Pacific
Journal of
Mathematics

ON MAXIMAL LINDENSTRAUSS SPACES

Petr Petráček and Jiří Spurný

Volume 276 No. 1 July 2015

PACIFIC JOURNAL OF MATHEMATICS
Vol. 276, No. 1, 2015
dx.doi.org/10.2140/pjm.2015.276.249

ON MAXIMAL LINDENSTRAUSS SPACES

PETR PETRÁČEK AND JIŘÍ SPURNÝ

We solve a problem of Lacey (1973) by showing that there exist a metrizable compact space K and a closed space $\mathcal{H} \subset \mathcal{C}(K)$ containing constants with $\overline{\partial_{\mathcal{H}} K} = K$ such that $\mathcal{H}$ is maximal with respect to $\partial_{\mathcal{H}} K$ and $\mathcal{H}$ is not a Lindenstrauss space.

1. Introduction

Let X be a compact convex subset of a real locally convex space and let $\mathfrak{A}^c(X)$ denote the space of all affine continuous functions on X. Denote by $\operatorname{ext} X$ the set of all extreme points of X.

Let K be a compact Hausdorff topological space and $\mathcal{H} \subset \mathcal{C}(K)$ a closed subspace of $\mathcal{C}(K)$ containing constants and separating points of K. The space $\mathcal{H}$ can be identified with $\mathfrak{A}^c(X)$, where

$$X = \{s^* \in \mathcal{H}^* : s^*(1) = \|s^*\| = 1\}$$

with the weak* topology. Consider the set

$$\partial_{\mathcal{H}} K = \{x \in K : \varepsilon_x|_{\mathcal{H}} \text{ is an extreme point of the unit ball of } \mathcal{H}^*\},$$

where ε_x denotes the Dirac measure at $x \in K$. Then $\operatorname{ext} X$ is homeomorphic to $\partial_{\mathcal{H}} K$ via the evaluation mapping (see Theorem 2.1 and [LMNS 2010, Proposition 4.26]).

The space $\mathcal{H}$ is called *maximal with respect to* $\partial_{\mathcal{H}} K$ if for every closed space $\mathcal{G}$ with $\mathcal{H} \subset \mathcal{G} \subset \mathcal{C}(K)$ we have $\mathcal{H} = \mathcal{G}$ provided $\partial_{\mathcal{H}} K = \partial_{\mathcal{G}} K$.

(In [Lacey 1973], the property of separating points is not a part of the definition of a function space. Nevertheless, in our opinion, this property is necessary for $\partial_{\mathcal{H}} K$ to be homeomorphic to $\operatorname{ext} X$. Indeed, consider $\mathcal{H} = \operatorname{span}\{1\}$ on $[0, 1]$. Then X is a singleton, and thus $\partial_{\mathcal{H}}[0, 1] = [0, 1]$. Obviously, $[0, 1]$ is not homeomorphic to $\operatorname{ext} X$.)

It is shown in [Edwards and Vincent-Smith 1968] that $\mathcal{H}$ is maximal with respect to $\partial_{\mathcal{H}} K$ whenever $\mathcal{H}$ is a Lindenstrauss space; see Theorems 2.1 and 2.3 below.

The research was supported by grant GAČR P201/12/0290. Spurný was also supported by the Neuron Fund for Support of Science.
MSC2010: 46B25.
Keywords: Lindenstrauss space, L_1-predual, function space.

(A real Banach space X is called a *Lindenstrauss space*, or an *L_1-predual*, if its dual space X^* is isometric to a space $L_1(X, \mathcal{S}, \mu)$ for some measure space $(X, \mathcal{S}, \mu)$.) This result serves as a motivation for the following problem, stated as Question 5 in [Lacey 1973, p. 144] (see also [Lacey 1974, p. 198]).

Question 1.1. Let K be a compact space and $\mathcal{H} \subset \mathcal{C}(K)$ a closed subspace containing constants and separating points of K such that $\overline{\partial_{\mathcal{H}} K} = K$. Let $\mathcal{H}$ be maximal with respect to $\partial_{\mathcal{H}} K$. Is $\mathcal{H}$ then a Lindenstrauss space?

The aim of our paper is to show that the answer to Question 1.1 is in general negative by proving the following theorem.

Theorem 1.2. *There exist a metrizable compact space K and a closed space $\mathcal{H} \subset \mathcal{C}(K)$ containing constants and separating points of K with $\overline{\partial_{\mathcal{H}} K} = K$ such that $\mathcal{H}$ is maximal with respect to $\partial_{\mathcal{H}} K$ and $\mathcal{H}$ is not a Lindenstrauss space.*

2. Function spaces

Let K be a compact space (we consider all topological spaces as Hausdorff). We identify the dual of $\mathcal{C}(K)$ with the space $\mathcal{M}(K)$ of all signed Radon measures on K. By a positive Radon measure on K we mean a finite complete inner regular measure defined at least on all Borel subsets of K. Let $\mathcal{M}^1(K)$ denote the set of all probability Radon measures on K, $\mathcal{M}^+(K)$ the set of all positive Radon measures on K, and ε_x the Dirac measure at $x \in K$.

By a *function space* $\mathcal{H}$ on K we mean a subspace $\mathcal{H}$ of $\mathcal{C}(K)$ containing constants and separating points of K. Assuming $\mathcal{H}$ is a function space on K we assign to each $x \in K$ the set

$$\mathcal{M}_x(\mathcal{H}) = \{\mu \in \mathcal{M}^1(K) : \mu(h) = h(x), h \in \mathcal{H}\}$$

of all *$\mathcal{H}$-representing measures*. Clearly, $\varepsilon_x \in \mathcal{M}_x(\mathcal{H})$ for each $x \in K$. We call

$$\operatorname{Ch}_{\mathcal{H}} K = \{x \in K : \mathcal{M}_x(\mathcal{H}) = \{\varepsilon_x\}\}$$

the *Choquet boundary* of $\mathcal{H}$. If $h \in \mathcal{H}$ attains its strict minimum at some $x \in K$, we call h an *$\mathcal{H}$-exposing function* and x an *$\mathcal{H}$-exposed point*. It is easy to see that any $\mathcal{H}$-exposed point belongs to the Choquet boundary of $\mathcal{H}$.

We define the space $\mathcal{A}^c(\mathcal{H})$ of all continuous *$\mathcal{H}$-affine functions* to be the family of all continuous functions f on K satisfying

$$f(x) = \int_K f \, d\mu \quad \text{for each } x \in K \text{ and } \mu \in \mathcal{M}_x(\mathcal{H}).$$

$\mathcal{A}^c(\mathcal{H})$ is a closed function space containing $\mathcal{H}$ and satisfying $\mathcal{M}_x(\mathcal{H}) = \mathcal{M}_x(\mathcal{A}^c(\mathcal{H}))$ for every $x \in K$. Thus $\operatorname{Ch}_{\mathcal{H}} K = \operatorname{Ch}_{\mathcal{A}^c(\mathcal{H})} K$. We define the *state space* of $\mathcal{H}$ as

$$\boldsymbol{S}(\mathcal{H}) = \{s^* \in \mathcal{H}^* : s(1) = \|s\| = 1\}$$

endowed with the weak* topology. The state space $\boldsymbol{S}(\mathcal{H})$ is a compact convex set and K is homeomorphically embedded into $\boldsymbol{S}(\mathcal{H})$ via $\phi : K \to \boldsymbol{S}(\mathcal{H})$, where

$$\phi(x) : h \to h(x), \quad h \in \mathcal{H},\ x \in K.$$

Let $B_{\mathcal{H}^*}$ stand for the unit ball of $\mathcal{H}^*$. Following the notation in [Lacey 1973, p. 143] mentioned in the introduction,

$$\partial_{\mathcal{H}} K = \{x \in K : \phi(x) \in \operatorname{ext} B_{\mathcal{H}^*}\}.$$

The next assertion shows that our definition of the Choquet boundary coincides with Lacey's definition of $\partial_{\mathcal{H}} K$.

Theorem 2.1. *If $\mathcal{H}$ is a function space on a compact space K, then* $\operatorname{Ch}_{\mathcal{H}} K = \partial_{\mathcal{H}} K$.

Proof. By [LMNS 2010, Proposition 4.26(d)], $\phi(\operatorname{Ch}_{\mathcal{H}} K) = \operatorname{ext} \boldsymbol{S}(\mathcal{H})$. Since $\boldsymbol{S}(\mathcal{H})$ is a face of $B_{\mathcal{H}^*}$ (see [LMNS 2010, Section 2.3.A]), we have $\operatorname{ext} \boldsymbol{S}(\mathcal{H}) = \operatorname{ext} B_{\mathcal{H}^*} \cap \boldsymbol{S}(\mathcal{H})$. Thus, given any $x \in K$, we have $\phi(x) \in \operatorname{ext} \boldsymbol{S}(\mathcal{H})$ if and only if $\phi(x) \in \operatorname{ext} B_{\mathcal{H}^*}$. □

The *Choquet ordering* on $\mathcal{M}^+(K)$ is given as follows: $\mu \prec \nu$ if $\mu(k) \le \nu(k)$ for each function k of the form $k = \max\{h_1, \dots, h_n\}$, where $n \in \mathbb{N}$ and $h_1, \dots, h_n \in \mathcal{H}$ (see [LMNS 2010, Definition 3.19 and Proposition 3.56]). A measure μ in $\mathcal{M}^+(K)$ is called $\mathcal{H}$*-maximal* if it is $\prec$-maximal. By [LMNS 2010, Theorem 3.65], there exists an $\mathcal{H}$-maximal measure $\mu \in \mathcal{M}_x(\mathcal{H})$ for every $x \in K$. Furthermore, if K is metrizable, the set $\operatorname{Ch}_{\mathcal{H}} K$ is G_δ (see [LMNS 2010, Theorem 3.42 and Proposition 3.43]) and $\mathcal{H}$-maximal measures are precisely those measures carried by $\operatorname{Ch}_{\mathcal{H}} K$ (see [LMNS 2010, Corollary 3.62]).

If for each $x \in K$ there exists only one $\mathcal{H}$-maximal measure in $\mathcal{M}_x(\mathcal{H})$, the function space $\mathcal{H}$ is called *simplicial* (see [LMNS 2010, Chapter 6]). A compact convex set X is called a *simplex* if the function space $\mathfrak{A}^c(X)$ is simplicial. The relation between simplicial function spaces and Lindenstrauss spaces is given by the following result.

Theorem 2.2. *Let $\mathcal{H}$ be a function space on a compact space K. Then $\mathcal{H}$ is simplicial if and only if the Banach space $\mathcal{A}^c(\mathcal{H})$ is a Lindenstrauss space.*

Proof. Let $\mathcal{A}^c(\mathcal{H})$ be a Lindenstrauss space. Since $\mathfrak{A}^c\big(\boldsymbol{S}(\mathcal{A}^c(\mathcal{H}))\big)$ is isometric to the space $\mathcal{A}^c(\mathcal{H})$ (see [LMNS 2010, Proposition 4.26]), it is a Lindenstrauss space as well. By [Fonf et al. 2001, Proposition 3.23], $\boldsymbol{S}(\mathcal{A}^c(\mathcal{H}))$ is a simplex. Thus it follows from [LMNS 2010, Theorem 6.54] that $\mathcal{H}$ is simplicial.

Conversely, if $\mathcal{H}$ is simplicial, $\boldsymbol{S}(\mathcal{A}^c(\mathcal{H}))$ is a simplex by [LMNS 2010, Theorem 6.54]. Using [Fonf et al. 2001, Proposition 3.23] we conclude that $\mathcal{A}^c(\mathcal{H})$, being isometric to $\mathfrak{A}^c\big(\boldsymbol{S}(\mathcal{A}^c(\mathcal{H}))\big)$, is a Lindenstrauss space. □

The next result asserts two important properties of closed function spaces that are Lindenstrauss spaces. As mentioned above, it can be considered a motivation for the question this paper aims to answer.

Theorem 2.3. *Let $\mathcal{H}$ be a closed function space on a compact space K such that $\mathcal{H}$ is a Lindenstrauss space. Then $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$ and $\mathcal{H}$ is maximal with respect to* $\mathrm{Ch}_{\mathcal{H}} K$.

Proof. To prove the first assertion notice that $\mathfrak{A}^c(\boldsymbol{S}(\mathcal{H}))$, being isometric to $\mathcal{H}$ (see [LMNS 2010, Proposition 4.26]), is a Lindenstrauss space. By [Fonf et al. 2001, Proposition 3.23], $\boldsymbol{S}(\mathcal{H})$ is a simplex. This implies that $\mathfrak{A}^c(\boldsymbol{S}(\mathcal{H}))$ is simplicial and thus, by [LMNS 2010, Theorem 6.16(vi)], $\mathfrak{A}^c(\boldsymbol{S}(\mathcal{H}))$ has the so-called weak Riesz interpolation property. This, however, implies that $\mathcal{H}$ has the weak Riesz interpolation property according to [LMNS 2010, Proposition 4.26]. To finish the proof it is enough to consult [LMNS 2010, Exercise 6.78].

To prove the second assertion, let $\mathcal{G} \supset \mathcal{H}$ be a closed function space with $\mathrm{Ch}_{\mathcal{H}} K = \mathrm{Ch}_{\mathcal{G}} K$. Since $\mathcal{G} \subset \mathcal{A}^c(\mathcal{G})$ and $\mathrm{Ch}_{\mathcal{G}} K = \mathrm{Ch}_{\mathcal{A}^c(\mathcal{G})} K$, we can assume without loss of generality that $\mathcal{A}^c(\mathcal{G}) = \mathcal{G}$. Using [LMNS 2010, Theorem 10.60] we infer that $\mathcal{G} = \mathcal{A}^c(\mathcal{H})$. Since $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$, we get $\mathcal{G} = \mathcal{H}$, finishing the proof. □

3. Proof of Theorem 1.2

We consider a compact subset of $\mathbb{R}^2$ defined as follows. Let $\{s, s^1, s^2, t^1, t^2\}$ be distinct points in $\mathbb{R}^2$. Let (s_n^i) and (t_n^i), $i = 0, 1, 2$, be sequences of points in $\mathbb{R}^2$ such that

- $s_n^0 \to s,\ t_n^0 \to s,$
- $s_n^i \to s^i,\ t_n^i \to t^i,\ i = 1, 2,$
- all the elements of these sequences are pairwise distinct and not contained in $\{s, s^1, s^2, t^1, t^2\}$.

Let $B(x, r)$ denote the closed ball in $\mathbb{R}^2$ with center $x \in \mathbb{R}^2$ and diameter $r > 0$. Let further $r_n > 0$, $n \in \mathbb{N}$, be numbers such that

- $r_n \to 0$,
- the family

$$\mathcal{K} = \big\{\{s\}, \{s^1\}, \{s^2\}, \{t^1\}, \{t^2\}\big\} \cup \{B(s_n^0, r_n) : n \in \mathbb{N}\} \cup \{B(t_n^0, r_n) : n \in \mathbb{N}\}$$

is disjoint.

We define the compact space K as $K = \bigcup \mathcal{K}$. Furthermore, we set $\mathcal{H}$ to be

$$\mathcal{H} = \Big\{h \in \mathcal{C}(K) : h(s) = \tfrac{1}{2}\big(h(s^1) + h(s^2)\big) = \tfrac{1}{2}\big(h(t^1) + h(t^2)\big),\\ h(s_n^0) = \tfrac{1}{2}\big(h(s_n^1) + h(s_n^2)\big),\ h(t_n^0) = \tfrac{1}{2}\big(h(t_n^1) + h(t_n^2)\big),\ n \in \mathbb{N}\Big\}.$$

Lemma 3.1. *The space $\mathcal{H}$ is a well defined function space with $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$. Let $L = \{s\} \cup \{s_n^0 : n \in \mathbb{N}\} \cup \{t_n^0 : n \in \mathbb{N}\}$. Then* $\operatorname{Ch}_{\mathcal{H}} K = K \setminus L$. *In particular*, $\operatorname{Ch}_{\mathcal{H}} K$ *is dense in K.*

Proof. Obviously, $\mathcal{H}$ contains constant functions. The fact that $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$ follows immediately from the definition of $\mathcal{H}$. To verify that $\mathcal{H}$ separates points of K it is enough to consider elementary constructions of functions from $\mathcal{H}$. Given $n \in \mathbb{N}$ and $z \in B(s_n^0, r_n) \setminus \{s_n^0\}$, we consider a continuous function $g : B(s_n^0, r_n) \to [0, 1]$ attaining 0 precisely at z and 1 at s_n^0. Then the function

$$h_z(x) = \begin{cases} g(x) & \text{if } x \in B(s_n^0, r_n),\\ 1 & \text{otherwise} \end{cases}$$

separates z from the remaining points of K. It also $\mathcal{H}$-exposes z, and thus $z \in \operatorname{Ch}_{\mathcal{H}} K$.

We can further construct functions h_{s_n} and h_s in $\mathcal{H}$ as follows:

$$h_{s_n}(x) = \begin{cases} 0 & \text{if } x = s_n^1,\\ 2 & \text{if } x \in B(s_n^0, r_n),\\ 4 & \text{if } x = s_n^2,\\ 1 & \text{otherwise,} \end{cases}$$

$$h_s(x) = \begin{cases} 0 & \text{if } x = s^1,\\ 2 & \text{if } x = s^2,\\ \frac{1}{2n} & \text{if } x = s_n^1,\\ 2 - \frac{1}{2n} & \text{if } x = s_n^2,\\ 1 & \text{otherwise.} \end{cases}$$

The function h_{s_n} then separates the points s_n^1, s_n^2 from any point in K and it separates s_n^0 from any point in $K \setminus B(s_n^0, r_n)$. Its construction also shows that the points s_n^1, s_n^2 are $\mathcal{H}$-exposed and thus lie in $\operatorname{Ch}_{\mathcal{H}} K$. Similarly, the function h_s separates points s^1, s, s^2 from each other and it separates s from every point in $\{s_n^i : n \in \mathbb{N}, i \in \{1, 2\}\}$. Furthermore, the construction of h_s shows that the points s^1, s^2 are $\mathcal{H}$-exposed and thus belong to $\operatorname{Ch}_{\mathcal{H}} K$.

Analogously we can construct functions h_{t_n}, $\tilde{h}_s$ and h_y for any $n \in \mathbb{N}$ and $y \in B(t_n^0, r_n) \setminus \{t_n^0\}$ to show that $\mathcal{H}$ indeed separates points of K and that all points in

$$\{t_1, t_2\} \cup \{t_n^i : n \in \mathbb{N}, i \in \{1, 2\}\} \cup \bigcup_{n \in \mathbb{N}} (B(t_n^0, r_n) \setminus \{t_n^0\})$$

lie in $\operatorname{Ch}_{\mathcal{H}} K$.

Overall, we have

$$\{s^1, s^2, t^1, t^2\} \cup \{s_n^i : n \in \mathbb{N}, i \in \{1,2\}\} \cup \{t_n^i : n \in \mathbb{N}, i \in \{1,2\}\} \subset \operatorname{Ch}_{\mathcal{H}} K$$

and

$$\bigcup_{n\in\mathbb{N}} (B(s_n^0, r_n) \setminus \{s_n^0\}) \cup \bigcup_{n\in\mathbb{N}} (B(t_n^0, r_n) \setminus \{t_n^0\}) \subset \operatorname{Ch}_{\mathcal{H}} K.$$

Clearly, any point in L has a nontrivial $\mathcal{H}$-representing measure. This together with the inclusions above yields $\operatorname{Ch}_{\mathcal{H}} K = K \setminus L$. □

Lemma 3.2. *Let $n \in \mathbb{N}$. Then*

$$\mathcal{M}_{s_n^0}(\mathcal{H}) = \operatorname{conv}\left\{\varepsilon_{s_n^0}, \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})\right\}$$

and

$$\mathcal{M}_{t_n^0}(\mathcal{H}) = \operatorname{conv}\left\{\varepsilon_{t_n^0}, \tfrac{1}{2}(\varepsilon_{t_n^1} + \varepsilon_{t_n^2})\right\}.$$

Proof. Let $n \in \mathbb{N}$ and $\mu \in \mathcal{M}_{s_n^0}(\mathcal{H})$ be fixed. Pick a continuous function

$$g : B(s_n^0, r_n) \to [0,1]$$

such that $g(s_n^0) = 0$ and $g(x) > 0$ otherwise. Using the function

$$h(x) = \begin{cases} 0 & \text{if } x \in \{s_n^1, s_n^2\}, \\ g(x) & \text{if } x \in B(s_n^0, r_n), \\ 1 & \text{otherwise,} \end{cases}$$

we infer that the support of μ is contained in $\{s_n^0, s_n^1, s_n^2\}$.

Further, let $a = \mu(\{s_n^0\})$. Assume first that $a = 0$, i.e., $\mu = b\varepsilon_{s_n^1} + (1-b)\varepsilon_{s_n^2}$ for some $b \in [0,1]$. Then the function

$$h(x) = \begin{cases} 0 & \text{if } x = s_n^1, \\ 1 & \text{if } x = s_n^0, \\ 2 & \text{if } x = s_n^2, \\ 1 & \text{otherwise} \end{cases}$$

shows that

$$1 = h(s_n^0) = \mu(h) = (1-b)h(s_n^2) = (1-b)2.$$

In other words, $b = \frac{1}{2}$ and $\mu = \frac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})$.

If $a \in (0,1)$, then the measure $\nu = \mu - a\varepsilon_{s_n^0}$ satisfies

$$h(s_n^0) = \mu(h) = \nu(h) + ah(s_n^0), \quad h \in \mathcal{H}.$$

Hence $\dfrac{1}{1-a}\nu$ is in $\mathcal{M}_{s_n^0}(\mathcal{H})$ and is carried by $\{s_n^1, s_n^2\}$. By the first part of the proof,

$$\frac{1}{1-a}\nu = \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2}).$$

Thus

$$\mu = \nu + a\varepsilon_{s_n^0} = (1-a)\frac{1}{1-a}\nu + a\varepsilon_{s_n^0} = (1-a)\tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2}) + a\varepsilon_{s_n^0}$$

is in $\operatorname{conv}\{\varepsilon_{s_n^0}, \frac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})\}$. If $a = 1$, obviously

$$\mu = \varepsilon_{s_n^0} \in \operatorname{conv}\{\varepsilon_{s_n^0}, \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})\}.$$

Thus

$$\mu \in \operatorname{conv}\{\varepsilon_{s_n^0}, \tfrac{1}{2}(\varepsilon_{s_n^1} + \varepsilon_{s_n^2})\}$$

holds in all cases.

The second part of the assertion can be proved analogously. □

Lemma 3.3. *The space $\mathcal{H}$ is not simplicial.*

Proof. The measures $\frac{1}{2}(\varepsilon_{s^1} + \varepsilon_{s^2})$, $\frac{1}{2}(\varepsilon_{t^1} + \varepsilon_{t^2})$ are different, they $\mathcal{H}$-represent s and, by Lemma 3.1, both are carried by $\operatorname{Ch}_{\mathcal{H}} K$. Hence there exist two $\mathcal{H}$-maximal measures representing s, which implies that $\mathcal{H}$ is not simplicial. □

Lemma 3.4. *The space $\mathcal{H}$ is maximal with respect to* $\operatorname{Ch}_{\mathcal{H}} K$. *That is, $\mathcal{G} = \mathcal{H}$ for any closed function space $\mathcal{H} \subset \mathcal{G}$ such that* $\operatorname{Ch}_{\mathcal{G}} K = \operatorname{Ch}_{\mathcal{H}} K$.

Proof. Fix an index $m \in \mathbb{N}$. Let $\tau \in \mathcal{M}_{s_m^0}(\mathcal{G})$ be a measure carried by $\operatorname{Ch}_{\mathcal{G}} K$. We aim to show that

$$\tau = \tfrac{1}{2}(\varepsilon_{s_m^1} + \varepsilon_{s_m^2}). \tag{3-1}$$

Since $\mathcal{M}_{s_m^0}(\mathcal{G}) \subset \mathcal{M}_{s_m^0}(\mathcal{H})$, we obtain by virtue of Lemma 3.2 that

$$\tau \in \operatorname{conv}\{\varepsilon_{s_m^0}, \tfrac{1}{2}(\varepsilon_{s_m^1} + \varepsilon_{s_m^2})\}.$$

This and the fact that τ is carried by $\operatorname{Ch}_{\mathcal{G}} K = \operatorname{Ch}_{\mathcal{H}} K \subset K \setminus \{s_m^0\}$ imply (3-1).

Pick $\mu_n \in \mathcal{M}_{s_n^0}(\mathcal{G})$, $n \in \mathbb{N}$, such that the measures μ_n are carried by $\operatorname{Ch}_{\mathcal{G}} K$ for all $n \in \mathbb{N}$. The sequence (s_n^0) converges to s, while the sequence (μ_n) converges to $\mu = \frac{1}{2}(\varepsilon_{s^1} + \varepsilon_{s^2})$. Thus $\mu \in \mathcal{M}_s(\mathcal{G})$. Analogously we infer that any measure ν_n in $\mathcal{M}_{t_n^0}(\mathcal{G})$ carried by $\operatorname{Ch}_{\mathcal{G}} K$ satisfies $\nu_n = \frac{1}{2}(\varepsilon_{t_n^1} + \varepsilon_{t_n^2})$, and thus $\nu = \frac{1}{2}(\varepsilon_{t^1} + \varepsilon_{t^2})$ is in $\mathcal{M}_s(\mathcal{G})$.

We want to show that $\mathcal{G} \subset \mathcal{H}$. To this end, let $g \in \mathcal{G}$ be given. We have to verify the conditions defining the space $\mathcal{H}$. Using the arguments above we get

$$g(s_n^0) = \mu_n(g) = \tfrac{1}{2}(g(s_n^1) + g(s_n^2)) \quad \text{and} \quad g(s) = \mu(g) = \tfrac{1}{2}(g(s^1) + g(s^2)),$$

while simultaneously

$$g(t_n^0) = \nu_n(g) = \tfrac{1}{2}(g(t_n^1) + g(t_n^2)) \quad \text{and} \quad g(s) = \nu(g) = \tfrac{1}{2}(g(t^1) + g(t^2)).$$

Hence $g \in \mathcal{H}$ by definition. This concludes the proof. □

Thus we have proved Theorem 1.2. Indeed, considering the compact space K and the closed function space $\mathcal{H} \subset \mathcal{C}(K)$ defined above, we have by Lemma 3.1 that $\mathrm{Ch}_{\mathcal{H}}\, K$ is dense in K. Furthermore, $\mathcal{H}$ is maximal with respect to $\mathrm{Ch}_{\mathcal{H}}\, K$ by Lemma 3.4. Since $\mathcal{H}$ is not simplicial according to Lemma 3.3, Theorem 2.2 asserts that $\mathcal{A}^c(\mathcal{H})$ is not a Lindenstrauss space. Since $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$ by Lemma 3.1, it follows that $\mathcal{H}$ is not a Lindenstrauss space.

References

[Edwards and Vincent-Smith 1968] D. A. Edwards and G. Vincent-Smith, "A Weierstrass–Stone theorem for Choquet simplexes", *Ann. Inst. Fourier* (*Grenoble*) **18**:1 (1968), 261–282. MR 39 #6060 Zbl 0172.15604

[Fonf et al. 2001] V. P. Fonf, J. Lindenstrauss, and R. R. Phelps, "Infinite dimensional convexity", pp. 599–670 in *Handbook of the geometry of Banach spaces*, vol. I, edited by W. B. Johnson and J. Lindenstrauss, North-Holland, Amsterdam, 2001. MR 2003c:46014 Zbl 1086.46004

[Lacey 1973] H. E. Lacey, "On the classification of Lindenstrauss spaces", *Pacific J. Math.* **47** (1973), 139–145. MR 50 #5443 Zbl 0251.46030

[Lacey 1974] H. E. Lacey, *The isometric theory of classical Banach spaces*, Die Grundlehren der mathematischen Wissenschaften **208**, Springer, New York, 1974. MR 58 #12308 Zbl 0285.46024

[LMNS 2010] J. Lukeš, J. Malý, I. Netuka, and J. Spurný, *Integral representation theory: applications to convexity, Banach spaces and potential theory*, de Gruyter Studies in Mathematics **35**, de Gruyter, Berlin, 2010. MR 2011e:46002 Zbl 1216.46003

Received May 5, 2014.

PETR PETRÁČEK
DEPARTMENT OF MATHEMATICAL ANALYSIS
FACULTY OF MATHEMATICS AND PHYSICS
CHARLES UNIVERSITY
SOKOLOVSKÁ 83
186 75 PRAHA 8
CZECH REPUBLIC
petracek@karlin.mff.cuni.cz

JIŘÍ SPURNÝ
DEPARTMENT OF MATHEMATICAL ANALYSIS
FACULTY OF MATHEMATICS AND PHYSICS
CHARLES UNIVERSITY
SOKOLOVSKÁ 83
186 75 PRAHA 8
CZECH REPUBLIC
spurny@karlin.mff.cuni.cz

PACIFIC JOURNAL OF MATHEMATICS

msp.org/pjm

Founded in 1951 by E. F. Beckenbach (1906–1982) and F. Wolf (1904–1989)

See inside back cover or msp.org/pjm for submission instructions.

The subscription price for 2015 is US $420/year for the electronic version, and $570/year for print and electronic.
Subscriptions, requests for back issues and changes of subscribers address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. The Pacific Journal of Mathematics is indexed by Mathematical Reviews, Zentralblatt MATH, PASCAL CNRS Index, Referativnyi Zhurnal, Current Mathematical Publications and Web of Knowledge (Science Citation Index).

The Pacific Journal of Mathematics (ISSN 0030-8730) at the University of California, c/o Department of Mathematics, 798 Evans Hall #3840, Berkeley, CA 94720-3840, is published twelve times a year. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163.

PJM peer review and production are managed by EditFLOW® from Mathematical Sciences Publishers.

PUBLISHED BY

mathematical sciences publishers

nonprofit scientific publishing

http://msp.org/

PACIFIC JOURNAL OF MATHEMATICS

Volume 276 No. 1 July 2015