Triple point numbers of surface-links and symmetric quandle cocycle invariants

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For any positive integer \( n \), we give a 2–component surface-link \( F = F_1 \cup F_2 \) such that \( F_1 \) is orientable, \( F_2 \) is non-orientable and the triple point number of \( F \) is equal to \( 2n \). To give lower bounds of the triple point numbers, we use symmetric quandle cocycle invariants.

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1 Introduction

A surface-link is a closed surface smoothly embedded in \( \mathbb{R}^4 \). Two surface-links \( F \) and \( F' \) are assumed to be the same if and only if there exists an ambient isotopy \( \{ h_t \} \) of \( \mathbb{R}^4 \) such that \( h_1(F) = F' \). When \( F \) and \( F' \) are oriented, it is assumed that \( h_1|_F: F \to F' \) is an orientation-preserving homeomorphism. In particular, when a surface-link is connected, we call it a surface-knot.

The triple point number of a surface-knot \( F \) is defined by the smallest number of the triple points among all the diagrams of \( F \), and we denote it by \( t(F) \). There are several studies on triple point numbers. For example, quandle cocycle invariants (see Carter, Jelsovsky, Kamada, Langford and Saito [1]) are used to give lower bounds of triple point numbers of orientable surface-links; for example, Satoh and Shima [14] determined the triple point number of the 2–twist-spun trefoil to be four, and Hatakenaka [4] gave a lower bound for the triple point number of the 2–twist-spun figure-eight knot. By a geometric argument about normal Euler numbers, Satoh [12] gave the following theorem:

**Theorem 1.1** (Satoh [12]) For any positive integer \( n \), there exists a 2–component surface-link \( F = F_1 \cup F_2 \) such that (i) each \( F_i \) is a non-orientable surface-knot with the Euler characteristic \( \chi(F_i) = 2 - n \), and (ii) \( t(F) = 2n \).

In Section 4, we show a method which gives lower bounds for the triple point numbers of surface-links by using the symmetric quandle cocycle invariants (see Kamada [7].
We remark that by the symmetric quandle cocycle invariants, we can give alternative proof of Theorem 1.1. Using new examples of surface-links, we can also prove the following theorem which is analogous to Theorem 1.1:

**Theorem 1.2** For any positive integer \( n \), there exists a 2–component surface-link \( F = F_1 \cup F_2 \) such that

(i) \( F_1 \) is an orientable surface-knot with \( \chi(F_1) = 0 \),

(ii) \( F_2 \) is a non-orientable surface-knot with \( \chi(F_2) = 2 - 2n \), and

(iii) \( t(F) = 2n \).

By a connected sum of the surface-link which Satoh used for Theorem 1.1 and an orientable surface-knot, the following was given in [8]: For any positive integers \( m \) and \( n \) with \( m \equiv n \pmod{2} \) and \( m \geq n \), there is a surface-link \( F = F_1 \cup F_2 \) such that \( F_1 \) is a non-orientable surface with \( \chi(F_1) = 2 - m \), \( F_2 \) is a non-orientable surface with \( \chi(F_2) = 2 - n \) and \( t(F) = 2n \). For surface-links composed of two non-orientable surfaces, we give the following theorem:

**Theorem 1.3** For any positive integer \( n \) and for any integer \( m \) with \( m \geq 3 \), there is a surface-link \( F = F_1 \cup F_2 \) such that

(i) \( F_1 \) is a non-orientable surface with \( \chi(F_1) = 2 - m \),

(ii) \( F_2 \) is a non-orientable surface with \( \chi(F_2) = 2 - 2n \), and

(iii) \( t(F) = 2n \).

The paper is organized as follows. In Sections 2 and 3, we recall symmetric quandles, symmetric quandle 3–cocycles, and surface-link invariants with symmetric quandles introduced in [7; 8]. In Section 4, we show a method to estimate the triple point numbers of surface-links by using the symmetric quandle invariants. Theorems 1.2 and 1.3 are proved by giving new examples of surface-links in Section 5. In Section 6, we show several results which can be obtained by using our method for estimating triple point numbers.

### 2 Symmetric quandles and their cocycles

A **quandle** (see Fenn and Rourke [3], Joyce [5] or Matveev [10]) is a set \( X \) with a binary operation \( (x, y) \mapsto x^y \) such that

(i) for any \( x \in X \), it holds that \( x^x = x \),

(ii) \( x^{y^z} = (x^y)^z \),

(iii) \( x^{y^z} = x^{y^z} \).

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We denote by \( x \) the dihedral quandle \( n \). Any \( x \) is the symmetric quandle \( A \). Let \( \hat{X} \) be the symmetric quandle, and any element of \( X \). For any element \( k \), we use the same symbol \( x \) and \( \hat{X} \). And any element of \( X \), \( x^{\hat{y}} = x^{-1} \).

We denote by \( x^{\hat{y}} \) the element \( z \) given in the condition (ii). For a quandle \( X \), a good involution \( \rho \) of \( X \) [7, 8] means an involution of \( X \) such that:

(i) for any \( x, y \in X \), \( \rho(x^y) = \rho(x)^y \), and

(ii) for any \( x, y \in X \), \( x^{\rho(y)} = x^{y^{-1}} \).

A pair of a quandle and a good involution is called a symmetric quandle.

Let \( (X, \rho) \) be a symmetric quandle, and \( A \) an abelian group. A homomorphism \( \theta: \mathbb{Z}(X^3) \rightarrow A \) is a symmetric quandle 3-cocycle of \( (X, \rho) \) if the following conditions are satisfied:

(i) For any \( (a, b, c, d) \in X^4 \),

\[
\theta(a, c, d) - \theta(a^b, c, d) - \theta(a, b, d) + \theta(a^c, b^c, d) + \theta(a, b, c) - \theta(a^d, b^d, c^d) = 0,
\]

(ii) for any \( (a, b) \in X^2 \), \( \theta(a, a, b) = 0 \) and \( \theta(a, b, b) = 0 \), and

(iii) for any \( (a, b, c) \in X^3 \),

\[
\theta(a, b, c) + \theta(a^b, b, c) = 0, \quad \theta(a, b, c) + \theta(a^b, c, b) = 0
\]

and

\[
\theta(a, b, c) + \theta(a^c, b^c, c) = 0.
\]

Here, \( \mathbb{Z}(X^3) \) is the free \( \mathbb{Z} \)-module generated by all the elements of \( X^3 = X \times X \times X \). Notice that a symmetric quandle 3-cocycle of \( (X, \rho) \) is a 3-cocycle of the cochain complex defined for the symmetric quandle \( (X, \rho) \) in [7, 8].

For any element \( k \) in \( \mathbb{Z} \), we use the same symbol \( k \) to indicate the element [\( k \)] in \( \mathbb{Z}_2 \), and any element of \( \mathbb{Z}_2 \oplus \mathbb{Z} \) is denoted by a form \( \alpha \oplus \beta \), where \( \alpha \) is the entry of \( \mathbb{Z}_2 \), and \( \beta \) is the entry of \( \mathbb{Z} \).

Example 2.1 The set \{0, 1, \cdots, n - 1\} with the operation \( x^y \equiv 2^y - x \pmod{n} \) for any \( x, y \in \{0, 1, \cdots, n - 1\} \) is a quandle, which is called a dihedral quandle of order \( n \). All of the good involutions of a dihedral quandle are determined in [8]. Let \( X \) be the dihedral quandle \{0, 1, 2, 3\} of order 4. The involution \( \rho: X \rightarrow X \) defined by \( \rho(0) = 2 \) and \( \rho(1) = 3 \), is a good involution of \( X \). Define a map \( \theta: X^3 \rightarrow \mathbb{Z}_2 \oplus \mathbb{Z} \) such that

\[
\theta(a, b, c) = \begin{cases} 
0 \oplus 1 & (a, b, c) = (0, 1, 0), (0, 3, 0), (2, 1, 2), (2, 3, 2), \\
& (1, 0, 3), (1, 2, 3), (3, 0, 1), (3, 2, 1), \\
0 \oplus (-1) & (a, b, c) = (0, 1, 2), (0, 3, 2), (2, 1, 0), (2, 3, 0), \\
& (1, 0, 1), (1, 2, 1), (3, 0, 3), (3, 2, 3), \\
0 \oplus 0 & \text{otherwise}.
\end{cases}
\]
Then the linear extension $\theta: \mathbb{Z}(X^3) \to \mathbb{Z}_2 \oplus \mathbb{Z}$ is a symmetric quandle 3–cocycle of $(X, \rho)$.

**Example 2.2** Let $X = \{0, 1, 2\}$ be the quandle such that

\begin{align*}
0^0 &= 0, \quad 0^1 = 0, \quad 0^2 = 0, \\
1^0 &= 2, \quad 1^1 = 1, \quad 1^2 = 1, \\
2^0 &= 1, \quad 2^1 = 2, \quad 2^2 = 2.
\end{align*}

The involution $\rho: X \to X$ defined by $\rho(0) = 0$ and $\rho(1) = 2$, is a good involution of $X$. Define a map $\theta: X^3 \to \mathbb{Z}_2 \oplus \mathbb{Z}$ such that

$$
\theta(a, b, c) = \begin{cases} 
1 \oplus 0 & (a, b, c) = (0, 1, 0), (0, 2, 0) \\
0 \oplus 1 & (a, b, c) = (1, 0, 2), (2, 0, 1) \\
0 \oplus (-1) & (a, b, c) = (1, 0, 1), (2, 0, 2) \\
0 \oplus 0 & \text{otherwise}.
\end{cases}
$$

Then the linear extension $\theta: \mathbb{Z}(X^3) \to \mathbb{Z}_2 \oplus \mathbb{Z}$ is a symmetric quandle 3–cocycle of $(X, \rho)$.

### 3 Symmetric quandle cocycle invariants

Let $D$ be a diagram in $\mathbb{R}^3$ of a surface-link $F$ in $\mathbb{R}^4$, where the lower sheets are divided along double point curves to indicate crossing information. We divide over-sheets along the double point curves and we call the sheets of the result *semi-sheets* of $D$. Note that every semi-sheet is orientable even if $F$ is non-orientable, see Kamada [6].

For a symmetric quandle $(X, \rho)$, we say that an assignment of a normal orientation and an element of $X$ to each semi-sheet of $D$ satisfies the *coloring conditions* if it satisfies the following:

(i) Suppose that two adjacent semi-sheets coming from an over-sheet of $D$ about a double point curve are labeled by $x_1$ and $x_2$. If the normal orientations are coherent then $x_1 = x_2$, otherwise $x_1 = \rho(x_2)$. See the top row of Figure 1.

(ii) Suppose that two adjacent semi-sheets $S_1$ and $S_2$ coming from under-sheets about a double point curve are labeled by $x_1$ and $x_2$, and that one of the two semi-sheets coming from an over-sheet of $D$, say $S_3$, is labeled by $x_3$. We assume that the normal orientation of $S_3$ points from $S_1$ to $S_2$. If the normal orientations of $S_1$ and $S_2$ are coherent, then $x_1^{x_3} = x_2$, otherwise $x_1^{x_3} = \rho(x_2)$. See the bottom row of Figure 1.
An \((X, \rho)\)–coloring of \(D\) is the equivalence class of an assignment of normal orientations and elements of \(X\) to the semi-sheets of \(D\) satisfying the coloring conditions. Here, the equivalence relation is generated by basic inversions, that is, a basic inversion reverses the normal orientation of a semi-sheet and changes the element \(x\) assigned the sheet by \(\rho(x)\). See Figure 2.

We call a diagram with an \((X, \rho)\)–coloring \(C_D\) an \((X, \rho)\)–colored diagram and denote it by \((D, C_D)\).
Let \((D, C_D)\) and \((D', C_{D'})\) be \((X, \rho)\)-colored diagrams of a surface-link \(F\). We say that \((D, C_D)\) and \((D', C_{D'})\) (or the \((X, \rho)\)-colorings \(C_D\) of \(D\) and \(C_{D'}\) of \(D'\)) are equivalent if they are related by a finite sequence of Roseman moves (see Roseman [11], and also Carter and Saito [2]) over which the colorings extend. We call the equivalence class of \((D, C_D)\) an \((X, \rho)\)-coloring of \(F\). An \((X, \rho)\)-colored surface-link \((F, C)\) is a surface-link \(F\) equipped with an \((X, \rho)\)-coloring \(C\).

Let \((D, C_D)\) be an \((X, \rho)\)-colored diagram of an \((X, \rho)\)-colored surface-link \((F, C)\). Let \(\theta : \mathbb{Z}(X^3) \to A\) be a symmetric quandle 3–cocycle of \((X, \rho)\). For a triple point of \(D\), define the \(\theta\)-weight as follows: Choose one of eight 3–dimensional complementary regions around the triple point and call the region a specified region. There exist 12 semi-sheets around the triple points. Let \(S_T, S_M\) and \(S_B\) be the three of them that face the specified region, where \(S_T, S_M\) and \(S_B\) are in the top sheet, the middle sheet and the bottom sheet at the triple point, respectively. Let \(n_T, n_M\) and \(n_B\) be the normal orientations of \(S_T, S_M\) and \(S_B\) which point away from the specified region. Let \(x, y\) and \(z\) be the elements of \(X\) assigned to the semi-sheets \(S_T, S_M\) and \(S_B\) with the normal orientations \(n_T, n_M\) and \(n_B\), respectively. The \(\theta\)-weight of the triple point is defined by \(\varepsilon \theta(z, y, x)\), where \(\varepsilon\) is +1 (or –1) if the triple of the normal orientations \((n_T, n_M, n_B)\) does (or does not) match with the orientation of \(\mathbb{R}^3\). The sign of the triple point as shown in Figure 3 is positive.

Define \(\theta(D, C_D)\) by
\[
\theta(D, C_D) = \sum_\tau (\theta\text{-weight of } \tau) \in A,
\]
where \(\tau\) runs over all the triple points of \(D\).

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\[\text{Figure 3}\]
Theorem 3.1 (Kamada and Oshiro [8]) The value \( \theta(D, C_D) \) is an invariant of an \((X, \rho)\)-colored surface-link \((F, C)\).

We denote \( \theta(D, C_D) \) by \( \theta(F, C) \).

4 Estimates of triple point numbers

For non-negative integers \( s \) and \( t \), let \( A_{s,t} \) denote the direct sum of \( s \) copies of \( \mathbb{Z}_2 \) and \( t \) copies of \( \mathbb{Z} \), that is, \( A_{s,t} = (\mathbb{Z}_2)^s \oplus (\mathbb{Z})^t \). Every element of \( A_{s,t} \) has a form \((\alpha_1 \oplus \cdots \oplus \alpha_s) \oplus (\beta_1 \oplus \cdots \oplus \beta_t)\), where \( \alpha_i \) is the entry of \( i \) th \( \mathbb{Z}_2 \) \((1 \leq i \leq s)\), and \( \beta_j \) is the entry of \( j \) th \( \mathbb{Z} \) \((1 \leq j \leq t)\). We denote by \( p_i \) and \( q_j \) the elements of \( A_{s,t} \) whose entries are all zeros except \( \alpha_i = 1 \) and \( \beta_j = 1 \), respectively.

Let \((X, \rho)\) be a symmetric quandle, and \( \theta : \mathbb{Z}(X^3) \rightarrow A_{s,t} \) a 3–cocycle of \((X, \rho)\). We consider the following condition for \( \theta \):

\((*)\) For any generator \((a, b, c) \in X^3 \) of \( \mathbb{Z}(X^3) \), it holds that

\[
\theta(a, b, c) \in \{0, p_i, \pm q_j \mid 1 \leq i \leq s, 1 \leq j \leq t\}.
\]

We remark that the symmetric quandle 3–cocycles given in Examples 2.1 and 2.2 satisfy the condition \((*)\).

Theorem 4.1 Let \( \theta \) be a 3–cocycle of a symmetric quandle \((X, \rho)\) with the condition \((*)\). If the invariant \( \theta(F, C) \) of a surface-link \( F \) with an \((X, \rho)\)–coloring \( C \) is equal to \((\alpha_1 \oplus \cdots \oplus \alpha_s) \oplus (\beta_1 \oplus \cdots \oplus \beta_t)\), then we have \( \theta(F) \geq \sum_{i=1}^{s} \alpha_i + \sum_{j=1}^{t} |\beta_j| \), where the sum is taken in \( \mathbb{Z} \) by regarding \( \alpha_k = 0 \) or 1 as an element of \( \mathbb{Z} \).

Proof We take any \((X, \rho)\)–colored diagram \((D, C_D)\) of \((F, C)\). Let \( t(D) \) denote the number of triple points of \( D \), and \( m_i \) \((1 \leq i \leq s)\), \( n_j \) and \( n_j' \) \((1 \leq j \leq t)\) the number(s) of triple points whose \( \theta \)–weights are \( p_i \), \( q_j \) and \( -q_j \), respectively.

Since the \( \theta \)–weight of any triple point of \( D \) is one of 0, \( p_i \), \( q_j \), and \( -q_j \), it holds that

\[
\theta(D, C_D) = \sum_{i=1}^{s} m_i p_i + \sum_{j=1}^{t} n_j q_j + \sum_{j=1}^{t} n'_j (-q_j) = (m_1 \oplus \cdots \oplus m_s) \oplus ((n_1 - n'_1) \oplus \cdots \oplus (n_t - n'_t)).
\]

Hence, we have \( \alpha_i \equiv m_i \pmod{2} \) and \( \beta_j = n_j - n'_j \) by assumption. Since \( \alpha_i \leq m_i \) and \( |\beta_j| \leq n_j + n'_j \), it holds that

\[
\sum_{i=1}^{s} \alpha_i + \sum_{j=1}^{t} |\beta_j| \leq \sum_{i=1}^{s} m_i + \sum_{j=1}^{t} (n_j + n'_j) \leq t(D). \quad \square
\]
5 Proofs of Theorems 1.2 and 1.3

In this section, we give surface-links which satisfy Theorems 1.2 and 1.3.

Let $F$ be a surface-link in $\mathbb{R}^4 = \{(x, y, z, t) \in \mathbb{R}^4\}$ whose motion picture is given in Figure 4. It is composed of an unknotted torus $F_1$ and an unknotted, non-orientable surface $F_2$ with $\chi(F_2) = 2 - 2n$. Notice that in Figure 4, the deformations from (i) to (ii) and from (iii) to (iv) are the isotopic deformations corresponding to $n$ Reidemeister moves of type III, respectively. The other isotopies are obtained by Reidemeister moves I and II only.

Let $D$ be the diagram obtained by the projection $\pi: \mathbb{R}^4 \to \mathbb{R}^3$ with $\pi(x, y, z, t) \mapsto (x, y, 0, t)$. Instead of illustrating the whole of $D$, we use the one-parameter family $\{D \cap \mathbb{R}^2[t]\}_{t \in \mathbb{R}}$, where $\mathbb{R}^2[t] = \{(x, y, 0, t) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$.

**Proof of Theorem 1.2** We will prove that the surface-link $F$ constructed as above satisfies $t(F) = 2n$. It is not difficult to see that $\chi(F_2) = 2 - 2n$.

Let $(X, \rho)$ and $\theta$ be the symmetric quandle and the symmetric quandle $3$–cocycle given in Example 2.2. We define an $(X, \rho)$–coloring $C$ for $D$ such that (i) any semi-sheet of $F_1$ is assigned by $0 \in X$ with any normal orientation, and (ii) the semi-sheet of $F_2$ marked by $*$ in $\mathbb{R}^2[-2]$ is assigned by $1 \in X$ with the orientation as in the figure, which can be extended to any other semi-sheets of $F_2$ uniquely.

Between the stills (i) and (ii) in Figure 4, the Reidemeister moves of type III arise $n$ times and each move is depicted in Figure 5. Each Reidemeister move of type III corresponds to a triple point whose $\theta$–weight is $-\theta(2, 0, 2) = 0 \oplus 1$. Between the stills (ii) and (iv) in Figure 4, the Reidemeister moves of type III arise $n$ times and each move is depicted in Figure 6. Each Reidemeister move of type III corresponds to a triple point whose $\theta$–weight is $\theta(1, 0, 2) = 0 \oplus 1$. Therefore, $\theta(E^{(n)}, C)$ is equal to $0 \oplus 2n$. By Theorem 4.1, $t(E^{(n)}) \geq 2n$.  

**Proof of Theorem 1.3** Let $F = F_1 \cup F_2$ be the surface-link as above, and $K$ an unknotted non-orientable surface-knot with $\chi(K) = 4 - m$ ($m \geq 3$). We denote by $F \natural K = (F_1 \natural K) \cup F_2$ the connected sum of $F_1 \subset F$ and $K$. It follows by definition that $\chi(F_1 \natural K) = 2 - m$ and $t(F_1 \natural K) \leq 2n$.

On the other hand, the $(X, \rho)$–coloring $C$ for $F$ in the proof of Theorem 1.2 is extended to that for $F \natural K$ with the same $\theta$–weight. Hence, we have $t(F \natural K) = 2n$ by a similar argument to the previous proof.  

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Remark 5.1 For the surface-link $F$ as above, we can also use Satoh's method [12] to prove that $t(F) = 2n$. However, for the surface-link $F^\# K$ which is constructed in the proof of Theorem 1.3, we cannot prove $t(F^\# K) = 2n$ by his method since the surface-link is $P^2$–reducible.
In this section, we show some results which can be obtained as an application of Theorem 4.1.

For the positive integer $n$, let $G = G_1 \cup G_2$ be a surface-link in $\mathbb{R}^4 = \{(x, y, z, t) \in \mathbb{R}^4\}$ whose motion picture is given in Figure 7. Each component of $G_i$ is a non-orientable surface with $\chi(G_i) = 2 - n$. This is the surface-link which Satoh used for proving Theorem 1.1.

The following theorem is a generalization of Theorem 1.1. We can give alternative proofs by a symmetric quandle 3-cocycle similarly to the proof of Theorem 1.2, or by a geometric argument used in [12]. We say that a surface-link is pseudo-ribbon if it has a diagram without triple points (see Kawauchi [9]).
Theorem 6.1  (Kamada and Oshiro [8])  Let $G$ be the surface-link as above. For any orientable surface-knot $K$, the connected sum $G^+_1K = (G_1^+K) \cup G_2$ satisfies
\[ t(G^+_1K) \geq 2n. \]
In particular, if $K$ is pseudo-ribbon, then the equality holds.

For a non-orientable surface-knot $K$, the connected sum $G^+_1K$ is not necessarily $P^2$–irreducible. Hence we can not apply the Satoh’s argument to the surface-link. In this case, we have the following.

Theorem 6.2  Let $G$ be the surface-link as above. For any non-orientable surface-knot $K$, it holds that
\[ t(G^+_1K) \geq \begin{cases} n + 1 & \text{if } n \text{ is an odd number,} \\ n & \text{if } n \text{ is an even number.} \end{cases} \]

Proof  Let $(X, \rho)$ and $\theta$ be the symmetric quandle and the symmetric quandle 3–cocycle given in Example 2.2, respectively. By the definition,
\[ \theta(a, b, c) \in \{0 \oplus 0, 1 \oplus 0, 0 \oplus 1, 0 \oplus (-1)\} \]
for any $(a, b, c) \in X^3$.

Let $D$ be the diagram of $G$ corresponding to the motion picture and $C$ the $(X, \rho)$–coloring for $G$ as shown in Figure 7. Between the stills (i) and (ii), the Reidemeister moves III arise $2n$ times. More precisely, a pair of moves III is depicted in Figure 8. The sum of the $\theta$–weights is equal to
\[ -\theta(1, 0, 1) + \theta(0, 2, 0) = 0 \oplus 1 + 1 \oplus 0 = 1 \oplus 1, \]
and hence, we have $\theta(G, C) = n \oplus n$.

For any non-orientable surface-knot $K$, we extend the $(X, \rho)$–coloring $C$ for $G$ to that for $G^+_1K$ such that $K$ is colored trivially. Then it follows by definition that $\theta(G^+_1K, C) = \theta(G, C) = n \oplus n$, and we have the conclusion by Theorem 4.1.  \[ \square \]

![Figure 8](image_url)
The equality given in Theorem 6.2 holds for \( n = 1 \).

**Question 6.3** Does the equality in Theorem 6.2 hold for any \( n \geq 2 \)?

We remark that the triple point number is generally not additive with respect to the connected sum (see Satoh [13]).

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