} u\Big(\frac{\lambda^2 x}{|x|^2}\Big) - \bar\lambda^{N-2} u\Big(\frac{\bar\lambda^2 x}{|x|^2}\Big)\right| < \frac{cR^{N-2}}{2} \quad \text{for } |x| \ge R.$$

It follows from (2-2) and the above inequality that

$$u(x) - u_\lambda(x) > 0 \quad \text{for } \bar\lambda \le \lambda \le \bar\lambda + \varepsilon \text{ and } |x| \ge R.$$

Thus, (2-1) and (2-2) are in contradiction with the definition of $\bar\lambda$. Similarly, we can prove that $v_{\bar\lambda} \equiv v$ in $\Sigma_{\bar\lambda}$. □

**Lemma 2.3.** $\bar\lambda < +\infty$.

*Proof.* Suppose by way of contradiction that $\bar\lambda = +\infty$. By the definition of $\bar\lambda$,

$$u(x) \ge u_\lambda(x) \quad \text{for } |x| \ge \lambda \text{ for all } \lambda > 0. \tag{2-3}$$

Set $|x| = \lambda^2$. Then, from (2-3),

$$u(x) \ge \lambda^{-(N-2)} u(x/\lambda^2) \ge c_1 |x|^{-(N-2)/2} \quad \text{for } |x| \ge 1,$$

where $c_1 = \min_{\partial B_1} u(x)$. Similarly, we have

$$v(x) \ge c_2 |x|^{-(N-2)/2} \quad \text{for } |x| \ge 1,$$

where $c_2 = \min_{\partial B_1} v(x)$. Rewrite system (1-1) into

$$\Delta u + \tilde a(x) v = 0 \quad \text{and} \quad \Delta v + \tilde b(x) u = 0,$$

where $\tilde a(x) = a(x) v^{p-1}$ and $\tilde b(x) = b(x) u^{q-1}$. For any $M > 0$, by condition (ii) of Theorem 1.1, there exists $R_0 \ge 1$ such that

$$\tilde a(x)|x|^2 \ge M c_2^{1-p} |x|^{2-\delta_1} (c_2 |x|^{-(N-2)/2})^{p-1} = M \quad \text{for } |x| \ge R_0.$$

Thus

$$\lim_{|x|\to+\infty} \tilde a(x)|x|^2 = +\infty. \tag{2-4}$$

Similarly, we have

$$\lim_{|x|\to+\infty} \tilde{b}(x)|x|^2 = +\infty. \tag{2-5}$$

For $|y| \le 1/2$, set $\tilde{u}(y) = u(x+|x|y)$ and $\tilde{v}(y) = v(x+|x|y)$. Then $\tilde{u}$ and $\tilde{v}$ satisfy

$$\left.\begin{aligned} \Delta\tilde{u} + |x|^2\tilde{a}(x+|x|y)\tilde{v} = 0 \\ \Delta\tilde{v} + |x|^2\tilde{b}(x+|x|y)\tilde{u} = 0 \end{aligned}\right\} \quad \text{in } B_{1/2}. \tag{2-6}$$

in $B_{1/2}$. Set $f(x) = \inf_{|y|\le 1/2}|x|^2\tilde{a}(x+|x|y)$ and $g(x) = \inf_{|y|\le 1/2}|x|^2\tilde{b}(x+|x|y)$. From (2-6),

$$\left.\begin{aligned} -\Delta\tilde{u} \ge f(x)\tilde{v} \\ -\Delta\tilde{v} \ge g(x)\tilde{u} \end{aligned}\right\} \quad \text{in } B_{1/2}. \tag{2-7}$$

Let $\phi \in H_0^1(B_{1/2})$ be the positive eigenfunction corresponding to the first eigenvalue $\lambda_1$ of $(-\Delta, H_0^1(B_{1/2}))$. Multiplying both sides of the first inequality in (2-7) by $\phi$ and integrating the obtained inequality over $B_{1/2}$, we have

$$\begin{aligned} f(x)\int_{B_{1/2}} \tilde{v}\phi\,dy &\le \int_{B_{1/2}} -\Delta\tilde{u}\cdot\phi\,dy \\ &\le \int_{B_{1/2}} -\Delta\phi\cdot\tilde{u}\,dy = \lambda_1\int_{B_{1/2}} \tilde{u}\phi\,dy. \end{aligned} \tag{2-8}$$

Similarly, by the second inequality in (2-7), we obtain

$$g(x)\int_{B_{1/2}} \tilde{u}\phi\,dy \le \lambda_1\int_{B_{1/2}} \tilde{v}\phi\,dy. \tag{2-9}$$

From (2-8) and (2-9), $f(x)g(x) \le \lambda_1^2$. However, by (2-4) and (2-5),

$$\lim_{|x|\to+\infty} |x|^2\tilde{a}(x+|x|y) = \lim_{|x|\to+\infty} |x|^2\tilde{b}(x+|x|y) = +\infty,$$

uniformly for $|y| \le 1/2$. This yields a contradiction. □

## 3. Proof of Theorem 1.1

By Lemmas 2.2 and 2.3, we have

$$u_{\bar{\lambda}}(x) \equiv u(x) \quad \text{and} \quad v_{\bar{\lambda}}(x) \equiv v(x) \quad \text{in } \mathbb{R}^N \setminus \{0\}.$$

Since

$$\begin{aligned} -\Delta u_\lambda &= \left(\frac{\lambda}{|x|}\right)^{N+2-p(N-2)} a\left(\frac{\lambda^2}{|x|^2}x\right)v_\lambda^p, \\ -\Delta v_\lambda &= \left(\frac{\lambda}{|x|}\right)^{N+2-q(N-2)} b\left(\frac{\lambda^2}{|x|^2}x\right)u_\lambda^q, \end{aligned}$$

we have by (1-1)

$$\left(\frac{\bar{\lambda}}{|x|}\right)^{N+2-p(N-2)} a\left(\frac{\bar{\lambda}^2}{|x|^2}x\right) \equiv a(x) \quad \text{in } \mathbb{R}^N \setminus \{0\},$$

that is,

$$\left(\frac{\bar{\lambda}^2}{|x|}\right)^{\delta_1} a\left(\frac{\bar{\lambda}^2}{|x|^2}x\right) \equiv |x|^{\delta_1} a(x) \quad \text{in } \mathbb{R}^N \setminus \{0\}. \tag{3-1}$$

For large $|x|$, we have, by condition (ii) of Theorem 1.1,

$$\left(\frac{\bar{\lambda}^2}{|x|}\right)^{\delta_1} a\left(\frac{\bar{\lambda}^2}{|x|^2}x\right) < |x|^{\delta_1} a(x),$$

which contradicts (3-1). □

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