CORRECTION TO THE ARTICLE
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PREPROJECTIVE ALGEBRA

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## QUIVER GRASSMANNIANS, QUIVER VARIETIES AND THE PREPROJECTIVE ALGEBRA

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**For quivers not of finite or affine type, certain isomorphisms asserted in the article under correction do not hold, as pointed out by Sarah Scherotzke. This note describes the affected results briefly. A corrected version of the paper can be found at arXiv 0909.3746.**

The original published version of this paper contained the following errors. We thank Sarah Scherotzke for bringing this to our attention.

**Error 1.** If $\mathfrak{g}$ is not of finite or affine type, then the Nakajima quiver variety $\Lambda(v, w)$ is not actually isomorphic to the variety $\mathrm{Gr}(v, q^w)$ of all $v$-dimensional subrepresentations of the injective module $q^w$. In fact, beyond affine type, $\mathrm{Gr}(v, q^w)$ does not have a natural variety structure, or, at least, is not finite-dimensional. This is because there are continuous families of nonisomorphic modules, all of which have a nontrivial extension with some one-dimensional simple module $S_i$.

There are two ways to modify the statement to make it true, and, with either of these modifications, the work in the original paper does prove the correct result. One must replace $\mathrm{Gr}(v, q^w)$ with either the variety $\mathrm{NGr}(v, q^w)$ of nilpotent $v$-dimensional subrepresentations of $q^w$, or with the variety $\mathrm{Gr}(v, \tilde{q}^w)$ of all $v$-dimensional subrepresentations, but where the injective hull $q^w$ in the category of all representations of the preprojective algebra has been replaced with the injective hull $\tilde{q}^w$ in the category of locally nilpotent representations. Our work shows that these are naturally isomorphic, and are also isomorphic to $\Lambda(v, w)$.

**Error 2.** Lemma 2.9 (which essentially asserted that $\mathrm{Gr}(v, q^w)$ and $\mathrm{NGr}(v, q^w)$ were isomorphic) is false beyond affine type, and should be removed. The proof is simply incorrect. In fact, this caused most of the issues in Error 1.

A corrected version of the paper that addresses these points can be found at arXiv 0909.3746. We show that both of the fixes to Error 1 discussed above work, although we mainly work with $\mathrm{Gr}(v, \tilde{q}^w)$. The reason is that $\tilde{q}^w$ is a direct limit of finite-dimensional varieties, and each quiver grassmannian is contained in one of these. Thus, with this viewpoint, the quiver grassmannians are naturally subvarieties of ordinary grassmannians, which we find helpful.

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