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FENG RONG

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ABSOLUTELY ISOLATED SINGULARITIES OF HOLOMORPHIC MAPS OF $\mathbb{C}^n$ TANGENT TO THE IDENTITY

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Let f be a holomorphic map of $\mathbb{C}^n$ tangent to the identity, with an absolutely isolated singularity. We show that there exists a finite blow-up sequence which reduces f to a map with only simple singularities.

1. Introduction

In discrete local holomorphic dynamics, an often-studied case is when a holomorphic map f of $\mathbb{C}^n$ is tangent to the identity at a fixed point p, that is, $df_p = \mathrm{id}$. When $n = 1$, there is the well-known Leau–Fatou flower theorem [Milnor 2006]. Abate [2001] generalized this theorem to dimension two when p is an isolated fixed point of f. There are three main ingredients in his proof. The first is a positive result on generic maps [Hakim 1998]. The second is a reduction theorem that reduces the singularities of a map into simpler and irreducible ones. The third is an index associated to a singularity of a map. The last two ingredients are inspired by studies in continuous local holomorphic dynamics [Camacho and Sad 1982].

Here, we prove a similar reduction theorem for holomorphic maps in higher dimensions having only absolutely isolated singularities (or AIS; see Section 2 for the definition). More precisely, we have the following theorem (see Section 3 for the definition of a simple singularity).

Theorem 1.1. *Let f be a holomorphic map of $\mathbb{C}^n$ tangent to the identity at an isolated fixed point p. Assume that p is an absolutely isolated singularity of f. Then after finitely many blow-ups, we have a map with only finitely many simple singularities.*

Absolutely isolated singularities of holomorphic vector fields have been studied by Camacho, Cano and Sad [1989] and Tome [1997].

In Section 2, we introduce basic concepts and definitions and finish with the first stage of the reduction. In Section 3, we give the definition of a simple singularity and finish with the second stage of the reduction, thus proving Theorem 1.1.

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2. Nonnilpotent reduction

Let M be an n-dimensional complex manifold, and let f be a holomorphic self-map of M with $p \in M$ as a fixed point. Assume that f is tangent to the identity at p, that is $df_p = \mathrm{id}$. In local coordinates centered at p, write $f = (f_1, \dots, f_n)$, with $f_j(z) = z_j + g_j(z)$ for $1 \le j \le n$. Let $g_j = P_{0,j} + P_{1,j} + \cdots$, with $\deg P_{i,j} = i$ or $P_{i,j} \equiv 0$, be the homogeneous expansion of g_j for $1 \le j \le n$. The *order* of f at p is $\nu(f) = \min\{\nu(g_1), \dots, \nu(g_n)\}$, where $\nu(g_j)$ is the least $i \ge 0$ such that $P_{i,j}$ is not identically zero. We always assume that $\nu(f) < \infty$. Set $l = \gcd(g_1, \dots, g_n)$ and $g_j = l g_j^{\mathrm{o}}$, with both l and g_j^{o} defined up to units in $\mathbb{O}_{M,p}$. Let $g_j^{\mathrm{o}} = P_{0,j}^{\mathrm{o}} + P_{1,j}^{\mathrm{o}} + \cdots$ be the homogeneous expansion of g_j^{o} for $1 \le j \le n$. The *pure order* of f at p is $\nu_{\mathrm{o}}(f) = \min\{\nu(g_1^{\mathrm{o}}), \dots, \nu(g_n^{\mathrm{o}})\}$. We say that p is a *singular point* or a *singularity* of f if $\nu_{\mathrm{o}}(f) \ge 1$.

Let $P = (P_1, \dots, P_n)$ be an n-tuple of homogeneous polynomials of degree ν in $\mathbb{C}^n$. A *characteristic direction* for P is a vector $v \in \mathbb{P}^{n-1}$ such that $P(v) = \lambda v$ for some $\lambda \in \mathbb{C}$. A *characteristic direction* for f at p is a characteristic direction for $P_{\nu(f)} = (P_{\nu(f),1}, \dots, P_{\nu(f),n})$. A *singular direction* for f at p is a characteristic direction for $P_{\nu_{\mathrm{o}}(f)}^{\mathrm{o}} = (P_{\nu_{\mathrm{o}}(f),1}^{\mathrm{o}}, \dots, P_{\nu_{\mathrm{o}}(f),n}^{\mathrm{o}})$. The set of singular directions is clearly an algebraic subvariety of $\mathbb{P}^{n-1}$. If the maximal dimension of the irreducible components of this subvariety is k, we say that f is *k-dicritical* at p. If $k = 0$, we say that f is *nondicritical* at p. If $k = n-1$, we say that f is *dicritical* at p.

Let $\pi : \tilde{M} \to M$ be the blow-up of M at p. Then there exists a unique map $\tilde{f}$, the *blow-up* of f at p, such that $\pi \circ \tilde{f} = f \circ \pi$; see [Abate 2000].

Definition 2.1. Let $p \in M$, and write $p = p(0)$, $M = M(0)$ and $f = f(0)$. If, for any sequence

$$M(0) \xleftarrow{\pi(1)} M(1) \longleftarrow \cdots \xleftarrow{\pi(N)} M(N)$$

of blow-ups, where $f(i)$ is the blow-up of $f(i-1)$ and the center of each $\pi(i)$ is a singularity $p(i-1)$ of $f(i-1)$, the last blow-up map $f(N)$ has finitely many singularities, then we say p is an *absolutely isolated singularity* (or AIS) of f.

By [Abate and Tovena 2003, Lemma 2.2], if p is not dicritical then a direction $v \in \mathbb{P}^{n-1}$ is singular for f if and only if it is a singularity of $\tilde{f}$. Therefore if $p = p(0)$ is an AIS, then each $p(i)$ for $i \ge 0$ is either nondicritical or dicritical.

Remark 2.2. It follows from the definition that if $\nu(g_j^{\mathrm{o}}) > \nu_{\mathrm{o}}(f)$ for more than one j at p, then p is not an absolutely isolated singularity.

We define *pure intersection index* of f at p by $I(f; p) := I(g_1^{\mathrm{o}}, \dots, g_n^{\mathrm{o}}; p)$, where $I(\cdot, \dots, \cdot; p)$ denotes the intersection multiplicity for germs in $\mathbb{O}_{M,p}$; see [Fulton 1998]. If f is the blow-up map at a nondicritical singularity, one can choose local coordinates such that the exceptional divisor S is given by $\{z_1 = 0\}$ and $g_1^{\mathrm{o}}(z) = z_1 h_1(z)$. Then we define the *adapted intersection index* of f at p

by $I(f, S; p) := I(h_1, g_2^o, \dots, g_n^o; p)$ and the *adapted multiplicity* of f at p by $\mu(f, S, p) := I(z_1, g_2^o, \dots, g_n^o; p)$. As in [Abate 2001, Lemma 2.2], one readily checks that the numerical invariants above are well defined.

Lemma 2.3 [Abate and Tovena 2003, Lemma 2.1]. *Let M be an n-dimensional complex manifold, and let f be a holomorphic self-map of M with $p \in M$ as an isolated singularity. If f is nondicritical at p, then*

$$\nu^{n-1} + \nu^{n-2} + \cdots + 1 = \sum_{q \in S} \mu(\tilde{f}, S; q),$$

where $\nu = \nu_o(f)$, $\tilde{f}$ is the blow-up map at p and S is the exceptional divisor.

The following proposition generalizes [Abate 2001, Lemma 2.3].

Proposition 2.4. *With the same assumptions and notations as in Lemma 2.3*,

$$I(f; p) = \nu^n - \nu^{n-1} - \cdots - 1 + \sum_{q \in S} I(\tilde{f}; q).$$

Proof. Since p is nondicritical, we can assume, up to a linear change of coordinates, that $\nu(g_1^o) = \cdots = \nu(g_n^o) = \nu$ and all the singularities of $\tilde{f}$ are contained in the chart $w_1 = z_1$ and $w_j = z_j/z_1$ for $2 \le j \le n$. Let π be the blow-up and write $\hat{g}_j^o = g_j^o \circ \pi / w_1^\nu$ for $1 \le j \le n$. Then

$$\tilde{g}_1^o = w_1 \hat{g}_1^o \quad \text{and} \quad \tilde{g}_j^o = (\hat{g}_j^o - w_j \hat{g}_1^o)/(1 + w_1^{\nu-1} \hat{g}_1^o) \quad \text{for } 2 \le j \le n.$$

By the basic properties of the intersection multiplicity,

$$\begin{aligned} I(\tilde{f}; q) &= I(\tilde{g}_1^o, \tilde{g}_2^o, \dots, \tilde{g}_n^o; q) \\ &= I(\hat{g}_1^o, \tilde{g}_2^o, \dots, \tilde{g}_n^o; q) + I(w_1, \tilde{g}_2^o, \dots, \tilde{g}_n^o; q) \\ &= I(\tilde{f}, S; q) + \mu(\tilde{f}, S; q) \end{aligned} \tag{2-1}$$

and

$$\begin{aligned} I(f; p) &= I(g_1^o, g_2^o, \dots, g_n^o; p) \\ &= \nu^n + \textstyle\sum_{q \in S} I(\hat{g}_1^o, \hat{g}_2^o, \dots, \hat{g}_n^o; q) \\ &= \nu^n + \textstyle\sum_{q \in S} I(\hat{g}_1^o, \hat{g}_2^o - w_2 \hat{g}_1^o, \dots, \hat{g}_n^o - w_n \hat{g}_1^o; q) \\ &= \nu^n + \textstyle\sum_{q \in S} I(\tilde{f}, S; q). \end{aligned} \tag{2-2}$$

The desired equality then follows from (2-1), (2-2) and Lemma 2.3. □

Lemma 2.5. *Let p be a dicritical singularity of f, and let $\tilde{f}$ be the blow-up of f at p. Let S be the exceptional divisor of the blow-up.*

(a) *$P^o_{\nu_o(f),j} = z_j \cdot R$ for $1 \le j \le n$, where R is a homogeneous polynomial of degree $\nu_o(f) - 1$.*

(b) *The singularities of $\tilde{f}$ in $S \simeq \mathbb{P}^{n-1}$ are contained in the subset*

$$\{[w_1 : \cdots : w_n] \in \mathbb{P}^{n-1} : \ R(w_1, \ldots, w_n) = 0\}.$$

(c) *The singularities of $\tilde{f}$ in S are not dicritical.*

(d) *The pure order of $\tilde{f}$ at any of its singularities in S is less than or equal to $\nu_o(f) - 1$. In particular, if $\nu_o(f) = 1$, then $\tilde{f}$ has no singularities in S.*

Proof. Set $\nu = \nu_o(f)$. In the canonical coordinates $[w_1 : \cdots : w_n]$ centered at $[1 : 0 : \cdots : 0]$, we have

$$\tilde{f}_j(w) = \begin{cases} w_1 + \tilde{l} w_1^{\nu}\big(P^{o}_{\nu,1}(1, w_2, \ldots, w_n) + O(w_1)\big) & \text{if } j = 1, \\ w_j + \tilde{l} w_1^{\nu-1}(P^{o}_{\nu,j}(1, w_2, \ldots, w_n) - w_j P^{o}_{\nu,1}(1, w_2, \ldots, w_n) + O(w_1)), & \\ & \text{if } j \neq 1. \end{cases}$$

By definition, p is a dicritical singularity of f if and only if

$$P^{o}_{\nu,j}(1, w_2, \ldots, w_n) - w_j P^{o}_{\nu,1}(1, w_2, \ldots, w_n) \equiv 0 \quad \text{for all } 2 \leq j \leq n.$$

This proves (a).

We now have $\tilde{g}^{o}_1(w) = R(1, w_2, \ldots, w_n) + O(w_1)$. Then (b) and (d) are evident. Since $w_1 \nmid R(1, w_2, \ldots, w_n)$, (c) follows from (a). □

Proposition 2.6. *Let p be an absolutely isolated singularity of f. Then there exists a finite sequence of blow-ups such that the final blow-up map only has isolated singularities of pure order equal to one.*

Proof. The pure order is strictly decreasing if p is nondicritical and $\nu_o(f) > 1$ by Proposition 2.4, or if p is dicritical by Lemma 2.5(d). □

We can now focus our attention on singularities of pure order one. The *eigenvalues* of f at a singularity p are by definition the eigenvalues of the linear part of $g^o = (g^o_1, \ldots, g^o_n)$. It is easy to see that they are uniquely determined up to a nonzero scalar multiple and are independent of the coordinates once l is chosen. We say that p is a *nonnilpotent* singularity of f if f has at least one nonzero eigenvalue at p. Otherwise, we say that p is *nilpotent*.

Proposition 2.7. *Let p be an isolated singularity of f with pure order one. If p is nilpotent, then p is not an absolutely isolated singularity.*

Proof. Since p is not nonnilpotent, we can choose local coordinates $(z_1, \ldots, z_n)$ such that the linear part P^o_1 of g^o is in Jordan canonical form, that is,

$$P^o_{1,j} = \epsilon_j z_{j+1} \quad \text{for } 1 \leq j < n \quad \text{and} \quad P^o_{1,n} = 0,$$

where $\epsilon_j \in \{0, 1\}$ for $1 \leq j < n$.

By Remark 2.2 we can assume that $\epsilon_j = 1$ for each j. In this case it is easy to see that $\tilde{p} = [1 : 0 : \cdots : 0] \in \mathbb{P}^{n-1}$ in the chart $w_1 = z_1$ and $w_j = z_j / z_1$ for $2 \leq j \leq n$

is the unique singularity of $\tilde{f}$, the blow-up of f at p. It is also easy to see that the linear part $\tilde{P}_1^{\mathrm{o}}$ of $\tilde{g}^{\mathrm{o}}$ is of the form

$$\tilde{P}_{1,1}^{\mathrm{o}} = 0, \qquad \tilde{P}_{1,j}^{\mathrm{o}} = \alpha_j w_1 + w_{j+1} \quad \text{for } 2 \le j < n, \qquad \tilde{P}_{1,n}^{\mathrm{o}} = \alpha_n w_1,$$

where $\alpha_j = P_{2,j}^{\mathrm{o}}(1, 0, \dots, 0)$ for $2 \le j \le n$. Note that $w_1 \,|\, \tilde{g}_1^{\mathrm{o}}$.

By Remark 2.2 we can assume that $\alpha_n \ne 0$. Consider the change of coordinates

$$\varphi : w_1 = (1/\alpha_n) t_n, \quad w_2 = t_1, \quad w_j = t_{j-1} - (\alpha_{j-1}/\alpha_n) t_n \quad \text{for } 3 \le j \le n,$$

and

$$\varphi^{-1} : t_1 = w_2, \quad t_j = \alpha_j w_1 + w_{j+1} \quad \text{for } 2 \le j < n, \quad t_n = \alpha_n w_1.$$

In the local coordinates $(t_1, \dots, t_n)$, we have

$$Q_{1,j}^{\mathrm{o}} = t_{j+1} \quad \text{for } 1 \le j < n, \quad Q_{1,n}^{\mathrm{o}} = 0,$$

where $\sum_{k \ge 1} Q_{k,j}$ for $1 \le j \le n$ is the homogeneous expansion of $\varphi^{-1} \circ \tilde{g}_j^{\mathrm{o}} \circ \varphi$.

As above, we see that $\tilde{\tilde{p}} = [1 : 0 : \dots : 0] \in \mathbb{P}^{n-1}$ in the chart $u_1 = t_1$ and $u_j = t_j / t_1$ for $2 \le j \le n$ is the unique singularity of $\tilde{\tilde{f}}$, the blow-up of $\tilde{f}$ at $\tilde{p}$, and that the linear part $\tilde{\tilde{P}}_1^{\mathrm{o}}$ of $\tilde{\tilde{g}}^{\mathrm{o}}$ is of the form

$$\tilde{\tilde{P}}_{1,1}^{\mathrm{o}} = 0, \quad \tilde{\tilde{P}}_{1,j}^{\mathrm{o}} = \beta_j u_1 + u_{j+1} \quad \text{for } 2 \le j < n, \quad \tilde{\tilde{P}}_{1,n}^{\mathrm{o}} = \beta_n u_1,$$

where $\beta_j = Q_{2,j}^{\mathrm{o}}(1, 0, \dots, 0)$, $2 \le j \le n$. Since $w_1 \,|\, \tilde{g}_1^{\mathrm{o}}$, we have

$$\beta_n = Q_{2,n}^{\mathrm{o}}(1, 0, \dots, 0) = \alpha_n \tilde{g}_1^{\mathrm{o}}(0, 1, 0, \dots, 0) = 0.$$

Therefore, $\tilde{\tilde{p}}$ is not an AIS by Remark 2.2; thus neither is p. □

Combining Propositions 2.6 and 2.7, we have the following reduction theorem.

Theorem 2.8. *If p is an absolutely isolated singularity of f, then there exists a finite sequence of blow-ups such that the final blow-up map only has nonnilpotent singularities.*

3. Simple reduction

In this section we study nonnilpotent singularities. By Lemma 2.5(d) we will focus on nondicritical nonnilpotent singularities.

Let p be a nondicritical nonnilpotent singularity of f, the blow-up map after a finite sequence of blow-ups. Let $e = e(S, p)$ be the number of irreducible components of S through p, where S is the exceptional divisor. Let $\{S_i\}_{i=1}^{e}$ be the set of the irreducible components. We say that f is *nondicritical* (respectively *dicritical*) along S_i if S_i is created by blowing up at a nondicritical (respectively dicritical) singularity. If we choose local coordinates such that S_i is given by $z_i = 0$, then

f is nondicritical (respectively dicritical) along S_i if and only if $g_i^{\mathrm{o}}(z) = z_i h_i(z)$ (respectively $z_i \nmid g_i^{\mathrm{o}}(z)$).

Remark 3.1. We always have $1 \le e \le n$. By Lemma 2.5(c), f is dicritical along at most one S_i. If $e = 1$ and f is dicritical along S_1, then at any singularity q of $\tilde{f}$, the blow-up of f at p, we have either $e(\tilde{f}, q) = 2$ or $e(\tilde{f}, q) = 1$, and $\tilde{f}$ is nondicritical along the new S_1.

Remark 3.2. Our notion f being nondicritical (respectively dicritical) along S has equivalent definitions in other sources. In [Abate 2001], f is said to be nondegenerate (respectively degenerate) along S, and in [Abate et al. 2004], f is said to be tangential (respectively nontangential) along S.

When $e = 1$, we say that p is a *simple point* if f is nondicritical along S_1 and one of the following occurs:

(A) $h_1(0) = 0$ and the multiplicity of the eigenvalue 0 is one.

(B) $h_1(0) = \lambda \neq 0$, the multiplicity of the eigenvalue λ is one, and if μ is another eigenvalue of f at p, then $\mu/\lambda \notin \mathbb{Q}^+$.

When $e = 2$, we say that p is a *dicritical simple corner* if f is nondicritical along S_1, dicritical along S_2, and either (A) or (B) as occurs above.

When $e \ge 2$, we say that p is a *nondicritical simple corner* if (up to a permutation of the coordinates) f is nondicritical along S_1 and S_2, and we have $h_1(0) = \lambda \neq 0$, $h_2(0) = \mu$ and $\mu/\lambda \notin \mathbb{Q}^+$.

We say that p is a *simple* singularity of f if it is a simple point or a simple corner.

The next proposition shows that simple singularities persist under blow-ups.

Proposition 3.3. *If p is a simple singularity of f, then every singularity of $\tilde{f}$ in $\pi^{-1}(p)$ is simple, where π denotes the blow-up at p. More precisely,*

(a) *If p is a simple point, then exactly one singularity $\tilde{p}$ of $\tilde{f}$ in $\pi^{-1}(p)$ is a simple point and all others are nondicritical simple corners. Moreover, p and $\tilde{p}$ have the same type* (A) *or* (B).

(b) *If p is a dicritical simple corner, then exactly one singularity $\tilde{p}$ of $\tilde{f}$ in $\pi^{-1}(p)$ is a simple point or a dicritical simple corner and all others are nondicritical simple corners. Moreover, p and $\tilde{p}$ have the same type* (A) *or* (B).

(c) *If p is a nondicritical simple corner, then every singularity of $\tilde{f}$ in $\pi^{-1}(p)$ is a nondicritical simple corner.*

Proof. For (a) we can write f as

$$f_j(z) = \begin{cases} z_1 + z_1^a z_1(\lambda + O(1)) & \text{if } j = 1, \\ z_j + z_1^a\big(\alpha_j z_1 + \sum_{2\le k\le n} \beta_{j;k} z_k + O(2)\big) & \text{if } j \neq 1. \end{cases}$$

In the canonical coordinates $[w_1 : \cdots : w_n]$ centered at $q = [1 : q_2 : \cdots : q_n]$, $\tilde{f}$ is of the form

$$\tilde{f}_j(w) = \begin{cases} w_1 + w_1^a w_1(\lambda + O(w_1)) & \text{if } j = 1, \\ w_j + w_1^a\big(\alpha_j + \sum_{k\neq j} \beta_{j;k}(w_k + q_k) + (\beta_{j;j} - \lambda)(w_j + q_j) + O(w_1)\big) & \\ & \text{if } j \neq 1. \end{cases}$$

The point q is a singularity of $\tilde{f}$ if and only if $\alpha_j + \sum_{k\neq j} \beta_{j;k} q_k + (\beta_{j;j} - \lambda) q_j = 0$ for all $j \neq 1$. Set $\Lambda = (\beta_{j;k})_{2\le j,k\le n}$ and let $\{\mu_i\}_{2\le i\le n}$ be the eigenvalues of Λ. If $\lambda = 0$, then $\mu_i \neq 0$, and if $\lambda \neq 0$, then $\mu_i/\lambda \notin \mathbb{Q}^+$. In either case, the matrix $\Lambda - \lambda I_{n-1}$ is of full rank and it has eigenvalues $\{\mu_i - \lambda\}_{2\le i\le n}$. Therefore we have a unique singularity $\tilde{p} = [1 : q_2 : \cdots : q_n]$, where

$$(q_2, \ldots, q_n)^T = (\Lambda - \lambda I_{n-1})^{-1}(\alpha_2, \ldots, \alpha_n)^T.$$

It is easy to see that $\tilde{p}$ has the same type as p.

We now choose local coordinates such that f is of the form

$$f_j(z) = \begin{cases} z_1 + z_1^a z_1(\lambda + O(1)) & \text{if } j = 1, \\ z_j + z_1^a\big(\sum_{1\le k\le j} \beta_{j;k} z_k + O(2)\big) & \text{if } j \neq 1. \end{cases}$$

Then the eigenvalues of f are λ and $\{\beta_{j;j}\}_{2\le j\le n}$.

In the canonical coordinates $[w_1 : \cdots : w_n]$ centered at

$$q = [0 : \cdots : 0 : 1 : q_{j+1} : \cdots : q_n] \quad \text{for } 2 \le j \le n,$$

$\tilde{f}$ is of the form

$$\tilde{f}_l(w) = \begin{cases} w_1 + w_1^a w_j^a w_1\big(\lambda - \beta_{j;j} - \sum_{1\le k<j} \beta_{j;k} w_k + O(w_j)\big) & \text{if } l = 1, \\ w_j + w_1^a w_j^a w_j\big(\beta_{j;j} + \sum_{1\le k<j} \beta_{j;k} w_k + O(w_j)\big) & \text{if } l = j, \\ w_l + w_1^a w_j^a(\cdots) & \text{if } l \neq 1, j. \end{cases}$$

Assume that q is a singularity of $\tilde{f}$. If $\lambda = 0$, then $\beta_{j;j} \neq 0$ and $(\lambda - \beta_{j;j})/\beta_{j;j} = -1 \notin \mathbb{Q}^+$. If $\lambda \neq 0$, then $\beta_{j;j}/(\lambda - \beta_{j;j}) \notin \mathbb{Q}^+$. Therefore q is a nondicritical simple corner. This proves (a).

For (b) the argument is similar to above and we leave it to the reader.

For (c) see [Rong 2010, Proposition 2.3]. □

Remark 3.4. The simple example

$$f_j(z) = \begin{cases} z_1 + z_1^a z_2^b z_1(\lambda + O(1)) & \text{if } j = 1, \\ z_2 + z_1^a z_2^b (z_2 + z_3 + O(2)) & \text{if } j = 2, \\ z_3 + z_1^a z_2^b (z_3 + O(2)) & \text{if } j = 3, \end{cases}$$

where $\lambda \le 0$, shows we may not be able to get rid of dicritical simple corners.

Before proving Theorem 1.1, let us take a closer look at the behavior of nondicritical nonnilpotent singularities under blow-ups. To state our next result, let us single out a very special case in dimension two: in suitable local coordinates (z, w) around a nondicritical nonnilpotent singularity p, $f = (f_1, f_2)$ is given by

$$\begin{aligned} f_1(z, w) &= z + l(\lambda z + O(z^2, zw, w^2)), \\ f_2(z, w) &= w + l(2\lambda w + O(z^3, zw, w^2)). \end{aligned} \tag{3-1}$$

with $\lambda \ne 0$. One easily checks that the blow-up map $\tilde{f}$ has a dicritical singularity in the exceptional divisor S.

Proposition 3.5. *Let p be a nondicritical nonnilpotent singularity of f and let $\tilde{f}$ be the blow-up of f at p. Let S be the exceptional divisor of the blow-up. If p is an absolutely isolated singularity of f and is not as in (3-1), then the singularities of $\tilde{f}$ in S are all nondicritical and nonnilpotent.*

Proof. If f has an eigenvalue λ of multiplicity $k > 1$, then in suitable local coordinates around p, we can write f as

$$f_j(z) = \begin{cases} z_j + l(\lambda z_j + \epsilon_j z_{j+1} + O(2)) & \text{if } 1 \le j < k, \\ z_k + l(\lambda z_k + O(2)) & \text{if } j = k, \\ z_j + l g_j^{\mathrm{o}} & \text{if } j > k, \end{cases}$$

where $\epsilon_j \in \{0, 1\}$ for $1 \le j < k$.

We claim that if $\epsilon_{j_0} = 0$ for some j_0 with $1 \le j_0 < k$, then $\tilde{f}$ has infinitely many singularities. Assume the premise. In the canonical coordinates $[w_1 : \cdots : w_n]$ centered at $q = [1 : q_2 : \cdots : q_n]$, $\tilde{f}$ is of the form

$$\tilde{f}_j(w) = \begin{cases} w_1 + \tilde{l} w_1(\lambda + \epsilon_1(w_2 + q_2) + O(w_1)) & \text{if } j = 1, \\ w_j + \tilde{l}(\epsilon_j(w_{j+1} + q_{j+1}) - \epsilon_1(w_2 + q_2)(w_j + q_j) + O(w_1)) & \\ & \text{if } 2 \le j < k, \\ w_k + \tilde{l}(-\epsilon_1(w_2 + q_2)(w_k + q_k) + O(w_1)) & \text{if } j = k, \\ w_j + \tilde{l}(\cdots) & \text{if } j > k. \end{cases}$$

If q is a singularity of $\tilde{f}$, then $\epsilon_j q_{j+1} - \epsilon_1 q_2 q_j = 0$ for $2 \le j < k$, and $-\epsilon_1 q_2 q_k = 0$. It is easy to check that if $q_j = 0$ for $j \ne j_0 + 1$, then we are free to choose q_{j_0+1}. This proves the claim above.

If f has n distinct eigenvalues $\{\lambda_i\}_{1\le i\le n}$ at p, then in suitable local coordinates around p, we can write f as

$$f_j(z) = z_j + l(\lambda_j z_j + O(2)) \quad \text{for } 1 \le j \le n.$$

Let $q_k = [0 : \cdots : 0 : 1 : 0 : \cdots : 0]$ with the k-th entry nonzero for $1 \le k \le n$. It is easy to see that $\{q_k\}_{1\le k\le n}$ are the only singularities of $\tilde{f}$ in S, and $\tilde{f}$ takes the following form at q_k:

$$\tilde{f}_j(w) = \begin{cases} w_k + \tilde{l}\big(\lambda_k w_k + O(w_k^2)\big) & \text{if } j = k, \\ w_j + \tilde{l}((\lambda_j - \lambda_k) w_j + O(w_k)) & \text{if } j \ne k. \end{cases}$$

If $\lambda_j \ne 2\lambda_k$ for any j and k, then clearly $\{q_k\}_{1\le k\le n}$ are all nondicritical and nonnilpotent. If $\lambda_j = 2\lambda_k$ for some j and k, then $\tilde{f}$ at q_k has an eigenvalue of multiplicity greater than one. Therefore, by the argument above we know that $\tilde{\tilde{f}}$ has infinitely many singularities. (Note that p is not as in (3-1).)

Let $\{\lambda_i\}_{1\le i\le m}$ be the distinct eigenvalues of f at p. Assume that λ_i has multiplicity k_i and set

$$s_i = \sum_{j\le i} k_i \quad \text{for } 1 \le i \le m.$$

Set $s_0 = 0$. Since p is an absolutely isolated singularity of f, we can write f as

$$f_j(z) = \begin{cases} z_j + l(\lambda_i z_j + z_{j+1} + O(2)) & \text{if } s_{i-1} < j < s_i \text{ for } 1 \le i \le m, \\ z_{s_i} + l(\lambda_i z_{s_i} + O(2)) & \text{if } \qquad j = s_i \text{ for } 1 \le i \le m \end{cases}$$

In the canonical coordinates $[w_1 : \cdots : w_n]$ centered at $q = [1 : q_2 : \cdots : q_n]$, $\tilde{f}$ is of the form

$$\tilde{f}_j(w) = \begin{cases} w_1 + \tilde{l} w_1(\lambda_1 + (w_2 + q_2) + O(w_1)) & \text{if } j = 1, \\ w_j + \tilde{l}((w_{j+1} + q_{j+1}) \\ \qquad\qquad - (w_2 + q_2)(w_j + q_j) + O(w_1)) & \text{if } 2 \le j < s_1, \\ w_{s_1} + \tilde{l}(-(w_2 + q_2)(w_{s_1} + q_{s_1}) + O(w_1)) & \text{if } j = s_1, \\ w_j + \tilde{l}((\lambda_i - \lambda_1)(w_j + q_j) + (w_{j+1} + q_{j+1}) \\ \qquad\qquad - (w_2 + q_2)(w_j + q_j) + O(w_1)) & \text{if } s_{i-1} < j < s_i, \\ & \qquad 2 \le i \le m, \\ w_{s_i} + \tilde{l}((\lambda_i - \lambda_1)(w_{s_i} + q_{s_i}) \\ \qquad\qquad - (w_2 + q_2)(w_{s_i} + q_{s_i}) + O(w_1)) & \text{if } j = s_i,\ 2 \le i \le m. \end{cases}$$

One readily checks that $q = [1 : 0 : \cdots : 0]$ is the only singularity of $\tilde{f}$ in this chart, and it is nondicritical and nonnilpotent.

In the canonical coordinates $[w_1 : \cdots : w_n]$ centered at $q = [q_1 : \cdots : q_{k-1} : 1 : q_{k+1} : \cdots : q_n]$ for $1 < k < s_1$, $\tilde{f}$ is of the form

$$\tilde{f}_j(w) = \begin{cases} w_j + \tilde{l}((w_{j+1} + q_{j+1}) \\ \qquad - (w_{k+1} + q_{k+1})(w_j + q_j) + O(w_k)) & \text{if } j \neq k-1, \\ & \quad 1 \leq j < s_1, \\ w_{k-1} + \tilde{l}(1 - (w_{k+1} + q_{k+1})(w_{k-1} + q_{k-1}) \\ \qquad + O(w_k)) & \text{if } j = k-1, \\ w_k + \tilde{l} w_k(\lambda_1 + (w_{k+1} + q_{k+1}) + O(w_k)) & \text{if } j = k, \\ w_{s_1} + \tilde{l}(-(w_{k+1} + q_{k+1})(w_{s_1} + q_{s_1}) + O(w_k)) & \text{if } j = s_1, \\ w_j + \tilde{l}((\lambda_i - \lambda_1)(w_j + q_j) + (w_{j+1} + q_{j+1}) \\ \qquad - (w_{k+1} + q_{k+1})(w_j + q_j) + O(w_k)) & \text{if } s_{i-1} < j < s_i, \\ & \quad 2 \leq i \leq m, \\ w_{s_i} + \tilde{l}((\lambda_i - \lambda_1)(w_{s_i} + q_{s_i}) \\ \qquad - (w_{k+1} + q_{k+1})(w_{s_i} + q_{s_i}) + O(w_k)), & \text{if } j = s_i,\ 2 \leq i \leq m. \end{cases}$$

One readily checks that there are no singularities of $\tilde{f}$ in this chart.

In the canonical coordinates $[w_1 : \cdots : w_n]$ centered at $q = [q_1 : \cdots : q_{k-1} : 1 : q_{k+1} : \cdots : q_n]$, where $k = s_1$, $\tilde{f}$ is of the form

$$\tilde{f}_j(w) = \begin{cases} w_j + \tilde{l}((w_{j+1} + q_{j+1}) + O(w_{s_1})) & \text{if } 1 \leq j < s_1 - 1, \\ w_{s_1-1} + \tilde{l}(1 + O(w_{s_1})) & \text{if } j = s_1 - 1, \\ w_{s_1} + \tilde{l} w_{s_1}(\lambda_1 + O(w_{s_1})) & \text{if } j = s_1, \\ w_j + \tilde{l}((\lambda_i - \lambda_1)(w_j + q_j) \\ \qquad + (w_{j+1} + q_{j+1}) + O(w_{s_1})) & \text{if } s_{i-1} < j < s_i,\ 2 \leq i \leq m, \\ w_{s_i} + \tilde{l}((\lambda_i - \lambda_1)(w_{s_i} + q_{s_i}) + O(w_{s_1})) & \text{if } j = s_i,\ 2 \leq i \leq m. \end{cases}$$

It is obvious that there are no singularities of $\tilde{f}$ in this chart.

Let $q_i = [0 : \cdots : 0 : 1 : 0 : \cdots : 0]$ with the $(s_{i-1}+1)$-st entry nonzero, for $1 \leq i \leq m$. Then a similar argument as above shows that they are the only singularities of $\tilde{f}$ in S. Moreover, they are all nondicritical and nonnilpotent. □

Proof of Theorem 1.1. Let p be an absolutely isolated singularity of f. By Theorem 2.8 we can assume that p is nonnilpotent. By Proposition 3.5 we need to show we can reduce nondicritical nonnilpotent singularities to simple singularities.

By Remark 3.1 we can assume that f is nondicritical along S_1 at p. If $h_1(0) = 0$ and there exists another S_2 such that f is nondicritical along S_2, then we have $h_2(0) \neq 0$ by Remark 2.2. In this case we can switch S_1 and S_2 and assume that $h_1(0) \neq 0$. Therefore we consider two cases:

(a) f is nondicritical along only S_1 and $h_1(0) = 0$;

(b) f is nondicritical along S_1 and $h_1(0) \neq 0$.

For (a) we claim that p is a type (A) simple point or dicritical simple corner. First, if $e(S, p) \geq 3$, then f is nondicritical along at least one more irreducible component S_2 by Remark 3.1 and $h_2(0) \neq 0$ by Remark 2.2. Therefore $1 \leq e(S, p) \leq 2$. Now it suffices to show that if the eigenvalue 0 has multiplicity greater than one, then p is not an AIS.

Assume that the linear part P_1^{o} of g^{o} is of the form

$$P_{1,1}^{\mathrm{o}} = 0, \quad P_{1,i}^{\mathrm{o}} = z_{i-1} \quad \text{for } 2 \leq i \leq k, \quad P_{1,j}^{\mathrm{o}} = P_{1,j}^{\mathrm{o}}(z_{k+1}, \dots, z_n) \quad \text{for } j > k,$$

where $k \geq 2$ is the multiplicity of the eigenvalue 0. Consider the chart $w_k = z_k$ and $w_j = z_j/z_k$ for $j \neq k$. It is easy to check that the blow-up map $\tilde{f}$ in this chart has $\nu(\tilde{g}_j^{\mathrm{o}}) > 1$ for $j = 1$ and $j = k$. Therefore p is not an AIS by Remark 2.2.

For (b) we consider an invariant $\mathrm{Inv}(f, S, p)$, which we now define (compare with [Cano 1987]).

Set $d(S, p) = \#\{S_i : f \text{ is nondicritical along } S_i\}$. Let $\{\alpha_i\}_{1 \leq i \leq n}$ be the set of eigenvalues of f at p counted with multiplicity, with $\alpha_i = h_i(0)$ for $1 \leq i \leq d(S, p)$. If $\alpha_i \neq 0$ for some i in $1 \leq i \leq d(S, p)$, then we set

$$c_i(f, S, p) = \#\{\alpha_j/\alpha_i \in \mathbb{Q}^+, \ j \neq i\}.$$

Define $c(f, S, p) = \min\{c_i(f, S, p) : \alpha_i \neq 0, \ 1 \leq i \leq d(S, p)\}$.

If $d(S, p) = 1$, then we set

$$J = \{j : \alpha_j/\alpha_1 \in \mathbb{Q}^+\} \quad \text{and} \quad m = \min\{r \in \mathbb{Z}^+ : r\alpha_j/\alpha_1 \in \mathbb{Z}^+, \ j \in J\}.$$

Define $n(f, S, p) = m \sum_{j \in J} \alpha_j/\alpha_1$.

If $d(S, p) \geq 2$ and $\alpha_i \neq 0$ for some $1 \leq i \leq d(S, p)$, then we set

$$J_i = \{j : \alpha_j/\alpha_i \in \mathbb{Q}^+, 1 \leq j \leq d(S, p)\}, \quad m_i = \min\{r \in \mathbb{Z}^+ : r\alpha_j/\alpha_i \in \mathbb{Z}^+, j \in J_i\}.$$

Define $n_i(f, S, p) = m_i \sum_{j \in J_i} \alpha_j/\alpha_i$. If p is not a simple corner, then it is easy to see that $n_i(f, S, p) = n_j(f, S, p)$ for $1 \leq i, j \leq d(S, p)$, and we define $n(f, S, p)$ to be this common value.

If p is not a simple singularity, define

$$\mathrm{Inv}(f, S, p) = (c(f, S, p), n - d(S, p), n(f, S, p)) \in \mathbb{N}^3.$$

Otherwise, define $\mathrm{Inv}(f, S, p) = (0, 0, 0)$.

We claim that

$$(\star) \qquad \mathrm{Inv}(\tilde{f}, \tilde{S}, q) < \mathrm{Inv}(f, S, p),$$

where $\tilde{S}$ is the strict transform of S under the blow-up π with center p, and q is a singularity of $\tilde{f}$ in $\pi^{-1}(p)$. Here we compare the invariants above in the lexicographic order of $\mathbb{N}^3$.

Choose local coordinates such that f is of the form

$$f_j(z) = z_j + l\Big(\sum_{1\le k<j} \beta_{j;k} z_k + \alpha_j z_j + O(2)\Big) \quad \text{for } 1 \le j \le n.$$

In the canonical coordinates $[w_1 : \cdots : w_n]$ centered at $q = [0 : \cdots : 0 : 1 : q_{i+1} : \cdots : q_n]$ for $1 \le i \le n$, $\tilde f$ is of the form

$$\tilde f_j(w) = \begin{cases} w_j + \tilde l\big(\sum_{1\le k<j} \beta_{j,k} w_k + (\alpha_j - \alpha_i) w_j + O(w_i)\big) & \text{if } 1 \le j < i, \\ w_i + \tilde l w_i\big(\alpha_i + \sum_{1\le k<i} \beta_{i,k} w_k + O(w_i)\big) & \text{if } j = i, \\ w_j + \tilde l\big(\sum_{1\le k<i} \beta_{j,k} w_k + \beta_{j,i} + \sum_{i<k<j} \beta_{j,k}(w_k + q_k) & \\ \qquad\qquad + (\alpha_j - \alpha_i)(w_j + q_j) + O(w_i)\big) & \text{if } i < j \le n. \end{cases}$$

First assume that $d(S, p) = 1$. Set $c = c(f, S, p)$ and assume without loss of generality that $\{\alpha_i\}_{1\le i\le c+1}$ are the eigenvalues with $\alpha_i/\alpha_1 \in \mathbb{Q}^+$.

If $q = [1 : q_2 : \cdots : q_n]$ is a singularity of $\tilde f$, then the eigenvalues of $\tilde f$ at q are α_1 and $\{\alpha_j - \alpha_1\}_{2\le j\le n}$. Clearly, $c(\tilde f, \tilde S, q) \le c(f, S, p)$ and $d(\tilde S, q) = d(S, p) = 1$. If $c(\tilde f, \tilde S, q) = c(f, S, p)$, then $\alpha_i/\alpha_1 > 1$ for $2 \le i \le c+1$. Set $\alpha_i/\alpha_1 = r_i/s_i$ with $\gcd(r_i, s_i) = 1$ for $2 \le i \le c+1$. Then the value m is the same at p and q, and is equal to $\operatorname{lcm}(s_2, \dots, s_{c+1})$. Set $t_i = m/s_i$ for $2 \le i \le c+1$. Then

$$n(\tilde f, \tilde S, q) = n(f, S, p) - \sum_{2\le i\le c+1} t_i s_i = n(f, S, p) - mc < n(f, S, p).$$

If $q = [0 : \cdots : 0 : 1 : q_{i+1} : \cdots : q_n]$ for $2 \le i \le n$ is a singularity of $\tilde f$, then the eigenvalues of $\tilde f$ at q are $\{\alpha_i\}$ and $\{\alpha_j - \alpha_i\}_{j\ne i}$. Note that $d(\tilde S, q) = 2 > d(S, p)$. If $i > c+1$, then q is a simple corner since $\alpha_i/(\alpha_1 - \alpha_i) \notin \mathbb{Q}^+$. If $2 \le i \le c+1$, then $\alpha_1/\alpha_i \in \mathbb{Q}^+$. Since $(\alpha_j - \alpha_i)/\alpha_i \in \mathbb{Q}^+$ implies $\alpha_j/\alpha_1 \in \mathbb{Q}^+$ for each $j \ne 1, i$, we have $c(\tilde f, \tilde S, q) \le c(f, S, p)$.

Suppose $d = d(S, p) \ge 2$. If p is not a simple singularity, $c = c(f, S, p) \ge d-1$. Assume without loss of generality that $f_j(z) = z_j + l z_j(\alpha_j + O(1))$ for $1 \le j \le d$ and that $\{\alpha_i\}_{1\le i\le c+1}$ are the eigenvalues with $\alpha_i/\alpha_1 \in \mathbb{Q}^+$.

If $q = [0 : \cdots : 0 : 1 : q_{i+1} : \cdots : q_n]$ for $1 \le i \le d$ is a singularity of $\tilde f$, then the eigenvalues of $\tilde f$ at q are $\{\alpha_i\}$ and $\{\alpha_j - \alpha_i\}_{j\ne i}$. Clearly, $c(\tilde f, \tilde S, q) \le c(f, S, p)$. If $c(\tilde f, \tilde S, q) = c(f, S, p)$, then $\alpha_j - \alpha_i \ne 0$ and $q_j = 0$ for $i+1 \le j \le d$. Therefore, $d(\tilde S, q) = d(S, p)$. Set $\alpha_j/\alpha_i = r_j/s_j$ with $\gcd(r_j, s_j) = 1$ for $1 \le j \le d$. Then the value m_i is the same at p and q, and is equal to $\operatorname{lcm}(s_1, \dots, s_d)$. Set $t_j = m_i/s_j$ for $1 \le j \le d$. Then

$$n(\tilde f, \tilde S, q) = n_i(\tilde f, \tilde S, q) = n_i(f, S, p) - \sum_{1\le j\le d,\, j\ne i} t_j s_j < n_i(f, S, p) = n(f, S, p).$$

If $q = [0 : \cdots : 0 : 1 : q_{i+1} : \cdots : q_n]$ for $d+1 \le i \le n$ is a singularity of $\tilde f$, the eigenvalues of $\tilde f$ at q are $\{\alpha_i\}$ and $\{\alpha_j - \alpha_i\}_{j\ne i}$. Now, $d(\tilde S, q) = d(S, p) + 1 > d(S, p)$.

If $i > c+1$, then q is a simple corner since $\alpha_i/(\alpha_1 - \alpha_i) \notin \mathbb{Q}^+$. If $d+1 \le i \le c+1$, then $\alpha_1/\alpha_i \in \mathbb{Q}^+$. Since $(\alpha_j - \alpha_i)/\alpha_i \in \mathbb{Q}^+$ implies $\alpha_j/\alpha_1 \in \mathbb{Q}^+$ for each $j \neq 1, i$, we have $c(\tilde{f}, \tilde{S}, q) \le c(f, S, p)$.

This completes the proof of the claim ($\star$), and thus the theorem. □

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References

[Abate 2000] M. Abate, "Diagonalization of nondiagonalizable discrete holomorphic dynamical systems", *Amer. J. Math.* **122**:4 (2000), 757–781. MR 2001m:32036 Zbl 0966.32018

[Abate 2001] M. Abate, "The residual index and the dynamics of holomorphic maps tangent to the identity", *Duke Math. J.* **107**:1 (2001), 173–207. MR 2003a:32028 Zbl 1015.37035

[Abate and Tovena 2003] M. Abate and F. Tovena, "Parabolic curves in $\mathbb{C}^3$", *Abstr. Appl. Anal.* **2003**:5 (2003), 275–294. MR 2004c:32035 Zbl 1018.37027

[Abate et al. 2004] M. Abate, F. Bracci, and F. Tovena, "Index theorems for holomorphic self-maps", *Ann. of Math.* (2) **159**:2 (2004), 819–864. MR 2005g:32044 Zbl 1056.37025

[Benazic Tome 1997] R. M. Benazic Tome, "A resolution theorem for absolutely isolated singularities of holomorphic vector fields", *Bol. Soc. Brasil. Mat.* (*N.S.*) **28** (1997), 211–231. MR 99c:32055 Zbl 0892.32026

[Camacho and Sad 1982] C. Camacho and P. Sad, "Invariant varieties through singularities of holomorphic vector fields", *Ann. of Math.* (2) **115**:3 (1982), 579–595. MR 83m:58062 Zbl 0503.32007

[Camacho et al. 1989] C. Camacho, F. Cano, and P. Sad, "Absolutely isolated singularities of holomorphic vector fields", *Invent. Math.* **98**:2 (1989), 351–369. MR 90i:58153 Zbl 0689.32008

[Cano 1987] F. Cano, "Final forms for a three-dimensional vector field under blowing-up", *Ann. Inst. Fourier* (*Grenoble*) **37**:2 (1987), 151–193. MR 88j:58105 Zbl 0607.58027

[Fulton 1998] W. Fulton, *Intersection theory*, 2nd ed., Ergebnisse der Mathematik und ihrer Grenzgebiete (3) **2**, Springer, Berlin, 1998. MR 99d:14003 Zbl 0885.14002

[Hakim 1998] M. Hakim, "Analytic transformations of $(\mathbf{C}^p, 0)$ tangent to the identity", *Duke Math. J.* **92**:2 (1998), 403–428. MR 99a:32036 Zbl 0952.32012

[Milnor 2006] J. Milnor, *Dynamics in one complex variable*, 3rd ed., Annals of Mathematics Studies **160**, Princeton University Press, 2006. MR 2006g:37070 Zbl 1085.30002

[Rong 2010] F. Rong, "Robust parabolic curves in $\mathbf{C}^m (m \geq 3)$", *Houston J. Math.* **36**:1 (2010), 147–155.

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FENG RONG
DEPARTMENT OF MATHEMATICS
SHANGHAI JIAO TONG UNIVERSITY
800 DONG CHUAN ROAD
SHANGHAI 200240
CHINA
frong@sjtu.edu.cn

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