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**TWO-DIMENSIONAL DISJOINT
MINIMAL GRAPHS**

LINFENG ZHOU

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Under the assumption of Gauss curvature vanishing at infinity, we prove Meeks' conjecture: the number of disjointly supported minimal graphs in \mathbb{R}^3 is at most two.

1. Introduction

Let Ω be an open subset in \mathbb{R}^2 and denote its boundary by $\partial\Omega$. As we know, if a function $u(x)$ defined on Ω satisfies the equation

$$(1) \quad \operatorname{div} \frac{\nabla u}{\sqrt{1 + |\nabla u|^2}} = 0,$$

$G = \{(x, u(x)) : x \in \Omega\}$ is called a minimal graph in \mathbb{R}^3 . We say the minimal graph G is supported on Ω if $u|_{\partial\Omega} = 0$ and $u \geq 0$.

Meeks [2005] has conjectured that the number of disjointly supported minimal graphs with zero boundary values over an open subset in \mathbb{R}^2 is at most 2. In fact, for arbitrary dimension, Meeks and Rosenberg [2005] proved if a set of disjointly supported minimal graphs have bounded gradient, then the number of the graphs must be finite. Later, Li and Wang [2001] gave an upper bound of the number of the graphs without any assumption on the growth rate of each graph. As a corollary, when minimal graphs are two dimensional in \mathbb{R}^3 , they obtained the number is at most 24. At the same time, Spruck [2002] proved that there are at most two admissible sublinear growth solution pairs of Equation (1) defined over disjoint domains. Recently, by using angular density, Tkachev [2009] showed the number of two dimensional disjointly supported minimal graphs is less than or equals 3.

Observing the similarity between disjoint d -massive sets and disjointly supported minimal graphs, we can apply the method for proving the finiteness theorem of disjoint d -massive sets in \mathbb{R}^2 [Li and Wang 1999] to study disjoint minimal graphs. We obtain the following theorem:

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Theorem 1.1. *Suppose $\{G_i = (\Omega_i, u_i)\}_{i=1}^k$ is a set of disjointly supported minimal graphs in \mathbb{R}^3 , where each Ω_i is an open subset in \mathbb{R}^2 . If the Gauss curvature $K_i(x)$ of each graph satisfies*

$$K_i(x) \rightarrow 0 \quad \text{as } |x| \rightarrow \infty,$$

then the number k is at most two.

By choosing a different region of integration, one obtains an improvement on a theorem of Spruck [2002]:

Corollary 1.2. *Suppose $\{G_i = (\Omega_i, u_i)\}_{i=1}^k$ is a set of disjointly supported minimal graphs in \mathbb{R}^3 , where each Ω_i is an open subset in \mathbb{R}^2 . If each graph has sublinear growth, then k is at most two.*

2. Proof of Theorem 1.1

We denote the 3-dimensional ball of radius R centered at the origin of \mathbb{R}^3 by $B^3(R)$ and the 2-dimensional sphere of radius R by $S^2(R)$. The key is to estimate the sum of all curves' length $\ell(G_i \cap S^2(R))$ when R is sufficiently large.

Theorem 2.1. *Suppose $\{G_i = (\Omega_i, u_i)\}_{i=1}^k$ is a set of disjointly supported minimal graphs in \mathbb{R}^3 , where the Gauss curvature $K_i(x)$ of each G_i satisfies*

$$K_i(x) \rightarrow 0 \quad \text{as } |x| \rightarrow \infty.$$

For a sufficiently large radius R , we have the bound

$$\sum_{i=1}^k \ell(G_i \cap S^2(R)) \leq \pi^2 R + o(1)R.$$

In the particular case when $k = 3$, we have the better estimate

$$\sum_{i=1}^3 \ell(G_i \cap S^2(R)) \leq 2\sqrt{2}\pi R + o(1)R.$$

Before proving this, we introduce a lemma.

Lemma 2.2. *Let $B_+^3(R)$ be a 3-dimensional upper half-ball with radius R and let $S_+^2(R)$ be a 2-dimensional upper half-sphere. Suppose $\pi_i : G_i \rightarrow \mathbb{R}^2$ is the natural projection map. If $\Sigma_1, \Sigma_2, \dots, \Sigma_s$ are planes in \mathbb{R}^3 such that the interiors of $\pi_i(\Sigma_i \cap B_+^3(R))$ are pairwise disjoint for sufficiently large R , we have*

$$\sum_{i=1}^s \ell(\Sigma_i \cap S_+^2(R)) \leq \pi^2 R.$$

Moreover, when $s = 3$, we have the better estimate

$$\sum_{i=1}^3 \ell(\Sigma_i \cap S_+^2(R)) \leq 2\sqrt{2}\pi R.$$

Proof. Suppose $D(R) = \{(x_1, x_2, 0) : x_1^2 + x_2^2 \leq R^2\}$ is a disk in \mathbb{R}^3 . Since each Σ_i is a plane, $\Sigma_i \cap D(R)$ is a chord; let θ_i be the corresponding central angle. Here we only need to consider the case that the union $\bigcup_{i=1}^s (\Sigma_i \cap D(R))$ is a polygon; otherwise, one can add more planes still satisfying the required conditions and such that the union of chords becomes a polygon.

If the center of the disk $D(R)$ is in the interior of the polygon or on one of the edges of the polygon, each central angle θ_i satisfies $0 < \theta_i \leq \pi$. Since the interiors of the $\pi_i(\Sigma_i \cap S_+^2(R))$ are pairwise disjoint, a simple computation yields the bound

$$\ell(\Sigma_i \cap S_+^2(R)) \leq \pi R \sin \frac{\theta_i}{2}.$$

on the length of the arc $\ell(\Sigma_i \cap S_+^2(R))$. The right-hand side achieves the maximum if and only if Σ_i is perpendicular to the disk $D(R)$. Thus

$$(2) \quad \begin{aligned} \sum_{i=1}^s \ell(\Sigma_i \cap S_+^2(R)) &\leq \sum_{i=1}^s \pi R \sin \frac{\theta_i}{2} \leq \pi R s \sin \left(\frac{1}{s} \sum_{i=1}^s \frac{\theta_i}{2} \right) \\ &\leq \pi R s \sin \frac{\pi}{s} \leq \pi^2 R. \end{aligned}$$

In the second inequality, we have used the concave property of the sine function on the interval $[0, \pi]$.

For the special case when $s = 3$, one gets from (2)

$$(3) \quad \sum_{i=1}^3 \ell(\Sigma_i \cap S_+^2(R)) \leq 3\pi R \sin \frac{\pi}{3} = \frac{3\sqrt{3}}{2} \pi R.$$

If the center of the disk $D(R)$ is outside the polygon, there exists an i_0 such that $\theta_{i_0} > \pi$. For simplicity, let us assume $i_0 = s$. A similar computation leads to

$$\ell(\Sigma_i \cap S_+^2(R)) \leq \begin{cases} \pi R \sin \frac{\theta_i}{2} & \text{for } 1 \leq i \leq s-1, \\ R\theta_s & \text{for } i = s. \end{cases}$$

In the first case, equality holds if and only if Σ_i is perpendicular to the disk, and in the second, if and only if Σ_s is in the same plane of the disk $D(R)$. Hence

$$(4) \quad \begin{aligned} \sum_{i=1}^s \ell(\Sigma_i \cap S_+^2(R)) &\leq \sum_{i=1}^{s-1} \pi R \sin \frac{\theta_i}{2} + R\theta_s \leq \sum_{i=1}^{s-1} \pi R \sin \frac{\theta_i}{2} + 2\pi R \sin \frac{\theta_s}{4} \\ &\leq \pi R(s+1) \sin \frac{\pi}{s+1} \leq \pi^2 R. \end{aligned}$$

If $s = 3$, by (4) we obtain that

$$(5) \quad \sum_{i=1}^3 \ell(\Sigma_i \cap S_+^2(R)) \leq 4\pi R \sin \frac{\pi}{4} = 2\sqrt{2}\pi R$$

The conclusion is derived from (2), (4) and (3), (5). □

Proof of Theorem 2.1. For each minimal graph G_i , since the Gauss curvature $K_i = 0$ at infinity, it means G_i is asymptotic to a flat plane. Therefore, we can use the intersection of a plane Σ_i and $S_+^2(R)$ to approximate the curve $G_i \cap S^2(R)$. By Lemma 2.2, one has

$$\ell(G_i \cap S^2(R)) \leq \ell(\Sigma_i \cap S_+^2(R)) + o(1)R.$$

Therefore

$$\sum_{i=1}^k \ell(G_i \cap S^2(R)) \leq \sum_{i=1}^k \ell(\Sigma_i \cap S_+^2(R)) + o(1)R \leq \pi^2 R + o(1)R. \quad \square$$

The following area growth estimate of a minimal graph is proved using a well-known argument; one can see [Li and Wang 2001] for the details.

Lemma 2.3. *If $G = (\Omega, u)$ is a minimal graph in \mathbb{R}^3 , the area of $G \cap B^3(R)$ satisfies*

$$A(G \cap B^3(R)) \leq 3\pi R^2.$$

Proof of Theorem 1.1. Let $B^3(R)$ be the ball of radius R in \mathbb{R}^3 . Since

$$\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2 \leq \int_{G_i \cap \partial B^3(R)} u_i \left(\tilde{\nabla} u_i \cdot \frac{\partial}{\partial r} \right),$$

where $\tilde{\nabla}$ means the gradient operator on G_i , one has

$$\begin{aligned} 2\lambda_1^{1/2}(G_i \cap \partial B^3(R)) \int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2 &\leq 2\lambda_1^{1/2} \int_{G_i \cap \partial B^3(R)} u_i \cdot \frac{\partial u_i}{\partial r} \\ &\leq \lambda_1 \int_{G_i \cap \partial B^3(R)} u_i^2 + \int_{G_i \cap \partial B^3(R)} \left(\frac{\partial u_i}{\partial r} \right)^2 \\ &\leq \int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2 + \int_{G_i \cap \partial B^3(R)} \left(\frac{\partial u_i}{\partial r} \right)^2 = \int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2. \end{aligned}$$

Here $\lambda_1^{1/2}(G_i \cap \partial B^3(R))$ denotes the first Dirichlet eigenvalue on $G_i \cap \partial B^3(R)$. We know that

$$\lambda_1^{1/2}(G_i \cap \partial B^3(R)) \geq \frac{\pi^2}{\ell^2(G_i \cap \partial B^3(R))}$$

in \mathbb{R}^3 . Therefore

$$\frac{\int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2}{\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2} \geq 2\lambda_1^{1/2} \geq \frac{2\pi}{\ell(\Gamma_i)},$$

where $\Gamma_i := G_i \cap \partial B^3(R)$. Thus we obtain

$$\sum_{i=1}^k \frac{\int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2}{\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2} \geq \sum_{i=1}^k \frac{2\pi}{\ell(\Gamma_i)}.$$

Notice that

$$k^2 \leq \left(\sum_{i=1}^k \ell(\Gamma_i) \right) \left(\sum_{i=1}^k \frac{1}{\ell(\Gamma_i)} \right).$$

According to [Theorem 2.1](#), one has

$$\sum_{i=1}^k \ell(\Gamma_i) \leq \pi^2 R + o(1)R$$

for a sufficiently large radius R . Then it can be concluded that

$$(6) \quad \sum_{i=1}^k \frac{\int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2}{\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2} \geq \frac{2\pi k^2}{R(\pi^2 + o(1))}.$$

Observing that

$$(7) \quad \int_{G_i \cap \partial B^3(r)} |\tilde{\nabla} u_i|^2 = \frac{\partial}{\partial r} \int_{G_i \cap B^3(r)} |\tilde{\nabla} u_i|^2,$$

we obtain from (6) that

$$(8) \quad \ln \prod_{i=1}^k \frac{\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2}{\int_{G_i \cap B^3(R_0)} |\tilde{\nabla} u_i|^2} \geq \frac{2\pi k^2}{\pi^2 + o(1)} \ln \frac{R}{R_0}.$$

Let $(x, y, u_i(x, y))$ be a parametrization of G_i , so the induced metric on G_i is

$$ds^2 = (1 + (u_i)_x)^2 dx^2 + 2(u_i)_x (u_i)_y dx dy + (1 + (u_i)_y)^2 dy^2.$$

We then have

$$|\tilde{\nabla} u_i| = \sqrt{u_{x^i} u_{x^j} g^{ij}} = \sqrt{\frac{|\nabla u_i|^2}{1 + |\nabla u_i|^2}} \leq 1,$$

from which one can deduce

$$(9) \quad \prod_{i=1}^k \int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2 \leq A^k (G_i \cap B^3(R)) \leq (3\pi R^2)^k.$$

Combining (8) and (9) implies

$$\frac{2\pi k^2}{\pi^2 + o(1)} (\ln R - \ln R_0) \leq 2k \ln R + c_1.$$

Letting $R \rightarrow +\infty$ we see that $k \leq \pi$; in particular, $k \leq 3$.

If $k = 3$, an analogous argument using the refined length estimate in Theorem 2.1 leads to $k \leq 2\sqrt{2}$, which is a contradiction. Thus k has to be at most 2. □

Remark. Tkachev [2009] has already proved the number of two dimensional disjointly supported minimal graphs is at most 3. Here a different approach can lead to a better estimate if assuming the Gauss curvature vanishes at infinity.

3. Proof of Corollary 1.2

Let $\pi_i : G_i \rightarrow \mathbb{R}^2$ be the natural projective map and $B^2(R)$ be the ball of radius R in \mathbb{R}^2 . By employing the same method in the proof of Theorem 1.1 except for using a different region of integration $\pi_i^{-1}(\Omega_i \cap B^2(R))$, one can conclude

Theorem 3.1. *Suppose $\{(\Omega_i, u_i)\}_{i=1}^k$ is a set of disjointly supported minimal graphs in R^3 where each Ω_i is an open subset in R^2 . If the gradient of each u_i is bounded, say $|\nabla u_i| \leq c$, then k satisfies $k \leq 2\sqrt{1 + c^2}$.*

Proof. By a similar argument, one can obtain that

$$\sum_{i=1}^k \frac{\int_{\pi_i^{-1}(\Omega_i \cap \partial B^2(R))} |\tilde{\nabla} u_i|^2}{\int_{\pi_i^{-1}(\Omega_i \cap B^2(R))} |\tilde{\nabla} u_i|^2} \geq \frac{2\pi k^2}{\sum_{i=1}^k \ell(\Gamma_i)}.$$

where $\Gamma_i := \pi_i^{-1}(\Omega_i \cap \partial B^2(R))$. If one chooses the parametrization

$$(R \cos \theta, R \sin \theta, u_i(R \cos \theta, R \sin \theta))$$

for the curve Γ_i and assume $|\nabla u_i| \leq c$, then

$$\begin{aligned} \ell(\Gamma_i) &= \int_{\theta_0}^{\theta_1} \sqrt{R^2 + (-(u_i)_x R \sin(\theta) + (u_i)_y R \cos(\theta))^2} d\theta \\ &\leq \int_{\theta_0}^{\theta_1} \sqrt{R^2 + ((u_i)_x^2 + (u_i)_y^2)(R^2 \sin^2(\theta) + R^2 \cos^2(\theta))} d\theta \\ &\leq (\theta_1 - \theta_0) R \sqrt{1 + c^2}. \end{aligned}$$

Since the minimal graphs are disjoint, we get

$$\sum_{i=1}^k \ell(\Gamma_i) \leq 2\pi R \sqrt{1 + c^2}.$$

Then it can be concluded that

$$(10) \quad \sum_{i=1}^k \frac{\int_{\pi_i^{-1}(\Omega_i \cap \partial B^2(R))} |\tilde{\nabla} u_i|^2}{\int_{\pi_i^{-1}(\Omega_i \cap B^2(R))} |\tilde{\nabla} u_i|^2} \geq \frac{k^2}{R\sqrt{1+c^2}}.$$

Integrating (10), one obtains

$$(11) \quad \ln \prod_{i=1}^k \frac{\int_{\pi_i^{-1}(\Omega_i \cap \partial B^2(R))} |\tilde{\nabla} u_i|^2}{\int_{\pi_i^{-1}(\Omega_i \cap B^2(R_0))} |\tilde{\nabla} u_i|^2} \geq \frac{k^2}{\sqrt{1+c^2}} \ln \frac{R}{R_0}.$$

On the other hand,

$$(12) \quad \prod_{i=1}^k \int_{\pi_i^{-1}(\Omega_i \cap B^2(R))} |\tilde{\nabla} u_i|^2 \leq A^k (\pi_i^{-1}(\Omega_i \cap B^2(R))) \\ = \left(\int_{\Omega_i \cap B^2(R)} \sqrt{1+|\nabla u|^2} \right)^k \leq (\sqrt{1+c^2} \pi R^2)^k.$$

Combining (11) and (12), we have

$$\frac{k^2}{\sqrt{1+c^2}} (\ln R - \ln R_0) \leq 2k \ln R + c_1.$$

Letting $R \rightarrow +\infty$ yields

$$k \leq 2\sqrt{1+c^2}. \quad \square$$

Obviously, [Corollary 1.2](#) follows from above theorem when each graph satisfies

$$|\nabla u_i| \rightarrow 0 \quad (|x| \rightarrow +\infty).$$

Remark. J. Spruck [2002] proved [Corollary 1.2](#) under the assumption of a certain decay rate at infinity for the Gauss curvature. However, here we do not need any restrictions on the Gauss curvature.

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Extending triangulations of the 2-sphere to the 3-disk preserving a 4-coloring	257
RUI PEDRO CARPENTIER	
Orthogonal quantum group invariants of links	267
LIN CHEN and QINGTAO CHEN	
Some properties of squeezing functions on bounded domains	319
FUSHENG DENG, QIAN GUAN and LIYOU ZHANG	
Representations of little q -Schur algebras	343
JIE DU, QIANG FU and JIAN-PAN WANG	
Renormalized weighted volume and conformal fractional Laplacians	379
MARÍA DEL MAR GONZÁLEZ	
The L_4 norm of Littlewood polynomials derived from the Jacobi symbol	395
JONATHAN JEDWAB and KAI-UWE SCHMIDT	
On a conjecture of Kaneko and Ohno	419
ZHONG-HUA LI	
Categories of unitary representations of Banach–Lie supergroups and restriction functors	431
STÉPHANE MERIGON, KARL-HERMANN NEEB and HADI SALMASIAN	
Odd Hamiltonian superalgebras and special odd Hamiltonian superalgebras of formal vector fields	471
LI REN, QIANG MU and YONGZHENG ZHANG	
Interior derivative estimates for the Kähler–Ricci flow	491
MORGAN SHERMAN and BEN WEINKOVE	
Two-dimensional disjoint minimal graphs	503
LINFENG ZHOU	