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Corrigendum to "Homotopy theory of modules over operads in symmetric spectra"

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Dmitri Pavlov and Jakob Scholbach have pointed out that part of Proposition 6.3, and hence Proposition 4.28(a), of Harper [2] are incorrect as stated. While all of the main results of that paper remain unchanged, this necessitates modifications to the statements and proofs of a few technical propositions.

55P43, 55P48; 55U35

1 Introduction

The author would like to thank Dmitri Pavlov and Jakob Scholbach for pointing out that the description of the cofibrations in the last sentence of Proposition 6.3 of Harper [2] is incorrect as stated; in general, to verify that a map is a cofibration, it is not enough to be a monomorphism such that $\Sigma_r^{\mathrm{op}} \times G$ acts freely on the simplices of the codomain not in the image.

It is well known that the cofibrations in S_*^G, equipped with the projective model structure, are precisely the monomorphisms such that G acts freely on the simplices of the codomain not in the image. One way to verify this is to (i) argue that the image of such a map is a subcomplex of the codomain (ie the codomain can be built from the image by attaching G–cells), and (ii) note that every monomorphism is isomorphic to its image, hence verifying that such maps are cofibrations, (iii) conversely, to note that every generating cofibration is such a map, and (iv) hence conclude that every cofibration is such a map, by using the fact that every cofibration is a retract of a (possibly transfinite) composition of pushouts of the generating cofibrations. The problem with our argument for the cofibration description in [2, Proposition 6.3] was a cavalier application of the subcomplex argument (i) above; we ignored the fact that $\Sigma_r^{\mathrm{op}} \times G$ and Σ_n might not act independently. Pavlov and Scholbach kindly pointed out this problem to the author, together with a helpful counterexample to focus one's attention. At the time they were working to generalize the main results in [2] to motivic settings (including Hornbostel's results [3]; see Remark 1.1). Their efforts have now appeared in Pavlov and Scholbach [5]; included in Appendix A therein is their helpful counterexample, together with further discussion related to these cofibrations.

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The following proposition corresponds to the corrected version of [2, Proposition 6.3].

Proposition 6.3* *Let G be a finite group and consider any $n, r \geq 0$. The diagram category $(\mathsf{S}_*^{\Sigma_n})^{\Sigma_r^{\mathrm{op}} \times G}$ inherits a corresponding projective model structure from the mixed Σ_n–equivariant model structure on $\mathsf{S}_*^{\Sigma_n}$. The weak equivalences (resp. fibrations) are the underlying weak equivalences (resp. fibrations) in $\mathsf{S}_*^{\Sigma_n}$.*

The consequence of the misunderstanding of the cofibrations in [2, Proposition 6.3] is that [2, Proposition 4.28(a)] is incorrect as stated. While all of the main results of that paper remain unchanged, this necessitates modifications to the statements and proofs of a few technical propositions.

Remark 1.1 This corrigendum also applies to the proof of the motivic generalization of our results provided by Hornbostel, namely [3, Theorems 3.6, 3.10 and 3.15].

The following proposition corresponds to the corrected version of [2, Proposition 4.28]. For a useful study of additional properties associated to tensor powers of cofibrations, see Pereira [6] and, more recently, Pavlov and Scholbach [5].

Proposition 4.28* *Let $B \in \mathsf{SymSeq}^{\Sigma_t^{\mathrm{op}}}$, $t \geq 1$, and $r, n \geq 0$. If $i \colon X \to Y$ is a cofibration between cofibrant objects in SymSeq with the positive flat stable model structure, then*

(a) *the map $B \check{\otimes} X^{\check{\otimes} t} \to B \check{\otimes} Y^{\check{\otimes} t}$, after evaluation at $[\mathbf{r}]_n$, is a cofibration in $\mathsf{S}_*^{\Sigma_t}$ with the projective model structure inherited from S_*,*

(b) *the map $B \check{\otimes}_{\Sigma_t} Q_{t-1}^t \to B \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t}$ is a monomorphism.*

Since Proposition 4.29 and Proposition 6.11 of [2] are no longer immediately applicable, we include below the closely related Proposition 4.29* and Proposition 6.11* which describe the technical properties that are actually used in the proofs of the main results in [2].

Proposition 4.29* *Let $t \geq 1$ and consider SymSeq and $\mathsf{SymSeq}^{\Sigma_t^{\mathrm{op}}}$ each with the positive flat stable model structure.*

(a) *If $B \in \mathsf{SymSeq}^{\Sigma_t^{\mathrm{op}}}$, then the functor*

$$B \check{\otimes}_{\Sigma_t} (-)^{\check{\otimes} t} \colon \mathsf{SymSeq} \to \mathsf{SymSeq}$$

preserves weak equivalences between cofibrant objects, and hence its total left derived functor exists.

(b) *If $Z \in \mathsf{SymSeq}$ is cofibrant, then the functor*

$$-\check{\otimes}_{\Sigma_t} Z^{\check{\otimes} t} \colon \mathsf{SymSeq}^{\Sigma_t^{\mathrm{op}}} \to \mathsf{SymSeq}$$

preserves weak equivalences.

Proposition 6.11* *Let $t \geq 1$ and consider SymSeq with the positive flat stable model structure. If $B \in \mathsf{SymSeq}^{\Sigma_t^{\mathrm{op}}}$, then the functor*

$$B \check{\otimes}_{\Sigma_t} (-)^{\check{\otimes} t} \colon \mathsf{SymSeq} \to \mathsf{SymSeq}$$

sends cofibrations between cofibrant objects to monomorphisms.

All references to Propositions 4.28, 4.29 and 6.11 in the proofs of the main results in [2] should be replaced by references to Propositions 4.28*, 4.29* and 6.11*, respectively, which are proved below in Section 2.

Propositions 1.6 and 7.7(a) of [2] are special cases of the statement of Proposition 4.28(a) of [2], and hence are incorrect as stated; the following propositions correspond to their corrected versions, respectively, and are special cases of Proposition 4.28* above.

Proposition 1.6* *Let $B \in (\mathsf{Sp}^{\Sigma})^{\Sigma_t^{\mathrm{op}}}$, $t \geq 1$, and $n \geq 0$. If $i \colon X \to Y$ is a cofibration between cofibrant objects in symmetric spectra with the positive flat stable model structure, then the map $B \wedge X^{\wedge t} \to B \wedge Y^{\wedge t}$, after evaluation at n, is a cofibration of Σ_t–diagrams in pointed simplicial sets.*

Proposition 7.7* *Let $B \in (\mathsf{Sp}^{\Sigma})^{\Sigma_t^{\mathrm{op}}}$, $t \geq 1$, and $n \geq 0$. If $i \colon X \to Y$ is a cofibration between cofibrant objects in Sp^{Σ} with the positive flat stable model structure, then*

(a) *the map $B \wedge X^{\wedge t} \to B \wedge Y^{\wedge t}$, after evaluation at n, is a cofibration in $\mathsf{S}_*^{\Sigma_t}$ with the projective model structure inherited from S_*,*

(b) *the map $B \wedge_{\Sigma_t} Q_{t-1}^t \to B \wedge_{\Sigma_t} Y^{\wedge t}$ is a monomorphism.*

2 Proofs

The purpose of this section is to prove Propositions 4.28*, 4.29* and 6.11*. The proofs follow closely our original arguments in [2].

The following proposition is a useful warm-up for the proof of Proposition 4.28*.

Proposition 2.1 *Let $B \in \mathsf{SymSeq}^{\Sigma_t^{\mathrm{op}}}$, $t \geq 2$ and $r, n \geq 0$. Let $\alpha \geq 1$, $q_0 \geq 0$ and $q_1, \dots, q_\alpha \geq 1$ such that $q_0 + q_1 + \cdots + q_\alpha = t$. If Z is a cofibrant object in* SymSeq *with the positive flat stable model structure, then the symmetric sequence*

$$B \check{\otimes} \big(\Sigma_t \cdot_{\Sigma_{q_0} \times \Sigma_{q_1} \times \cdots \times \Sigma_{q_\alpha}} Z^{\check{\otimes} q_0} \check{\otimes} X_1^{\check{\otimes} q_1} \check{\otimes} \cdots \check{\otimes} X_\alpha^{\check{\otimes} q_\alpha} \big)$$

equipped with the diagonal Σ_t–action, after evaluation at $[\mathbf{r}]_n$, is a cofibrant object in $\mathsf{S}_*^{\Sigma_t}$ with the projective model structure inherited from S_*. Here each $K_i \to L_i$ is a generating cofibration for S_* $(1 \leq i \leq \alpha)$, and each X_i is defined as

$$X_i := G_{p_i}(S \otimes G_{m_i}^{H_i}(L_i / K_i)), \quad 1 \leq i \leq \alpha,$$

by applying the indicated functors in [2, (4.1)] *to the pointed simplicial set L_i/K_i, where $m_i \geq 1$, $H_i \subset \Sigma_{m_i}$ is a subgroup and $p_i \geq 0$; in other words, each X_i is assumed to be the cofiber of a generating cofibration for* SymSeq *with the positive flat stable model structure.*

Proof This is an exercise left to the reader; the argument is by induction on q_0, together with (i) the filtrations described in [2, (4.14)] and (ii) the fact that every cofibration of the form $* \to Z$ in SymSeq is a retract of a (possibly transfinite) composition of pushouts of maps as in [2, (6.17)], starting with $Z_0 = *$. □

Proof of Proposition 4.28*(a) Let $m \geq 1$, $H \subset \Sigma_m$ a subgroup, and $k, p \geq 0$. Let $g\colon \partial\Delta[k]_+ \to \Delta[k]_+$ be a generating cofibration for S_* and consider the pushout diagram [2, (6.17)] in SymSeq with Z_0 cofibrant. It follows from [2, Proposition 6.13] that the diagrams

$$\begin{array}{ccc} Q^t_{t-1}(g_*) & \longrightarrow & Q^t_{t-1}(i_0) \\ \downarrow & & \downarrow \\ D^{\check{\otimes} t} & \longrightarrow & Z_1^{\check{\otimes} t} \end{array} \qquad \text{and} \qquad \begin{array}{ccc} B \check{\otimes} Q^t_{t-1}(g_*) & \longrightarrow & B \check{\otimes} Q^t_{t-1}(i_0) \\ \downarrow {\scriptstyle (*)} & & \downarrow {\scriptstyle (**)} \\ B \check{\otimes} D^{\check{\otimes} t} & \longrightarrow & B \check{\otimes} Z_1^{\check{\otimes} t} \end{array}$$

are pushout diagrams in $\mathsf{SymSeq}^{\Sigma_t}$; here, the right-hand diagram is obtained by applying $B \check{\otimes} -$ to the left-hand diagram. Since $m \geq 1$, it follows from [2, (3.7)] that $(*)$, after evaluation at $[\mathbf{r}]_n$, is a cofibration in $\mathsf{S}_*^{\Sigma_t}$; hence $(**)$, after evaluation at $[\mathbf{r}]_n$, is a cofibration in $\mathsf{S}_*^{\Sigma_t}$. Consider a sequence

$$Z_0 \xrightarrow{i_0} Z_1 \xrightarrow{i_1} Z_2 \xrightarrow{i_2} \cdots \tag{2.2}$$

of pushouts of maps as in [2, (6.17)] with Z_0 cofibrant, define $Z_\infty := \operatorname{colim}_q Z_q$, and consider the naturally occurring map $i_\infty\colon Z_0 \to Z_\infty$. Using [2, (4.14)] together with

Proposition 2.1, it is easy to verify that the maps

$$B \check{\otimes} Z_q^{\check{\otimes} t} \to B \check{\otimes} Q_{t-1}^t(i_q) \quad \text{and} \quad B \check{\otimes} Q_{t-1}^t(i_q) \to B \check{\otimes} Z_{q+1}^{\check{\otimes} t},$$

after evaluation at $[\mathbf{r}]_n$, are cofibrations in $\mathsf{S}_*^{\Sigma_t}$. It follows immediately that each

$$B \check{\otimes} Z_q^{\check{\otimes} t} \to B \check{\otimes} Z_{q+1}^{\check{\otimes} t},$$

after evaluation at $[\mathbf{r}]_n$, is a cofibration in $\mathsf{S}_*^{\Sigma_t}$, and hence the map

$$B \check{\otimes} Z_0^{\check{\otimes} t} \to B \check{\otimes} Z_\infty^{\check{\otimes} t},$$

after evaluation at $[\mathbf{r}]_n$, is a cofibration in $\mathsf{S}_*^{\Sigma_t}$. Noting that every cofibration between cofibrant objects in SymSeq with the positive flat stable model structure is a retract of a (possibly transfinite) composition of pushouts of maps as in [2, (6.17)] finishes the proof. □

The following proposition is an exercise left to the reader.

Proposition 2.3 *Let G be a finite group. Consider any pullback diagram*

$$\begin{array}{ccc} A & \longrightarrow & C \\ \downarrow & & \downarrow \\ B & \xrightarrow{f} & D \end{array}$$

of monomorphisms in S_*^G. If f is a cofibration in S_*^G, then the pushout corner map $B \amalg_A C \to D$ is a cofibration in S_*^G.

Definition 2.4 Let I be the poset $\{0 \to 1 \to 2\}$, $\mathsf{I} \to \mathsf{SymSeq}$ a diagram, and $t \geq 1$. Consider any subset $\mathcal{A} \subset \{0 \to 1 \to 2\}^{\times t} = \mathsf{I}^{\times t}$ closed under the canonical Σ_t–action on $\mathsf{I}^{\times t}$. Denote by

$$Q_{\mathcal{A}}^t := \operatorname{colim}(\mathcal{A} \subset \mathsf{I}^{\times t} \to \mathsf{SymSeq}^{\times t} \xrightarrow{\check{\otimes}} \mathsf{SymSeq})$$

the indicated colimit in SymSeq, equipped with the induced Σ_t–action.

The following proposition is proved in Pereira [6]. It provides a refinement of the filtrations for tensor powers of a single map $X \to Y$ in [2, Definition 4.13] to tensor powers of a composition of maps $X \to Y \to Z$, and will be used in the proof of Proposition 4.28*(b) below.

Proposition 2.5 *Let $X \xrightarrow{i} Y \xrightarrow{j} Z$ be morphisms in SymSeq and $t \geq 1$. Consider any convex subset $\mathcal{A} \subset \{0 \to 1 \to 2\}^{\times t} = \mathsf{I}^{\times t}$ closed under the canonical Σ_t–action on $\mathsf{I}^{\times t}$. Let $e \in \mathcal{A}$ be maximal and define*

$$\mathcal{A}' := \mathcal{A}\text{–orbit}(e), \quad \mathcal{A}_e := \{v \in \mathsf{I}^{\times t} : v \leq e,\ v \neq e\}.$$

Suppose $\mathcal{A}' \ni (0, \dots, 0)$. Then $\mathcal{A}_e \subset \mathcal{A}'$, and the following hold:

(a) *The induced map $Q^t_{\mathcal{A}'} \to Q^t_{\mathcal{A}}$ fits into a pushout diagram of the form*

$$\begin{array}{ccc} \Sigma_t \cdot_{\Sigma_p \times \Sigma_q \times \Sigma_r} Q^t_{\mathcal{A}_e} & \longrightarrow & Q^t_{\mathcal{A}'} \\ \downarrow & & \downarrow \\ \Sigma_t \cdot_{\Sigma_p \times \Sigma_q \times \Sigma_r} X^{\check{\otimes} p} \check{\otimes} Y^{\check{\otimes} q} \check{\otimes} Z^{\check{\otimes} r} & \longrightarrow & Q^t_{\mathcal{A}}. \end{array}$$

(b) *The induced map $Q^t_{\mathcal{A}_e} \to X^{\check{\otimes} p} \check{\otimes} Y^{\check{\otimes} q} \check{\otimes} Z^{\check{\otimes} r}$ is isomorphic to $X^{\check{\otimes} p} \check{\otimes} -$ applied to the pushout corner map of the commutative diagram*

$$\begin{array}{ccc} Q^q_{q-1}(i) \check{\otimes} Q^r_{r-1}(j) & \xrightarrow{i_* \check{\otimes} \mathrm{id}} & Y^{\check{\otimes} q} \check{\otimes} Q^r_{r-1}(j) \\ {\scriptstyle \mathrm{id} \check{\otimes} j_*} \downarrow & & \downarrow {\scriptstyle \mathrm{id} \check{\otimes} j_*} \\ Q^q_{q-1}(i) \check{\otimes} Z^{\check{\otimes} r} & \xrightarrow[i_* \check{\otimes} \mathrm{id}]{} & Y^{\check{\otimes} q} \check{\otimes} Z^{\check{\otimes} r}. \end{array}$$

Here, $p := l_0(e)$, $q := l_1(e)$, $r := l_2(e)$, where the "i–length of e", $l_i(e)$, denotes the number of i 's in the t–tuple e, and $Q^0_{-1} := *$.

Proof This follows from the fact that $\mathcal{A}_e = \mathcal{A}^1_e \cup \mathcal{A}^2_e$ can be written as the union of the convex subsets

$$\mathcal{A}^1_e := \{v \in \mathsf{I}^{\times t} : v \leq e, v_j < e_j = 1 \text{ for some } 1 \leq j \leq t\},$$
$$\mathcal{A}^2_e := \{v \in \mathsf{I}^{\times t} : v \leq e, v_j < e_j = 2 \text{ for some } 1 \leq j \leq t\}$$

of $\mathsf{I}^{\times t}$, together with the observation in Goodwillie [1, Claim 2.8] that convexity of $\mathcal{A}^1_e$ and $\mathcal{A}^2_e$ implies that the commutative diagram

$$\begin{array}{ccc} \operatorname{colim}_{\mathcal{A}^1_e \cap \mathcal{A}^2_e} \mathcal{X} & \longrightarrow & \operatorname{colim}_{\mathcal{A}^2_e} \mathcal{X} \\ \downarrow & & \downarrow \\ \operatorname{colim}_{\mathcal{A}^1_e} \mathcal{X} & \longrightarrow & \operatorname{colim}_{\mathcal{A}^1_e \cup \mathcal{A}^2_e} \mathcal{X} \end{array}$$

is a pushout diagram in SymSeq, for any functor $\mathcal{X}\colon \mathsf{I}^{\times t} \to \mathsf{SymSeq}$. □

Remark 2.6 For instance, the induced map $Q_2^3(ji) \to Q_2^3(j)$ is isomorphic to the composition of maps

$$Q^3_{\mathcal{B}_0} \to Q^3_{\mathcal{B}_1} \to Q^3_{\mathcal{B}_2} \to Q^3_{\mathcal{B}_3},$$

where

$$\begin{aligned} \mathcal{B}_0 &:= \{v \in \mathsf{I}^{\times 3} : l_0(v) \geq 1\}, \quad & \mathcal{B}_1 &:= \mathcal{B}_0 \cup \operatorname{orbit}((1,1,1)), \\ \mathcal{B}_2 &:= \mathcal{B}_1 \cup \operatorname{orbit}((1,1,2)), \quad & \mathcal{B}_3 &:= \mathcal{B}_2 \cup \operatorname{orbit}((1,2,2)). \end{aligned}$$

Proof of Proposition 4.28*(b) Proceed as above for part (a) and consider the commutative diagram

$$\begin{array}{ccccccccc} B\check{\otimes} Z_0^{\check{\otimes} t} & \longrightarrow & B\check{\otimes} Q^t_{t-1}(i_0) & \longrightarrow & B\check{\otimes} Q^t_{t-1}(i_1 i_0) & \longrightarrow & B\check{\otimes} Q^t_{t-1}(i_2 i_1 i_0) & \longrightarrow & \cdots \\ \| & & \downarrow & & \downarrow & & \downarrow & & \\ B\check{\otimes} Z_0^{\check{\otimes} t} & \longrightarrow & B\check{\otimes} Z_1^{\check{\otimes} t} & \longrightarrow & B\check{\otimes} Z_2^{\check{\otimes} t} & \longrightarrow & B\check{\otimes} Z_3^{\check{\otimes} t} & \longrightarrow & \cdots \end{array} \tag{2.7}$$

in $\mathsf{SymSeq}^{\Sigma_t}$. We know by part (a) that the bottom row, after evaluation at $[\mathbf{r}]_n$, is a diagram of cofibrations in $\mathsf{S}_*^{\Sigma_t}$. Using Propositions 2.5, 2.3 and 2.1, together with [2, (4.14)], it is easy to verify that each of the maps

$$\begin{aligned} &B\check{\otimes} Q^t_{t-1}(i_0) \to B\check{\otimes} Z_1^{\check{\otimes} t}, \\ &B\check{\otimes} Q^t_{t-1}(i_1 i_0) \to B\check{\otimes} Q^t_{t-1}(i_1) \to B\check{\otimes} Z_2^{\check{\otimes} t}, \\ &B\check{\otimes} Q^t_{t-1}(i_2 i_1 i_0) \to B\check{\otimes} Q^t_{t-1}(i_2 i_1) \to B\check{\otimes} Q^t_{t-1}(i_2) \to B\check{\otimes} Z_3^{\check{\otimes} t}, \quad \ldots \end{aligned}$$

and hence the vertical maps in (2.7), after evaluation at $[\mathbf{r}]_n$, are cofibrations in $\mathsf{S}_*^{\Sigma_t}$. It follows that applying $\operatorname{colim}_{\Sigma_t}(-)$ to (2.7) gives the commutative diagram [2, (6.20)] of monomorphisms, hence the induced map

$$B\check{\otimes}_{\Sigma_t} Q^t_{t-1}(i_\infty) \to B\check{\otimes}_{\Sigma_t} Z_\infty^{\check{\otimes} t}$$

is a monomorphism. The observation that every cofibration between cofibrant objects in SymSeq is a retract of a (possibly transfinite) composition of pushouts of maps as in [2, (6.17)], together with [2, Proposition 6.14], finishes the proof. □

The following proposition, which appeared in an early version of [7], can be thought of as a refinement of the arguments in [4, Lemma 15.5] and [8, Proposition 3.3].

Proposition 2.8 *Let G be a finite group, $Z' \to Z$ a morphism in $(\mathsf{Sp}^\Sigma)^G$, and $k \in \mathbb{Z} \cup \{\infty\}$. Assume that G acts freely on Z', Z away from the basepoint $*$, and consider the G–orbits spectrum $Z/G := \operatorname{colim}_G Z \cong S \wedge_G Z$. If Z (resp. $Z' \to Z$) is k–connected, then Z/G (resp. $Z'/G \to Z/G$) is k–connected.*

Proof Consider the contractible simplicial set $EG \xrightarrow{\simeq} *$ with free right G–action, given by realization of the usual simplicial bar construction with respect to Cartesian product $EG = |\mathrm{Bar}^{\times}(*, G, G)|$. Since G acts freely on Z away from the basepoint, the induced map

$$EG_+ \wedge_G Z \xrightarrow{\simeq} *_+ \wedge_G Z \cong S \wedge_G Z$$

of symmetric spectra is a weak equivalence. We need to verify that $S \wedge_G Z$ is k–connected; it suffices to verify that $EG_+ \wedge_G Z$ is k–connected. The symmetric spectrum $EG_+ \wedge_G Z$ is isomorphic to the realization of the usual simplicial bar construction with respect to smash product $|\mathrm{Bar}^{\wedge}(*_+, G_+, Z)|$. We know by assumption that Z is k–connected, hence $\mathrm{Bar}^{\wedge}(*_+, G_+, Z)$ is objectwise k–connected. The other case is similar. □

Proof of Proposition 4.29* Consider part (b). Suppose $A \to B$ in $\mathsf{SymSeq}^{\Sigma_t^{\mathrm{op}}}$ is a weak equivalence. Then it follows from Proposition 4.28*(a) and Proposition 2.8 (with $k = \infty$) that the induced map

$$A \check{\otimes}_{\Sigma_t} Z^{\check{\otimes} t} \to B \check{\otimes}_{\Sigma_t} Z^{\check{\otimes} t}$$

is a weak equivalence. Consider part (a). Suppose $X \to Y$ in SymSeq is a weak equivalence between cofibrant objects; we want to show that

$$B \check{\otimes}_{\Sigma_t} X^{\check{\otimes} t} \to B \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t}$$

is a weak equivalence. The map $* \to B$ factors in $\mathsf{SymSeq}^{\Sigma_t^{\mathrm{op}}}$ as $* \to B^c \to B$, a cofibration followed by an acyclic fibration, the diagram

$$\begin{array}{ccc} B^c \check{\otimes}_{\Sigma_t} X^{\check{\otimes} t} & \longrightarrow & B^c \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t} \\ \downarrow & & \downarrow \\ B \check{\otimes}_{\Sigma_t} X^{\check{\otimes} t} & \longrightarrow & B \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t} \end{array} \tag{2.9}$$

commutes, and since three of the maps are weak equivalences, so is the fourth; here, we have used [2, Proposition 4.29(b)]. □

Proof of Proposition 6.11* Suppose $X \to Y$ in SymSeq is a cofibration between cofibrant objects; we want to show that $B \check{\otimes}_{\Sigma_t} X^{\check{\otimes} t} \to B \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t}$ is a monomorphism. This follows immediately from Proposition 4.28*. □

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