Pacific Journal of Mathematics

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PETR PETRÁČEK AND JIŘÍ SPURNÝ

Volume 276 No. 1 July 2015

PACIFIC JOURNAL OF MATHEMATICS
Vol. 276, No. 1, 2015
dx.doi.org/10.2140/pjm.2015.276.249

ON MAXIMAL LINDENSTRAUSS SPACES

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We solve a problem of Lacey (1973) by showing that there exist a metrizable compact space K and a closed space $\mathcal{H} \subset \mathcal{C}(K)$ containing constants with $\overline{\partial_{\mathcal{H}} K} = K$ such that $\mathcal{H}$ is maximal with respect to $\partial_{\mathcal{H}} K$ and $\mathcal{H}$ is not a Lindenstrauss space.

1. Introduction

Let X be a compact convex subset of a real locally convex space and let $\mathfrak{A}^c(X)$ denote the space of all affine continuous functions on X. Denote by $\operatorname{ext} X$ the set of all extreme points of X.

Let K be a compact Hausdorff topological space and $\mathcal{H} \subset \mathcal{C}(K)$ a closed subspace of $\mathcal{C}(K)$ containing constants and separating points of K. The space $\mathcal{H}$ can be identified with $\mathfrak{A}^c(X)$, where

$$X = \{s^* \in \mathcal{H}^* : s^*(1) = \|s^*\| = 1\}$$

with the weak* topology. Consider the set

$$\partial_{\mathcal{H}} K = \{x \in K : \varepsilon_x|_{\mathcal{H}} \text{ is an extreme point of the unit ball of } \mathcal{H}^*\},$$

where ε_x denotes the Dirac measure at $x \in K$. Then $\operatorname{ext} X$ is homeomorphic to $\partial_{\mathcal{H}} K$ via the evaluation mapping (see Theorem 2.1 and [LMNS 2010, Proposition 4.26]).

The space $\mathcal{H}$ is called *maximal with respect to* $\partial_{\mathcal{H}} K$ if for every closed space $\mathcal{G}$ with $\mathcal{H} \subset \mathcal{G} \subset \mathcal{C}(K)$ we have $\mathcal{H} = \mathcal{G}$ provided $\partial_{\mathcal{H}} K = \partial_{\mathcal{G}} K$.

(In [Lacey 1973], the property of separating points is not a part of the definition of a function space. Nevertheless, in our opinion, this property is necessary for $\partial_{\mathcal{H}} K$ to be homeomorphic to $\operatorname{ext} X$. Indeed, consider $\mathcal{H} = \operatorname{span}\{1\}$ on $[0, 1]$. Then X is a singleton, and thus $\partial_{\mathcal{H}}[0, 1] = [0, 1]$. Obviously, $[0, 1]$ is not homeomorphic to $\operatorname{ext} X$.)

It is shown in [Edwards and Vincent-Smith 1968] that $\mathcal{H}$ is maximal with respect to $\partial_{\mathcal{H}} K$ whenever $\mathcal{H}$ is a Lindenstrauss space; see Theorems 2.1 and 2.3 below.

The research was supported by grant GAČR P201/12/0290. Spurný was also supported by the Neuron Fund for Support of Science.
MSC2010: 46B25.
Keywords: Lindenstrauss space, L_1-predual, function space.

(A real Banach space X is called a *Lindenstrauss space*, or an L_1*-predual*, if its dual space X^* is isometric to a space $L_1(X, \mathcal{S}, \mu)$ for some measure space $(X, \mathcal{S}, \mu)$.) This result serves as a motivation for the following problem, stated as Question 5 in [Lacey 1973, p. 144] (see also [Lacey 1974, p. 198]).

Question 1.1. Let K be a compact space and $\mathcal{H} \subset \mathcal{C}(K)$ a closed subspace containing constants and separating points of K such that $\overline{\partial_{\mathcal{H}} K} = K$. Let $\mathcal{H}$ be maximal with respect to $\partial_{\mathcal{H}} K$. Is $\mathcal{H}$ then a Lindenstrauss space?

The aim of our paper is to show that the answer to Question 1.1 is in general negative by proving the following theorem.

Theorem 1.2. *There exist a metrizable compact space K and a closed space $\mathcal{H} \subset \mathcal{C}(K)$ containing constants and separating points of K with $\overline{\partial_{\mathcal{H}} K} = K$ such that $\mathcal{H}$ is maximal with respect to $\partial_{\mathcal{H}} K$ and $\mathcal{H}$ is not a Lindenstrauss space.*

2. Function spaces

Let K be a compact space (we consider all topological spaces as Hausdorff). We identify the dual of $\mathcal{C}(K)$ with the space $\mathcal{M}(K)$ of all signed Radon measures on K. By a positive Radon measure on K we mean a finite complete inner regular measure defined at least on all Borel subsets of K. Let $\mathcal{M}^1(K)$ denote the set of all probability Radon measures on K, $\mathcal{M}^+(K)$ the set of all positive Radon measures on K, and ε_x the Dirac measure at $x \in K$.

By a *function space* $\mathcal{H}$ on K we mean a subspace $\mathcal{H}$ of $\mathcal{C}(K)$ containing constants and separating points of K. Assuming $\mathcal{H}$ is a function space on K we assign to each $x \in K$ the set

$$\mathcal{M}_x(\mathcal{H}) = \{\mu \in \mathcal{M}^1(K) : \mu(h) = h(x), h \in \mathcal{H}\}$$

of all $\mathcal{H}$*-representing measures*. Clearly, $\varepsilon_x \in \mathcal{M}_x(\mathcal{H})$ for each $x \in K$. We call

$$\operatorname{Ch}_{\mathcal{H}} K = \{x \in K : \mathcal{M}_x(\mathcal{H}) = \{\varepsilon_x\}\}$$

the *Choquet boundary* of $\mathcal{H}$. If $h \in \mathcal{H}$ attains its strict minimum at some $x \in K$, we call h an $\mathcal{H}$*-exposing function* and x an $\mathcal{H}$*-exposed point*. It is easy to see that any $\mathcal{H}$-exposed point belongs to the Choquet boundary of $\mathcal{H}$.

We define the space $\mathcal{A}^c(\mathcal{H})$ of all continuous $\mathcal{H}$*-affine functions* to be the family of all continuous functions f on K satisfying

$$f(x) = \int_K f \, d\mu \quad \text{for each } x \in K \text{ and } \mu \in \mathcal{M}_x(\mathcal{H}).$$

$\mathcal{A}^c(\mathcal{H})$ is a closed function space containing $\mathcal{H}$ and satisfying $\mathcal{M}_x(\mathcal{H}) = \mathcal{M}_x(\mathcal{A}^c(\mathcal{H}))$ for every $x \in K$. Thus $\operatorname{Ch}_{\mathcal{H}} K = \operatorname{Ch}_{\mathcal{A}^c(\mathcal{H})} K$. We define the *state space* of $\mathcal{H}$ as

$$\boldsymbol{S}(\mathcal{H}) = \{s^* \in \mathcal{H}^* : s(1) = \|s\| = 1\}$$

endowed with the weak* topology. The state space $\boldsymbol{S}(\mathcal{H})$ is a compact convex set and K is homeomorphically embedded into $\boldsymbol{S}(\mathcal{H})$ via $\phi : K \to \boldsymbol{S}(\mathcal{H})$, where

$$\phi(x) : h \to h(x), \quad h \in \mathcal{H},\ x \in K.$$

Let $B_{\mathcal{H}^*}$ stand for the unit ball of $\mathcal{H}^*$. Following the notation in [Lacey 1973, p. 143] mentioned in the introduction,

$$\partial_{\mathcal{H}} K = \{x \in K : \phi(x) \in \operatorname{ext} B_{\mathcal{H}^*}\}.$$

The next assertion shows that our definition of the Choquet boundary coincides with Lacey's definition of $\partial_{\mathcal{H}} K$.

Theorem 2.1. *If $\mathcal{H}$ is a function space on a compact space K, then* $\operatorname{Ch}_{\mathcal{H}} K = \partial_{\mathcal{H}} K$.

Proof. By [LMNS 2010, Proposition 4.26(d)], $\phi(\operatorname{Ch}_{\mathcal{H}} K) = \operatorname{ext} \boldsymbol{S}(\mathcal{H})$. Since $\boldsymbol{S}(\mathcal{H})$ is a face of $B_{\mathcal{H}^*}$ (see [LMNS 2010, Section 2.3.A]), we have $\operatorname{ext} \boldsymbol{S}(\mathcal{H}) = \operatorname{ext} B_{\mathcal{H}^*} \cap \boldsymbol{S}(\mathcal{H})$. Thus, given any $x \in K$, we have $\phi(x) \in \operatorname{ext} \boldsymbol{S}(\mathcal{H})$ if and only if $\phi(x) \in \operatorname{ext} B_{\mathcal{H}^*}$. □

The *Choquet ordering* on $\mathcal{M}^+(K)$ is given as follows: $\mu \prec \nu$ if $\mu(k) \le \nu(k)$ for each function k of the form $k = \max\{h_1, \dots, h_n\}$, where $n \in \mathbb{N}$ and $h_1, \dots, h_n \in \mathcal{H}$ (see [LMNS 2010, Definition 3.19 and Proposition 3.56]). A measure μ in $\mathcal{M}^+(K)$ is called *$\mathcal{H}$-maximal* if it is $\prec$-maximal. By [LMNS 2010, Theorem 3.65], there exists an $\mathcal{H}$-maximal measure $\mu \in \mathcal{M}_x(\mathcal{H})$ for every $x \in K$. Furthermore, if K is metrizable, the set $\operatorname{Ch}_{\mathcal{H}} K$ is G_δ (see [LMNS 2010, Theorem 3.42 and Proposition 3.43]) and $\mathcal{H}$-maximal measures are precisely those measures carried by $\operatorname{Ch}_{\mathcal{H}} K$ (see [LMNS 2010, Corollary 3.62]).

If for each $x \in K$ there exists only one $\mathcal{H}$-maximal measure in $\mathcal{M}_x(\mathcal{H})$, the function space $\mathcal{H}$ is called *simplicial* (see [LMNS 2010, Chapter 6]). A compact convex set X is called a *simplex* if the function space $\mathfrak{A}^c(X)$ is simplicial. The relation between simplicial function spaces and Lindenstrauss spaces is given by the following result.

Theorem 2.2. *Let $\mathcal{H}$ be a function space on a compact space K. Then $\mathcal{H}$ is simplicial if and only if the Banach space $\mathcal{A}^c(\mathcal{H})$ is a Lindenstrauss space.*

Proof. Let $\mathcal{A}^c(\mathcal{H})$ be a Lindenstrauss space. Since $\mathfrak{A}^c\big(\boldsymbol{S}(\mathcal{A}^c(\mathcal{H}))\big)$ is isometric to the space $\mathcal{A}^c(\mathcal{H})$ (see [LMNS 2010, Proposition 4.26]), it is a Lindenstrauss space as well. By [Fonf et al. 2001, Proposition 3.23], $\boldsymbol{S}(\mathcal{A}^c(\mathcal{H}))$ is a simplex. Thus it follows from [LMNS 2010, Theorem 6.54] that $\mathcal{H}$ is simplicial.

Conversely, if $\mathcal{H}$ is simplicial, $\boldsymbol{S}(\mathcal{A}^c(\mathcal{H}))$ is a simplex by [LMNS 2010, Theorem 6.54]. Using [Fonf et al. 2001, Proposition 3.23] we conclude that $\mathcal{A}^c(\mathcal{H})$, being isometric to $\mathfrak{A}^c\big(\boldsymbol{S}(\mathcal{A}^c(\mathcal{H}))\big)$, is a Lindenstrauss space. □

The next result asserts two important properties of closed function spaces that are Lindenstrauss spaces. As mentioned above, it can be considered a motivation for the question this paper aims to answer.

Theorem 2.3. *Let $\mathcal{H}$ be a closed function space on a compact space K such that $\mathcal{H}$ is a Lindenstrauss space. Then $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$ and $\mathcal{H}$ is maximal with respect to* $\mathrm{Ch}_{\mathcal{H}} K$.

Proof. To prove the first assertion notice that $\mathfrak{A}^c(\boldsymbol{S}(\mathcal{H}))$, being isometric to $\mathcal{H}$ (see [LMNS 2010, Proposition 4.26]), is a Lindenstrauss space. By [Fonf et al. 2001, Proposition 3.23], $\boldsymbol{S}(\mathcal{H})$ is a simplex. This implies that $\mathfrak{A}^c(\boldsymbol{S}(\mathcal{H}))$ is simplicial and thus, by [LMNS 2010, Theorem 6.16(vi)], $\mathfrak{A}^c(\boldsymbol{S}(\mathcal{H}))$ has the so-called weak Riesz interpolation property. This, however, implies that $\mathcal{H}$ has the weak Riesz interpolation property according to [LMNS 2010, Proposition 4.26]. To finish the proof it is enough to consult [LMNS 2010, Exercise 6.78].

To prove the second assertion, let $\mathcal{G} \supset \mathcal{H}$ be a closed function space with $\mathrm{Ch}_{\mathcal{H}} K = \mathrm{Ch}_{\mathcal{G}} K$. Since $\mathcal{G} \subset \mathcal{A}^c(\mathcal{G})$ and $\mathrm{Ch}_{\mathcal{G}} K = \mathrm{Ch}_{\mathcal{A}^c(\mathcal{G})} K$, we can assume without loss of generality that $\mathcal{A}^c(\mathcal{G}) = \mathcal{G}$. Using [LMNS 2010, Theorem 10.60] we infer that $\mathcal{G} = \mathcal{A}^c(\mathcal{H})$. Since $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$, we get $\mathcal{G} = \mathcal{H}$, finishing the proof. □

3. Proof of Theorem 1.2

We consider a compact subset of $\mathbb{R}^2$ defined as follows. Let $\{s, s^1, s^2, t^1, t^2\}$ be distinct points in $\mathbb{R}^2$. Let (s_n^i) and (t_n^i), $i = 0, 1, 2$, be sequences of points in $\mathbb{R}^2$ such that

- $s_n^0 \to s,\ t_n^0 \to s,$
- $s_n^i \to s^i,\ t_n^i \to t^i,\ i = 1, 2,$
- all the elements of these sequences are pairwise distinct and not contained in $\{s, s^1, s^2, t^1, t^2\}$.

Let $B(x, r)$ denote the closed ball in $\mathbb{R}^2$ with center $x \in \mathbb{R}^2$ and diameter $r > 0$. Let further $r_n > 0$, $n \in \mathbb{N}$, be numbers such that

- $r_n \to 0,$
- the family

$$\mathcal{K} = \big\{\{s\}, \{s^1\}, \{s^2\}, \{t^1\}, \{t^2\}\big\} \cup \{B(s_n^0, r_n) : n \in \mathbb{N}\} \cup \{B(t_n^0, r_n) : n \in \mathbb{N}\}$$

  is disjoint.

We define the compact space K as $K = \bigcup \mathcal{K}$. Furthermore, we set $\mathcal{H}$ to be

$$\mathcal{H} = \big\{h \in \mathcal{C}(K) : h(s) = \tfrac{1}{2}\big(h(s^1) + h(s^2)\big) = \tfrac{1}{2}\big(h(t^1) + h(t^2)\big),\\ h(s_n^0) = \tfrac{1}{2}\big(h(s_n^1) + h(s_n^2)\big),\ h(t_n^0) = \tfrac{1}{2}\big(h(t_n^1) + h(t_n^2)\big),\ n \in \mathbb{N}\big\}.$$

Lemma 3.1. *The space $\mathcal{H}$ is a well defined function space with $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$. Let $L = \{s\} \cup \{s_n^0 : n \in \mathbb{N}\} \cup \{t_n^0 : n \in \mathbb{N}\}$. Then $\operatorname{Ch}_{\mathcal{H}} K = K \setminus L$. In particular, $\operatorname{Ch}_{\mathcal{H}} K$ is dense in K.*

Proof. Obviously, $\mathcal{H}$ contains constant functions. The fact that $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$ follows immediately from the definition of $\mathcal{H}$. To verify that $\mathcal{H}$ separates points of K it is enough to consider elementary constructions of functions from $\mathcal{H}$. Given $n \in \mathbb{N}$ and $z \in B(s_n^0, r_n) \setminus \{s_n^0\}$, we consider a continuous function $g : B(s_n^0, r_n) \to [0, 1]$ attaining 0 precisely at z and 1 at s_n^0. Then the function

$$h_z(x) = \begin{cases} g(x) & \text{if } x \in B(s_n^0, r_n), \\ 1 & \text{otherwise} \end{cases}$$

separates z from the remaining points of K. It also $\mathcal{H}$-exposes z, and thus $z \in \operatorname{Ch}_{\mathcal{H}} K$.

We can further construct functions h_{s_n} and h_s in $\mathcal{H}$ as follows:

$$h_{s_n}(x) = \begin{cases} 0 & \text{if } x = s_n^1, \\ 2 & \text{if } x \in B(s_n^0, r_n), \\ 4 & \text{if } x = s_n^2, \\ 1 & \text{otherwise}, \end{cases}$$

$$h_s(x) = \begin{cases} 0 & \text{if } x = s^1, \\ 2 & \text{if } x = s^2, \\ \frac{1}{2n} & \text{if } x = s_n^1, \\ 2 - \frac{1}{2n} & \text{if } x = s_n^2, \\ 1 & \text{otherwise}. \end{cases}$$

The function h_{s_n} then separates the points s_n^1, s_n^2 from any point in K and it separates s_n^0 from any point in $K \setminus B(s_n^0, r_n)$. Its construction also shows that the points s_n^1, s_n^2 are $\mathcal{H}$-exposed and thus lie in $\operatorname{Ch}_{\mathcal{H}} K$. Similarly, the function h_s separates points s^1, s, s^2 from each other and it separates s from every point in $\{s_n^i : n \in \mathbb{N}, i \in \{1, 2\}\}$. Furthermore, the construction of h_s shows that the points s^1, s^2 are $\mathcal{H}$-exposed and thus belong to $\operatorname{Ch}_{\mathcal{H}} K$.

Analogously we can construct functions $h_{t_n}, \tilde{h}_s$ and h_y for any $n \in \mathbb{N}$ and $y \in B(t_n^0, r_n) \setminus \{t_n^0\}$ to show that $\mathcal{H}$ indeed separates points of K and that all points in

$$\{t_1, t_2\} \cup \{t_n^i : n \in \mathbb{N}, i \in \{1, 2\}\} \cup \bigcup_{n \in \mathbb{N}} (B(t_n^0, r_n) \setminus \{t_n^0\})$$

lie in $\operatorname{Ch}_{\mathcal{H}} K$.

Overall, we have

$$\{s^1, s^2, t^1, t^2\} \cup \{s_n^i : n \in \mathbb{N}, i \in \{1, 2\}\} \cup \{t_n^i : n \in \mathbb{N}, i \in \{1, 2\}\} \subset \operatorname{Ch}_{\mathcal{H}} K$$

and

$$\bigcup_{n\in\mathbb{N}} (B(s_n^0, r_n) \setminus \{s_n^0\}) \cup \bigcup_{n\in\mathbb{N}} (B(t_n^0, r_n) \setminus \{t_n^0\}) \subset \operatorname{Ch}_{\mathcal{H}} K.$$

Clearly, any point in L has a nontrivial $\mathcal{H}$-representing measure. This together with the inclusions above yields $\operatorname{Ch}_{\mathcal{H}} K = K \setminus L$. □

Lemma 3.2. *Let $n \in \mathbb{N}$. Then*

$$\mathcal{M}_{s_n^0}(\mathcal{H}) = \operatorname{conv}\left\{\varepsilon_{s_n^0}, \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})\right\}$$

and

$$\mathcal{M}_{t_n^0}(\mathcal{H}) = \operatorname{conv}\left\{\varepsilon_{t_n^0}, \tfrac{1}{2}(\varepsilon_{t_n^1} + \varepsilon_{t_n^2})\right\}.$$

Proof. Let $n \in \mathbb{N}$ and $\mu \in \mathcal{M}_{s_n^0}(\mathcal{H})$ be fixed. Pick a continuous function

$$g : B(s_n^0, r_n) \to [0, 1]$$

such that $g(s_n^0) = 0$ and $g(x) > 0$ otherwise. Using the function

$$h(x) = \begin{cases} 0 & \text{if } x \in \{s_n^1, s_n^2\}, \\ g(x) & \text{if } x \in B(s_n^0, r_n), \\ 1 & \text{otherwise,} \end{cases}$$

we infer that the support of μ is contained in $\{s_n^0, s_n^1, s_n^2\}$.

Further, let $a = \mu(\{s_n^0\})$. Assume first that $a = 0$, i.e., $\mu = b\varepsilon_{s_n^1} + (1-b)\varepsilon_{s_n^2}$ for some $b \in [0, 1]$. Then the function

$$h(x) = \begin{cases} 0 & \text{if } x = s_n^1, \\ 1 & \text{if } x = s_n^0, \\ 2 & \text{if } x = s_n^2, \\ 1 & \text{otherwise} \end{cases}$$

shows that

$$1 = h(s_n^0) = \mu(h) = (1-b)h(s_n^2) = (1-b)2.$$

In other words, $b = \frac{1}{2}$ and $\mu = \frac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})$.

If $a \in (0, 1)$, then the measure $\nu = \mu - a\varepsilon_{s_n^0}$ satisfies

$$h(s_n^0) = \mu(h) = \nu(h) + ah(s_n^0), \quad h \in \mathcal{H}.$$

Hence $\dfrac{1}{1-a}\nu$ is in $\mathcal{M}_{s_n^0}(\mathcal{H})$ and is carried by $\{s_n^1, s_n^2\}$. By the first part of the proof,

$$\frac{1}{1-a}\nu = \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2}).$$

Thus

$$\mu = \nu + a\varepsilon_{s_n^0} = (1-a)\frac{1}{1-a}\nu + a\varepsilon_{s_n^0} = (1-a)\tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2}) + a\varepsilon_{s_n^0}$$

is in $\operatorname{conv}\{\varepsilon_{s_n^0}, \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})\}$. If $a = 1$, obviously

$$\mu = \varepsilon_{s_n^0} \in \operatorname{conv}\{\varepsilon_{s_n^0}, \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})\}.$$

Thus

$$\mu \in \operatorname{conv}\{\varepsilon_{s_n^0}, \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})\}$$

holds in all cases.

The second part of the assertion can be proved analogously. □

Lemma 3.3. *The space $\mathcal{H}$ is not simplicial.*

Proof. The measures $\frac{1}{2}(\varepsilon_{s^1} + \varepsilon_{s^2})$, $\frac{1}{2}(\varepsilon_{t^1} + \varepsilon_{t^2})$ are different, they $\mathcal{H}$-represent s and, by Lemma 3.1, both are carried by $\operatorname{Ch}_{\mathcal{H}} K$. Hence there exist two $\mathcal{H}$-maximal measures representing s, which implies that $\mathcal{H}$ is not simplicial. □

Lemma 3.4. *The space $\mathcal{H}$ is maximal with respect to* $\operatorname{Ch}_{\mathcal{H}} K$. *That is*, $\mathcal{G} = \mathcal{H}$ *for any closed function space $\mathcal{H} \subset \mathcal{G}$ such that* $\operatorname{Ch}_{\mathcal{G}} K = \operatorname{Ch}_{\mathcal{H}} K$.

Proof. Fix an index $m \in \mathbb{N}$. Let $\tau \in \mathcal{M}_{s_m^0}(\mathcal{G})$ be a measure carried by $\operatorname{Ch}_{\mathcal{G}} K$. We aim to show that

$$\tau = \tfrac{1}{2}(\varepsilon_{s_m^1} + \varepsilon_{s_m^2}). \tag{3-1}$$

Since $\mathcal{M}_{s_m^0}(\mathcal{G}) \subset \mathcal{M}_{s_m^0}(\mathcal{H})$, we obtain by virtue of Lemma 3.2 that

$$\tau \in \operatorname{conv}\{\varepsilon_{s_m^0}, \tfrac{1}{2}(\varepsilon_{s_m^1} + \varepsilon_{s_m^2})\}.$$

This and the fact that τ is carried by $\operatorname{Ch}_{\mathcal{G}} K = \operatorname{Ch}_{\mathcal{H}} K \subset K \setminus \{s_m^0\}$ imply (3-1).

Pick $\mu_n \in \mathcal{M}_{s_n^0}(\mathcal{G})$, $n \in \mathbb{N}$, such that the measures μ_n are carried by $\operatorname{Ch}_{\mathcal{G}} K$ for all $n \in \mathbb{N}$. The sequence (s_n^0) converges to s, while the sequence (μ_n) converges to $\mu = \frac{1}{2}(\varepsilon_{s^1} + \varepsilon_{s^2})$. Thus $\mu \in \mathcal{M}_s(\mathcal{G})$. Analogously we infer that any measure ν_n in $\mathcal{M}_{t_n^0}(\mathcal{G})$ carried by $\operatorname{Ch}_{\mathcal{G}} K$ satisfies $\nu_n = \frac{1}{2}(\varepsilon_{t_n^1} + \varepsilon_{t_n^2})$, and thus $\nu = \frac{1}{2}(\varepsilon_{t^1} + \varepsilon_{t^2})$ is in $\mathcal{M}_s(\mathcal{G})$.

We want to show that $\mathcal{G} \subset \mathcal{H}$. To this end, let $g \in \mathcal{G}$ be given. We have to verify the conditions defining the space $\mathcal{H}$. Using the arguments above we get

$$g(s_n^0) = \mu_n(g) = \tfrac{1}{2}(g(s_n^1) + g(s_n^2)) \quad \text{and} \quad g(s) = \mu(g) = \tfrac{1}{2}(g(s^1) + g(s^2)),$$

while simultaneously

$$g(t_n^0) = \nu_n(g) = \tfrac{1}{2}(g(t_n^1) + g(t_n^2)) \quad \text{and} \quad g(s) = \nu(g) = \tfrac{1}{2}(g(t^1) + g(t^2)).$$

Hence $g \in \mathcal{H}$ by definition. This concludes the proof. □

Thus we have proved Theorem 1.2. Indeed, considering the compact space K and the closed function space $\mathcal{H} \subset \mathcal{C}(K)$ defined above, we have by Lemma 3.1 that $\mathrm{Ch}_{\mathcal{H}} K$ is dense in K. Furthermore, $\mathcal{H}$ is maximal with respect to $\mathrm{Ch}_{\mathcal{H}} K$ by Lemma 3.4. Since $\mathcal{H}$ is not simplicial according to Lemma 3.3, Theorem 2.2 asserts that $\mathcal{A}^c(\mathcal{H})$ is not a Lindenstrauss space. Since $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$ by Lemma 3.1, it follows that $\mathcal{H}$ is not a Lindenstrauss space.

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Received May 5, 2014.

PETR PETRÁČEK
DEPARTMENT OF MATHEMATICAL ANALYSIS
FACULTY OF MATHEMATICS AND PHYSICS
CHARLES UNIVERSITY
SOKOLOVSKÁ 83
186 75 PRAHA 8
CZECH REPUBLIC
petracek@karlin.mff.cuni.cz

JIŘÍ SPURNÝ
DEPARTMENT OF MATHEMATICAL ANALYSIS
FACULTY OF MATHEMATICS AND PHYSICS
CHARLES UNIVERSITY
SOKOLOVSKÁ 83
186 75 PRAHA 8
CZECH REPUBLIC
spurny@karlin.mff.cuni.cz

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The Pacific Journal of Mathematics (ISSN 0030-8730) at the University of California, c/o Department of Mathematics, 798 Evans Hall #3840, Berkeley, CA 94720-3840, is published twelve times a year. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163.

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PACIFIC JOURNAL OF MATHEMATICS

Volume 276 No. 1 July 2015

0030-8730(201507)276:1;1-Y