

Corrigendum to “Homotopy theory of modules over operads in symmetric spectra”

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Dmitri Pavlov and Jakob Scholbach have pointed out that part of Proposition 6.3, and hence Proposition 4.28(a), of Harper [2] are incorrect as stated. While all of the main results of that paper remain unchanged, this necessitates modifications to the statements and proofs of a few technical propositions.

[55P43](#), [55P48](#); [55U35](#)

1 Introduction

The author would like to thank Dmitri Pavlov and Jakob Scholbach for pointing out that the description of the cofibrations in the last sentence of Proposition 6.3 of Harper [2] is incorrect as stated; in general, to verify that a map is a cofibration, it is not enough to be a monomorphism such that $\Sigma_r^{\text{op}} \times G$ acts freely on the simplices of the codomain not in the image.

It is well known that the cofibrations in S_*^G , equipped with the projective model structure, are precisely the monomorphisms such that G acts freely on the simplices of the codomain not in the image. One way to verify this is to (i) argue that the image of such a map is a subcomplex of the codomain (ie the codomain can be built from the image by attaching G -cells), and (ii) note that every monomorphism is isomorphic to its image, hence verifying that such maps are cofibrations, (iii) conversely, to note that every generating cofibration is such a map, and (iv) hence conclude that every cofibration is such a map, by using the fact that every cofibration is a retract of a (possibly transfinite) composition of pushouts of the generating cofibrations. The problem with our argument for the cofibration description in [2, Proposition 6.3] was a cavalier application of the subcomplex argument (i) above; we ignored the fact that $\Sigma_r^{\text{op}} \times G$ and Σ_n might not act independently. Pavlov and Scholbach kindly pointed out this problem to the author, together with a helpful counterexample to focus one’s attention. At the time they were working to generalize the main results in [2] to motivic settings (including Hornbostel’s results [3]; see Remark 1.1). Their efforts have now appeared in Pavlov and Scholbach [5]; included in Appendix A therein is their helpful counterexample, together with further discussion related to these cofibrations.

The following proposition corresponds to the corrected version of [2, Proposition 6.3].

Proposition 6.3* *Let G be a finite group and consider any $n, r \geq 0$. The diagram category $(S_*^{\Sigma_n})^{\Sigma_r^{\text{op}} \times G}$ inherits a corresponding projective model structure from the mixed Σ_n -equivariant model structure on $S_*^{\Sigma_n}$. The weak equivalences (resp. fibrations) are the underlying weak equivalences (resp. fibrations) in $S_*^{\Sigma_n}$.*

The consequence of the misunderstanding of the cofibrations in [2, Proposition 6.3] is that [2, Proposition 4.28(a)] is incorrect as stated. While all of the main results of that paper remain unchanged, this necessitates modifications to the statements and proofs of a few technical propositions.

Remark 1.1 This corrigendum also applies to the proof of the motivic generalization of our results provided by Hornbostel, namely [3, Theorems 3.6, 3.10 and 3.15].

The following proposition corresponds to the corrected version of [2, Proposition 4.28]. For a useful study of additional properties associated to tensor powers of cofibrations, see Pereira [6] and, more recently, Pavlov and Scholbach [5].

Proposition 4.28* *Let $B \in \text{SymSeq}^{\Sigma_t^{\text{op}}}$, $t \geq 1$, and $r, n \geq 0$. If $i: X \rightarrow Y$ is a cofibration between cofibrant objects in SymSeq with the positive flat stable model structure, then*

- (a) *the map $B \check{\otimes} X^{\check{\otimes} t} \rightarrow B \check{\otimes} Y^{\check{\otimes} t}$, after evaluation at $[\mathbf{r}]_n$, is a cofibration in $S_*^{\Sigma_t}$ with the projective model structure inherited from S_* ,*
- (b) *the map $B \check{\otimes}_{\Sigma_t} Q_{t-1}^t \rightarrow B \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t}$ is a monomorphism.*

Since Proposition 4.29 and Proposition 6.11 of [2] are no longer immediately applicable, we include below the closely related **Proposition 4.29*** and **Proposition 6.11*** which describe the technical properties that are actually used in the proofs of the main results in [2].

Proposition 4.29* *Let $t \geq 1$ and consider SymSeq and $\text{SymSeq}^{\Sigma_t^{\text{op}}}$ each with the positive flat stable model structure.*

- (a) *If $B \in \text{SymSeq}^{\Sigma_t^{\text{op}}}$, then the functor*

$$B \check{\otimes}_{\Sigma_t} (-)^{\check{\otimes} t}: \text{SymSeq} \rightarrow \text{SymSeq}$$

preserves weak equivalences between cofibrant objects, and hence its total left derived functor exists.

(b) If $Z \in \text{SymSeq}$ is cofibrant, then the functor

$$-\check{\otimes}_{\Sigma_t} Z^{\check{\otimes} t}: \text{SymSeq}^{\Sigma_t^{\text{op}}} \rightarrow \text{SymSeq}$$

preserves weak equivalences.

Proposition 6.11* Let $t \geq 1$ and consider SymSeq with the positive flat stable model structure. If $B \in \text{SymSeq}^{\Sigma_t^{\text{op}}}$, then the functor

$$B \check{\otimes}_{\Sigma_t} (-)^{\check{\otimes} t}: \text{SymSeq} \rightarrow \text{SymSeq}$$

sends cofibrations between cofibrant objects to monomorphisms.

All references to Propositions 4.28, 4.29 and 6.11 in the proofs of the main results in [2] should be replaced by references to Propositions 4.28*, 4.29* and 6.11*, respectively, which are proved below in Section 2.

Propositions 1.6 and 7.7(a) of [2] are special cases of the statement of Proposition 4.28(a) of [2], and hence are incorrect as stated; the following propositions correspond to their corrected versions, respectively, and are special cases of Proposition 4.28* above.

Proposition 1.6* Let $B \in (\text{Sp}^{\Sigma})^{\Sigma_t^{\text{op}}}$, $t \geq 1$, and $n \geq 0$. If $i: X \rightarrow Y$ is a cofibration between cofibrant objects in symmetric spectra with the positive flat stable model structure, then the map $B \wedge X^{\wedge t} \rightarrow B \wedge Y^{\wedge t}$, after evaluation at n , is a cofibration of Σ_t -diagrams in pointed simplicial sets.

Proposition 7.7* Let $B \in (\text{Sp}^{\Sigma})^{\Sigma_t^{\text{op}}}$, $t \geq 1$, and $n \geq 0$. If $i: X \rightarrow Y$ is a cofibration between cofibrant objects in Sp^{Σ} with the positive flat stable model structure, then

- (a) the map $B \wedge X^{\wedge t} \rightarrow B \wedge Y^{\wedge t}$, after evaluation at n , is a cofibration in $S_*^{\Sigma_t}$ with the projective model structure inherited from S_* ,
- (b) the map $B \wedge_{\Sigma_t} Q_{t-1}^t \rightarrow B \wedge_{\Sigma_t} Y^{\wedge t}$ is a monomorphism.

2 Proofs

The purpose of this section is to prove Propositions 4.28*, 4.29* and 6.11*. The proofs follow closely our original arguments in [2].

The following proposition is a useful warm-up for the proof of Proposition 4.28*.

Proposition 2.1 *Let $B \in \text{SymSeq}^{\Sigma_t^{\text{op}}}$, $t \geq 2$ and $r, n \geq 0$. Let $\alpha \geq 1$, $q_0 \geq 0$ and $q_1, \dots, q_\alpha \geq 1$ such that $q_0 + q_1 + \dots + q_\alpha = t$. If Z is a cofibrant object in SymSeq with the positive flat stable model structure, then the symmetric sequence*

$$B \check{\otimes} (\Sigma_t \cdot \Sigma_{q_0} \times \Sigma_{q_1} \times \dots \times \Sigma_{q_\alpha} Z \check{\otimes}^{q_0} \check{\otimes} X_1 \check{\otimes}^{q_1} \check{\otimes} \dots \check{\otimes} X_\alpha \check{\otimes}^{q_\alpha})$$

equipped with the diagonal Σ_t -action, after evaluation at $[\mathbf{r}]_n$, is a cofibrant object in $S_^{\Sigma_t}$ with the projective model structure inherited from S_* . Here each $K_i \rightarrow L_i$ is a generating cofibration for S_* ($1 \leq i \leq \alpha$), and each X_i is defined as*

$$X_i := G_{p_i}(S \otimes G_{m_i}^{H_i}(L_i/K_i)), \quad 1 \leq i \leq \alpha,$$

by applying the indicated functors in [2, (4.1)] to the pointed simplicial set L_i/K_i , where $m_i \geq 1$, $H_i \subset \Sigma_{m_i}$ is a subgroup and $p_i \geq 0$; in other words, each X_i is assumed to be the cofiber of a generating cofibration for SymSeq with the positive flat stable model structure.

Proof This is an exercise left to the reader; the argument is by induction on q_0 , together with (i) the filtrations described in [2, (4.14)] and (ii) the fact that every cofibration of the form $* \rightarrow Z$ in SymSeq is a retract of a (possibly transfinite) composition of pushouts of maps as in [2, (6.17)], starting with $Z_0 = *$. □

Proof of Proposition 4.28*(a) Let $m \geq 1$, $H \subset \Sigma_m$ a subgroup, and $k, p \geq 0$. Let $g: \partial\Delta[k]_+ \rightarrow \Delta[k]_+$ be a generating cofibration for S_* and consider the pushout diagram [2, (6.17)] in SymSeq with Z_0 cofibrant. It follows from [2, Proposition 6.13] that the diagrams

$$\begin{array}{ccc} Q_{t-1}^t(g_*) & \longrightarrow & Q_{t-1}^t(i_0) \\ \downarrow & & \downarrow \\ D^{\check{\otimes}t} & \longrightarrow & Z_1^{\check{\otimes}t} \end{array} \quad \text{and} \quad \begin{array}{ccc} B \check{\otimes} Q_{t-1}^t(g_*) & \longrightarrow & B \check{\otimes} Q_{t-1}^t(i_0) \\ \downarrow (*) & & \downarrow (***) \\ B \check{\otimes} D^{\check{\otimes}t} & \longrightarrow & B \check{\otimes} Z_1^{\check{\otimes}t} \end{array}$$

are pushout diagrams in SymSeq^{Σ_t} ; here, the right-hand diagram is obtained by applying $B \check{\otimes} -$ to the left-hand diagram. Since $m \geq 1$, it follows from [2, (3.7)] that $(*)$, after evaluation at $[\mathbf{r}]_n$, is a cofibration in $S_*^{\Sigma_t}$; hence $(**)$, after evaluation at $[\mathbf{r}]_n$, is a cofibration in $S_*^{\Sigma_t}$. Consider a sequence

$$(2.2) \quad Z_0 \xrightarrow{i_0} Z_1 \xrightarrow{i_1} Z_2 \xrightarrow{i_2} \dots$$

of pushouts of maps as in [2, (6.17)] with Z_0 cofibrant, define $Z_\infty := \text{colim}_q Z_q$, and consider the naturally occurring map $i_\infty: Z_0 \rightarrow Z_\infty$. Using [2, (4.14)] together with

Proposition 2.1, it is easy to verify that the maps

$$B \check{\otimes} Z_q^{\check{\otimes} t} \rightarrow B \check{\otimes} Q_{t-1}^t(i_q) \quad \text{and} \quad B \check{\otimes} Q_{t-1}^t(i_q) \rightarrow B \check{\otimes} Z_{q+1}^{\check{\otimes} t},$$

after evaluation at $[\mathbf{r}]_n$, are cofibrations in $S_*^{\Sigma^t}$. It follows immediately that each

$$B \check{\otimes} Z_q^{\check{\otimes} t} \rightarrow B \check{\otimes} Z_{q+1}^{\check{\otimes} t},$$

after evaluation at $[\mathbf{r}]_n$, is a cofibration in $S_*^{\Sigma^t}$, and hence the map

$$B \check{\otimes} Z_0^{\check{\otimes} t} \rightarrow B \check{\otimes} Z_\infty^{\check{\otimes} t},$$

after evaluation at $[\mathbf{r}]_n$, is a cofibration in $S_*^{\Sigma^t}$. Noting that every cofibration between cofibrant objects in SymSeq with the positive flat stable model structure is a retract of a (possibly transfinite) composition of pushouts of maps as in [2, (6.17)] finishes the proof. \square

The following proposition is an exercise left to the reader.

Proposition 2.3 *Let G be a finite group. Consider any pullback diagram*

$$\begin{array}{ccc} A & \longrightarrow & C \\ \downarrow & & \downarrow \\ B & \xrightarrow{f} & D \end{array}$$

of monomorphisms in S_^G . If f is a cofibration in S_*^G , then the pushout corner map $B \amalg_A C \rightarrow D$ is a cofibration in S_*^G .*

Definition 2.4 Let l be the poset $\{0 \rightarrow 1 \rightarrow 2\}$, $l \rightarrow \text{SymSeq}$ a diagram, and $t \geq 1$. Consider any subset $\mathcal{A} \subset \{0 \rightarrow 1 \rightarrow 2\}^{\times t} = l^{\times t}$ closed under the canonical Σ_t -action on $l^{\times t}$. Denote by

$$Q_{\mathcal{A}}^t := \text{colim}(\mathcal{A} \subset l^{\times t} \rightarrow \text{SymSeq}^{\times t} \xrightarrow{\check{\otimes}} \text{SymSeq})$$

the indicated colimit in SymSeq , equipped with the induced Σ_t -action.

The following proposition is proved in Pereira [6]. It provides a refinement of the filtrations for tensor powers of a single map $X \rightarrow Y$ in [2, Definition 4.13] to tensor powers of a composition of maps $X \rightarrow Y \rightarrow Z$, and will be used in the proof of Proposition 4.28*(b) below.

Proposition 2.5 Let $X \xrightarrow{i} Y \xrightarrow{j} Z$ be morphisms in SymSeq and $t \geq 1$. Consider any convex subset $\mathcal{A} \subset \{0 \rightarrow 1 \rightarrow 2\}^{\times t} = l^{\times t}$ closed under the canonical Σ_t -action on $l^{\times t}$. Let $e \in \mathcal{A}$ be maximal and define

$$\mathcal{A}' := \mathcal{A}\text{-orbit}(e), \quad \mathcal{A}_e := \{v \in l^{\times t} : v \leq e, v \neq e\}.$$

Suppose $\mathcal{A}' \ni (0, \dots, 0)$. Then $\mathcal{A}_e \subset \mathcal{A}'$, and the following hold:

(a) The induced map $Q_{\mathcal{A}'}^t \rightarrow Q_{\mathcal{A}_e}^t$ fits into a pushout diagram of the form

$$\begin{array}{ccc} \Sigma_t \cdot \Sigma_p \times \Sigma_q \times \Sigma_r & Q_{\mathcal{A}_e}^t & \longrightarrow & Q_{\mathcal{A}'}^t \\ & \downarrow & & \downarrow \\ \Sigma_t \cdot \Sigma_p \times \Sigma_q \times \Sigma_r & X^{\check{\otimes} p} \check{\otimes} Y^{\check{\otimes} q} \check{\otimes} Z^{\check{\otimes} r} & \longrightarrow & Q_{\mathcal{A}}^t. \end{array}$$

(b) The induced map $Q_{\mathcal{A}_e}^t \rightarrow X^{\check{\otimes} p} \check{\otimes} Y^{\check{\otimes} q} \check{\otimes} Z^{\check{\otimes} r}$ is isomorphic to $X^{\check{\otimes} p} \check{\otimes} -$ applied to the pushout corner map of the commutative diagram

$$\begin{array}{ccc} Q_{q-1}^q(i) \check{\otimes} Q_{r-1}^r(j) & \xrightarrow{i_* \check{\otimes} \text{id}} & Y^{\check{\otimes} q} \check{\otimes} Q_{r-1}^r(j) \\ \text{id} \check{\otimes} j_* \downarrow & & \downarrow \text{id} \check{\otimes} j_* \\ Q_{q-1}^q(i) \check{\otimes} Z^{\check{\otimes} r} & \xrightarrow{i_* \check{\otimes} \text{id}} & Y^{\check{\otimes} q} \check{\otimes} Z^{\check{\otimes} r}. \end{array}$$

Here, $p := l_0(e)$, $q := l_1(e)$, $r := l_2(e)$, where the “ i -length of e ”, $l_i(e)$, denotes the number of i 's in the t -tuple e , and $Q_{-1}^0 := *$.

Proof This follows from the fact that $\mathcal{A}_e = \mathcal{A}_e^1 \cup \mathcal{A}_e^2$ can be written as the union of the convex subsets

$$\begin{aligned} \mathcal{A}_e^1 &:= \{v \in l^{\times t} : v \leq e, v_j < e_j = 1 \text{ for some } 1 \leq j \leq t\}, \\ \mathcal{A}_e^2 &:= \{v \in l^{\times t} : v \leq e, v_j < e_j = 2 \text{ for some } 1 \leq j \leq t\} \end{aligned}$$

of $l^{\times t}$, together with the observation in Goodwillie [1, Claim 2.8] that convexity of \mathcal{A}_e^1 and \mathcal{A}_e^2 implies that the commutative diagram

$$\begin{array}{ccc} \text{colim}_{\mathcal{A}_e^1 \cap \mathcal{A}_e^2} \mathcal{X} & \longrightarrow & \text{colim}_{\mathcal{A}_e^2} \mathcal{X} \\ \downarrow & & \downarrow \\ \text{colim}_{\mathcal{A}_e^1} \mathcal{X} & \longrightarrow & \text{colim}_{\mathcal{A}_e^1 \cup \mathcal{A}_e^2} \mathcal{X} \end{array}$$

is a pushout diagram in SymSeq , for any functor $\mathcal{X}: l^{\times t} \rightarrow \text{SymSeq}$. □

Remark 2.6 For instance, the induced map $Q_2^3(ji) \rightarrow Q_2^3(j)$ is isomorphic to the composition of maps

$$Q_{B_0}^3 \rightarrow Q_{B_1}^3 \rightarrow Q_{B_2}^3 \rightarrow Q_{B_3}^3,$$

where

$$\begin{aligned} B_0 &:= \{v \in I^{\times 3} : l_0(v) \geq 1\}, & B_1 &:= B_0 \cup \text{orbit}((1, 1, 1)), \\ B_2 &:= B_1 \cup \text{orbit}((1, 1, 2)), & B_3 &:= B_2 \cup \text{orbit}((1, 2, 2)). \end{aligned}$$

Proof of Proposition 4.28*(b) Proceed as above for part (a) and consider the commutative diagram

$$(2.7) \quad \begin{array}{ccccccc} B \check{\otimes} Q_0^{\check{\otimes} t} & \rightarrow & B \check{\otimes} Q_{t-1}^t(i_0) & \rightarrow & B \check{\otimes} Q_{t-1}^t(i_1 i_0) & \rightarrow & B \check{\otimes} Q_{t-1}^t(i_2 i_1 i_0) \rightarrow \dots \\ \parallel & & \downarrow & & \downarrow & & \downarrow \\ B \check{\otimes} Z_0^{\check{\otimes} t} & \longrightarrow & B \check{\otimes} Z_1^{\check{\otimes} t} & \longrightarrow & B \check{\otimes} Z_2^{\check{\otimes} t} & \longrightarrow & B \check{\otimes} Z_3^{\check{\otimes} t} \longrightarrow \dots \end{array}$$

in SymSeq^{Σ_t} . We know by part (a) that the bottom row, after evaluation at $[\mathbf{r}]_n$, is a diagram of cofibrations in $S_*^{\Sigma_t}$. Using Propositions 2.5, 2.3 and 2.1, together with [2, (4.14)], it is easy to verify that each of the maps

$$\begin{aligned} B \check{\otimes} Q_{t-1}^t(i_0) &\rightarrow B \check{\otimes} Z_1^{\check{\otimes} t}, \\ B \check{\otimes} Q_{t-1}^t(i_1 i_0) &\rightarrow B \check{\otimes} Q_{t-1}^t(i_1) \rightarrow B \check{\otimes} Z_2^{\check{\otimes} t}, \\ B \check{\otimes} Q_{t-1}^t(i_2 i_1 i_0) &\rightarrow B \check{\otimes} Q_{t-1}^t(i_2 i_1) \rightarrow B \check{\otimes} Q_{t-1}^t(i_2) \rightarrow B \check{\otimes} Z_3^{\check{\otimes} t}, \dots \end{aligned}$$

and hence the vertical maps in (2.7), after evaluation at $[\mathbf{r}]_n$, are cofibrations in $S_*^{\Sigma_t}$. It follows that applying $\text{colim}_{\Sigma_t}(-)$ to (2.7) gives the commutative diagram [2, (6.20)] of monomorphisms, hence the induced map

$$B \check{\otimes}_{\Sigma_t} Q_{t-1}^t(i_\infty) \rightarrow B \check{\otimes}_{\Sigma_t} Z_\infty^{\check{\otimes} t}$$

is a monomorphism. The observation that every cofibration between cofibrant objects in SymSeq is a retract of a (possibly transfinite) composition of pushouts of maps as in [2, (6.17)], together with [2, Proposition 6.14], finishes the proof. □

The following proposition, which appeared in an early version of [7], can be thought of as a refinement of the arguments in [4, Lemma 15.5] and [8, Proposition 3.3].

Proposition 2.8 *Let G be a finite group, $Z' \rightarrow Z$ a morphism in $(\text{Sp}^{\Sigma})^G$, and $k \in \mathbb{Z} \cup \{\infty\}$. Assume that G acts freely on Z', Z away from the basepoint $*$, and consider the G -orbits spectrum $Z/G := \text{colim}_G Z \cong S \wedge_G Z$. If Z (resp. $Z' \rightarrow Z$) is k -connected, then Z/G (resp. $Z'/G \rightarrow Z/G$) is k -connected.*

Proof Consider the contractible simplicial set $EG \xrightarrow{\simeq} *$ with free right G -action, given by realization of the usual simplicial bar construction with respect to Cartesian product $EG = |\text{Bar}^\times(*, G, G)|$. Since G acts freely on Z away from the basepoint, the induced map

$$EG_+ \wedge_G Z \xrightarrow{\simeq} *_{+} \wedge_G Z \cong S \wedge_G Z$$

of symmetric spectra is a weak equivalence. We need to verify that $S \wedge_G Z$ is k -connected; it suffices to verify that $EG_+ \wedge_G Z$ is k -connected. The symmetric spectrum $EG_+ \wedge_G Z$ is isomorphic to the realization of the usual simplicial bar construction with respect to smash product $|\text{Bar}^\wedge(*_+, G_+, Z)|$. We know by assumption that Z is k -connected, hence $\text{Bar}^\wedge(*_+, G_+, Z)$ is objectwise k -connected. The other case is similar. □

Proof of Proposition 4.29* Consider part (b). Suppose $A \rightarrow B$ in $\text{SymSeq}^{\Sigma_t^{\text{op}}}$ is a weak equivalence. Then it follows from Proposition 4.28*(a) and Proposition 2.8 (with $k = \infty$) that the induced map

$$A \check{\otimes}_{\Sigma_t} Z^{\check{\otimes} t} \rightarrow B \check{\otimes}_{\Sigma_t} Z^{\check{\otimes} t}$$

is a weak equivalence. Consider part (a). Suppose $X \rightarrow Y$ in SymSeq is a weak equivalence between cofibrant objects; we want to show that

$$B \check{\otimes}_{\Sigma_t} X^{\check{\otimes} t} \rightarrow B \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t}$$

is a weak equivalence. The map $*$ $\rightarrow B$ factors in $\text{SymSeq}^{\Sigma_t^{\text{op}}}$ as $*$ $\rightarrow B^c \rightarrow B$, a cofibration followed by an acyclic fibration, the diagram

$$(2.9) \quad \begin{array}{ccc} B^c \check{\otimes}_{\Sigma_t} X^{\check{\otimes} t} & \longrightarrow & B^c \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t} \\ \downarrow & & \downarrow \\ B \check{\otimes}_{\Sigma_t} X^{\check{\otimes} t} & \longrightarrow & B \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t} \end{array}$$

commutes, and since three of the maps are weak equivalences, so is the fourth; here, we have used [2, Proposition 4.29(b)]. □

Proof of Proposition 6.11* Suppose $X \rightarrow Y$ in SymSeq is a cofibration between cofibrant objects; we want to show that $B \check{\otimes}_{\Sigma_t} X^{\check{\otimes} t} \rightarrow B \check{\otimes}_{\Sigma_t} Y^{\check{\otimes} t}$ is a monomorphism. This follows immediately from Proposition 4.28*. □

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Received: 31 July 2014

