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Topologically slice knots that are not smoothly slice in any definite 4–manifold

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We prove that there exist infinitely many topologically slice knots which cannot bound a smooth null-homologous disk in any definite 4–manifold. Furthermore, we show that we can take such knots so that they are linearly independent in the knot concordance group.

57M25, 57M27

1 Introduction

A knot K in S^3 is called *smoothly slice* (resp. *topologically slice*) if K bounds a smooth disk (resp. topologically locally flat disk) in B^4. While any smoothly slice knot is obviously topologically slice, it has been known that there exist infinitely many topologically slice knots that are not smoothly slice; for instance, see Endo [3] and Gompf [6]. The purpose of this paper is to prove that there exist infinitely many topologically slice knots which cannot bound a null-homologous smooth disk not only in B^4 but also in any 4–manifold with definite intersection form.

For a 4–manifold V with boundary S^3, we call a knot K in S^3 *smoothly slice in V* if K bounds a smooth disk D in V such that $[D, \partial D] = 0 \in H_2(V, \partial V; \mathbb{Z})$. We call a 4–manifold V *definite* if the intersection form of V is either positive definite or negative definite. We denote the smooth knot concordance group by $\mathcal{C}$. Then our main theorem is stated as follows.

Theorem 1.1 *There exist infinitely many topologically slice knots which are not slice in any definite 4–manifold. Furthermore, we can take such knots so that they are linearly independent in $\mathcal{C}$.*

In order to prove Theorem 1.1, we use the Heegaard Floer τ–invariant and the V_k–invariants defined by Ni and Wu [13]. In particular, by combining Wu's cabling

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formula [21] and Bodnár, Celoria and Golla's connected sum inequality [1] for V_k–invariants, we prove the following proposition. Here we denote the mirror image of a knot K by K^*, the $(n,1)$–cable of K by $K_{n,1}$ and the connected sum of two knots K and J by $K\#J$. The symbol ε denotes Hom's ε–invariant [9].

Proposition 1.2 *Let K and J be knots. If $V_0(K) > V_0(J)$ and $\tau(K), \tau(J) > 0$, then for any positive integer n with $\tau(K) + \frac{1}{2}(1-\varepsilon(J)) < n\big(\tau(J) + \frac{1}{2}(1-\varepsilon(J))\big)$, the knot $K\#(J_{n,1})^*$ is not slice in any definite 4–manifold.*

Note that if both K and J are topologically slice, then $K\#(J_{n,1})^*$ is also topologically slice for any $n \in \mathbb{Z}\setminus\{0\}$. Furthermore, it follows from Hedden, Kim and Livingston [8, Proposition 6.1, Theorem B.1] that for any $m \in \mathbb{Z}_{>0}$, there exists a topologically slice knot K_m with $V_0(K_m) = m$ and $\tau(K_m) > 0$. Hence by taking $K_l\#((K_m)_{n,1})^*$ so that $l > m$ and n is sufficiently large, we immediately obtain infinitely many topologically slice knots which are not slice in any definite 4–manifold. Our proof of the linear independence of these topologically slice knots relies on Kim and Park's recent result [11].

The problem of smooth sliceness leads to the notion of the *kinkiness* of knots, as defined by Gompf [6]. Let K be a knot in $S^3 = \partial B^4$, and consider all self-transverse immersed disks in B^4 with boundary K. Then we define $k_+(K)$ (resp. $k_-(K)$) to be the minimal number of positive (resp. negative) self-intersection points occurring in such a disk. Gompf proved in [6] that for any $n \in \mathbb{Z}_{>0}$, there exists a topologically slice knot K such that $(k_+(K), k_-(K)) = (0, n)$. On the other hand, as far as the author knows, whether there exist topologically slice knots which satisfy $k_+ > 0$ and $k_- > 0$ has remained so far unsolved. In this paper, we give an affirmative answer to the question.

Theorem 1.3 *For any $m, n \in \mathbb{Z}_{>0}$, there exist infinitely many topologically slice knots with $k_+ \geq m$ and $k_- \geq n$.*

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2 Preliminaries

In this section, we recall some knot concordance invariants derived from Heegaard Floer homology theory and show that they give obstructions to the sliceness of knots in definite 4–manifolds.

2.1 Correction terms and d_1–invariant

Ozsváth and Szabó [15] introduced a $\mathbb{Q}$–valued invariant d (called the *correction term*) for rational homology 3–spheres endowed with a Spin^c structure. In particular, since any integer homology 3–sphere Y has a unique Spin^c structure, we may denote the correction term simply by $d(Y)$ in this case. Furthermore, for any integer homology 3–sphere Y, we note that $d(Y)$ is an even integer.

Let $S^3_1(K)$ denote the 1–surgery along a knot K in S^3. Then $S^3_1(K)$ is an integer homology 3–sphere, and hence we can define the d_1*–invariant* of K as $d_1(K) := d(S^3_1(K))$. It is known that $d_1(K)$ is a knot concordance invariant of K. For details, see [19]. Here we show that the d_1–invariant gives an obstruction to the sliceness in negative-definite 4–manifolds.

Lemma 2.1 *If a knot K is smoothly slice in some negative-definite 4–manifold, then we have $d_1(K) = 0$.*

Proof It is proved in [19] that $d_1(K) \leq 0$ for any knot K. Hence we only need to show that $d_1(K) \geq 0$.

Suppose that K is slice in a negative-definite 4–manifold V. Then there exists a properly embedded null-homologous disk D in V with boundary K. By attaching a $(+1)$–framed 2–handle h^2 along K, and gluing D with the core of h^2, we obtain an embedded 2–sphere S in $W := V \cup h^2$ with self-intersection $+1$. This implies that there exists a 4–manifold W' with boundary $S^3_1(K)$ such that $W = W' \# \mathbb{C}P^2$. Note that $\partial W' = \partial W = S^3_1(K)$. Since the number of positive eigenvalues of the intersection form of W is one, the intersection form of W' must be negative definite. Now we use the following theorem.

Theorem 2.2 (Ozsváth and Szabó [15, Corollary 9.8]) *If Y is an integer homology 3–sphere with $d(Y) < 0$, then there is no negative-definite 4–manifold X with $\partial X = Y$.*

By Theorem 2.2 and the existence of W', we have $d_1(K) = d(S^3_1(K)) \geq 0$. □

2.2 τ–invariant, V_k–invariant and ν^+–invariant

The τ*–invariant* τ is a famous knot concordance invariant defined by Ozsváth and Szabó [17] and Rasmussen [20]. It is known that τ is a group homomorphism from $\mathcal{C}$ to $\mathbb{Z}$, while d_1 is not a homomorphism.

The V_k*–invariant* is a family of $\mathbb{Z}_{\geq 0}$–valued knot concordance invariants $\{V_k(K)\}_{k\geq 0}$ defined by Ni and Wu [13]. In particular, $\nu^+(K) := \min\{k \geq 0 \mid V_k(K) = 0\}$ is known as the ν^+*–invariant* [10]. It is proved therein that for any knot K, the inequality $\tau(K) \leq \nu^+(K)$ holds.

In [13], Ni and Wu prove that the set $\{V_k(K)\}_{k\geq 0}$ determines all correction terms of p/q–surgeries along K for any coprime $p, q > 0$. Let $S^3_{p/q}(K)$ denote the p/q–surgery along K. Note that there is a canonical identification between the set of Spin^c structures over $S^3_{p/q}(K)$ and $\{i \mid 0 \leq i \leq p-1\}$. This identification can be made explicit by the procedure in [18, Sections 4 and 7].

Proposition 2.3 [13, Proposition 1.6] *Suppose $p, q > 0$, and fix $0 \leq i \leq p-1$. Then*

$$d(S^3_{p/q}(K), i) = d(S^3_{p/q}(O), i) - 2\max\{V_{\lfloor i/q \rfloor}(K), V_{\lfloor (p+q-1-i)/q \rfloor}(K)\},$$

where O denotes the unknot and $\lfloor \cdot \rfloor$ is the floor function.

As a corollary, the following lemma holds. Note that $\{V_k(K)\}_{k\geq 0}$ satisfy the inequalities $V_k(K) - 1 \leq V_{k+1}(K) \leq V_k(K)$ for each $k \geq 0$.

Lemma 2.4 *For any knot K, we have $d_1(K) = -2V_0(K)$.*

Here we show that the τ–invariant also gives an obstruction to the sliceness in negative-definite 4–manifolds.

Lemma 2.5 *If a knot K is slice in some negative-definite 4–manifold, then $\tau(K) \leq 0$.*

Proof Suppose that K is slice in some negative-definite 4–manifold. Then by Lemma 2.1, we have $d_1(K) = 0$. By Lemma 2.4, this implies that $V_0(K) = 0$ and $\nu^+(K) = 0$. Hence we have $\tau(K) \leq \nu^+(K) = 0$. □

By combining Lemmas 2.1, 2.4 and 2.5, we obtain the following obstruction to sliceness in definite 4–manifolds.

Proposition 2.6 *Let K be a knot. If $V_0(K) \neq 0$ and $\tau(K) < 0$, then K is not slice in any definite 4–manifold.*

Proof Assume that a knot K satisfies $V_0(K) \neq 0$ and $\tau(K) < 0$. Then it immediately follows from Lemmas 2.1 and 2.4 that K is not slice in any negative-definite 4–manifold.

Suppose that K is slice in a positive-definite 4–manifold V. Then by reversing the orientation of V, we obtain a slice disk in $-V$ with boundary K^*. Since $-V$ is negative definite and τ is a group homomorphism from $\mathcal{C}$ to $\mathbb{Z}$, Lemma 2.5 implies

$$\tau(K) = -\tau(K^*) \geq 0.$$

This contradicts the assumption $\tau(K) < 0$. □

2.3 Some formulas for V_k–invariants

In this subsection, we recall Wu's cabling formula and Bodnár, Celoria and Golla's connected sum inequality for V_k–invariants. Since the $(p,1)$–cable and the connected sum of topologically slice knots are also topologically slice, we can estimate the V_k–invariants of various topologically slice knots by using these formulas.

We first recall Wu's cabling formula for V_k. For coprime integers $p, q > 0$, let $T_{p,q}$ denote the (p,q)–torus knot and $K_{p,q}$ the (p,q)–cable of a knot K. We define a map

$$\phi_{p,q}\colon \left\{i \mid 0 \leq i \leq \tfrac{1}{2}pq\right\} \to \{i \mid 0 \leq i \leq q-1\}$$

by

$$\phi_{p,q}(i) \equiv i - \tfrac{1}{2}(p-1)(q-1) \bmod q.$$

Proposition 2.7 (Wu [21, Equation (13)]) *Given $p, q > 0$ and $\frac{1}{2}(p-1)(q-1) \leq i \leq \frac{1}{2}pq$, we have*

$$V_i(K_{p,q}) = V_{\lfloor \phi_{p,q}(i)/p \rfloor}(K).$$

If we consider the case where $q = 1$, then we have $\phi_{p,1}(i) = 0$ for any $0 \leq i \leq \frac{1}{2}p$. Hence Proposition 2.7 gives the following lemma.

Lemma 2.8 *Given $p > 0$ and $0 \leq i \leq \frac{1}{2}p$, we have*

$$V_i(K_{p,1}) = V_0(K).$$

Next we recall Bodnár, Celoria and Golla's connected sum inequality for V_k.

Proposition 2.9 [1, Proposition 6.1] *For any two knots K and J and any $m, n \in \mathbb{Z}_{\geq 0}$,*

$$V_{m+n}(K \# J) \leq V_m(K) + V_n(J).$$

In this paper, we only need Proposition 2.9 in the case where $m = n = 0$, which is stated as follows.

Proposition 2.10 *For any two knots K and J, we have*

$$V_0(K \# J) \leq V_0(K) + V_0(J).$$

We can use Proposition 2.10 to give a lower bound for V_0 of the connected sum of two knots as well. In particular, we have the following lemma.

Lemma 2.11 *For any two knots K and J, we have*

$$V_0(K \# J^*) \geq V_0(K) - V_0(J).$$

Proof For the inequality in Proposition 2.10, by replacing K with $K \# J^*$, we have

$$V_0(K \# J^* \# J) \leq V_0(K \# J^*) + V_0(J).$$

Since $K \# J^* \# J$ is concordant to K, we have $V_0(K \# J^* \# J) = V_0(K)$. This completes the proof. □

3 Proof of the main theorems

In this section, we prove Proposition 1.2, Theorem 1.1 and Theorem 1.3.

Proof of Proposition 1.2 Suppose that two given knots K and J satisfy $V_0(K) > V_0(J)$ and $\tau(K), \tau(J) > 0$. Fix a positive integer n satisfying $\tau(K) + \frac{1}{2}(1 - \varepsilon(J)) < n\big(\tau(J) + \frac{1}{2}(1 - \varepsilon(J))\big)$. Then Lemmas 2.8 and 2.11 imply that

$$V_0(K \# (J_{n,1})^*) \geq V_0(K) - V_0(J_{n,1}) = V_0(K) - V_0(J) > 0.$$

Furthermore, by [9, Theorem 1] and the assumption $\tau(J) > 0$, we have $\varepsilon(J) \neq 0$ and

$$\tau(K \# (J_{n,1})^*) = \tau(K) - \tau(J_{n,1}) = \tau(K) - n \cdot \tau(J) - \tfrac{1}{2}(n-1)(1 - \varepsilon(J)) < 0.$$

Hence it follows from Proposition 2.6 that $K \# (J_{n,1})^*$ is not slice in any definite 4–manifold. □

Proof of Theorem 1.1 For a knot K, let $\mathrm{Wh}(K)$ denote the positively clasped untwisted Whitehead double of K. Then we set

$$K_n := \big(\#_{i=1}^{3} \mathrm{Wh}(T_{2,3})\big) \# ((\mathrm{Wh}(T_{2,3}))_{n+3,1})^*$$

for any positive integer n. Since the Alexander polynomial of K_n equals 1, K_n is topologically slice for any n [4; 5].

We first prove that K_n is not slice in any definite 4–manifold. As mentioned in [11, Section 3], it follows from [8, Proposition 6.1, Theorem B.1] and [11, Lemma 3.1] that $\#_{i=1}^{k} \mathrm{Wh}(T_{2,3})$ is ν^+–equivalent to $T_{2,2k+1}$ for any $k > 0$. (Here, knots K and J being ν^+–equivalent means that the equalities $\nu^+(K \# J^*) = \nu^+(J \# K^*) = 0$ hold.) Furthermore, we can see that V_0 (more generally, all V_k) is invariant under ν^+–equivalence. Indeed, if two knots K and J are ν^+–equivalent, then Lemma 2.11 implies that

$$0 = -V_0(J \# K^*) \leq V_0(K) - V_0(J) \leq V_0(K \# J^*) = 0.$$

Hence, combining with [16, Corollary 1.5], we have

$$V_0\big(\#_{i=1}^{k} \mathrm{Wh}(T_{2,3})\big) = V_0(T_{2,2k+1}) = \left\lceil \tfrac{1}{2}k \right\rceil.$$

This implies that

$$V_0\big(\#_{i=1}^{3} \mathrm{Wh}(T_{2,3})\big) = \left\lceil \tfrac{3}{2} \right\rceil > \left\lceil \tfrac{1}{2} \right\rceil = V_0(\mathrm{Wh}(T_{2,3})).$$

Moreover, it follows from [7, Theorem 1.5] and [9, Section 1] that $\tau(\mathrm{Wh}(T_{2,3})) = 1$ and $\varepsilon(\mathrm{Wh}(T_{2,3})) = 1$, and we have

$$\tau\big(\#_{i=1}^{3}\mathrm{Wh}(T_{2,3})\big)+\tfrac{1}{2}(1-\varepsilon(\mathrm{Wh}(T_{2,3}))) < (n+3)\big(\tau(\mathrm{Wh}(T_{2,3}))+\tfrac{1}{2}(1-\varepsilon(\mathrm{Wh}(T_{2,3})))\big)$$

for any $n > 0$. Hence we can apply Proposition 1.2 to $\big(\#_{i=1}^{3} \mathrm{Wh}(T_{2,3}), \mathrm{Wh}(T_{2,3})\big)$ and conclude that for any $n > 0$, the knot K_n is not slice in any definite 4–manifold.

Next we prove that the knots $\{K_n\}_{n \in \mathbb{Z}_{>0}}$ are linearly independent in $\mathcal{C}$. Suppose that a linear combination $m_1[K_{n_1}] + \cdots + m_k[K_{n_k}]$ equals zero in $\mathcal{C}$ (where we may assume that $0 < n_1 < n_2 < \cdots < n_k$). Then we have the equality

$$(1)\quad 3\big(\textstyle\sum_{i=1}^{k} m_i\big)[\mathrm{Wh}(T_{2,3})] = m_1[(\mathrm{Wh}(T_{2,3}))_{n_1+3,1}] + \cdots + m_k[(\mathrm{Wh}(T_{2,3}))_{n_k+3,1}].$$

In the proof of [11, Therorem A], the authors define a homomorphism $\phi\colon \mathcal{C} \to \mathbb{Z}^\infty$ and show that $\phi([(\mathrm{Wh}(T_{2,3}))_{n+3,1}]) = (*, \ldots, *, 1, 0, 0, \ldots)$, where 1 is the $(n+2)^{\mathrm{nd}}$ coordinate. Hence we can see that the $(n_k+2)^{\mathrm{nd}}$ coordinate of ϕ(RHS of (1)) is m_k. On the other hand, we can verify that $\phi([\mathrm{Wh}(T_{2,3})]) = \phi([T_{2,3}]) = (0, 0, \ldots)$, and hence m_k must be 0. Inductively, we have $m_1 = \cdots = m_k = 0$. This completes the proof. □

Remark If one just wants to find infinitely many knots which are not smoothly slice in any simply connected definite 4–manifold, then we only need the following proposition, which immediately follows from [2, Proposition 1.2].

Proposition 3.1 *If the Levine–Tristram signature of a knot K has both positive and negative values, then K is not smoothly slice in any simply connected definite 4–manifold.*

Indeed, we can take $J_k := T_{2,2k+9} \# \big(\#_{i=1}^{k+5} T_{2,3}\big)^*$ $(k \in \mathbb{Z}_{>0})$ as the concrete sequence. To see that any J_k satisfies the assumption of Proposition 3.1, let us recall the Levine–Tristram signature and compute it for positive $(2,q)$–torus knots. Let A be a Seifert matrix for a knot K and $x \in (0,1)$ a real number. Then

$$A_x := (1-e^{2i\pi x})A + (1-e^{-2i\pi x})A^\tau$$

is a Hermitian matrix, and all of its eigenvalues are real numbers. We define

$$\sigma_K(x) := \#\{\text{positive eigenvalues of } A_x\} - \#\{\text{negative eigenvalues of } A_x\}.$$

Then the value $\sigma_K(x)$ is an invariant of K for any $x \in (0,1)$. Note that $\sigma_K\big(\frac{1}{2}\big)$ is equal to the knot signature, and $\sigma_K(x) = \sigma_K(1-x)$ for any $0 < x \leq \frac{1}{2}$.

Lemma 3.2 *Let $q > 1$ be an odd integer and $0 < x \leq \frac{1}{2}$ a real number. If x is not contained in $\{1/2q, 3/2q, \dots, (q-2)/2q\}$, then we have*

$$\sigma_{T_{2,q}}(x) = -2\big\lceil \tfrac{1}{2}(2qx-1)\big\rceil.$$

Proof Applying [12, Proposition 1] to the cases where $p=2$ and $0 < x \leq \frac{1}{2}$, we have

$$\sigma_{T_{2,q}}(x) = \#\big\{j \in \mathbb{Z} \;\big|\; x + \tfrac{1}{2} < j/q < 1\big\} - \#\big\{j \in \mathbb{Z} \;\big|\; 0 < j/q < x + \tfrac{1}{2}\big\}. \tag{2}$$

Furthermore, if $x \notin \{1/2q, 3/2q, \dots, (q-2)/2q\}$, then the equality

$$\#\big\{j \in \mathbb{Z} \;\big|\; 0 < j/q < x + \tfrac{1}{2}\big\} + \#\big\{j \in \mathbb{Z} \;\big|\; x + \tfrac{1}{2} < j/q < 1\big\} = q-1$$

holds, and hence (2) implies

$$\sigma_{T_{2,q}}(x) = q - 1 - 2\big(\#\big\{j \in \mathbb{Z} \;\big|\; 0 < j/q < x + \tfrac{1}{2}\big\}\big). \tag{3}$$

We denote the value $\#\big\{j \in \mathbb{Z} \mid 0 < j/q < x + \frac{1}{2}\big\}$ by n. Then the inequalities

$$n < q\big(x + \tfrac{1}{2}\big) = \tfrac{1}{2}(2qx-1) + \tfrac{1}{2}(q+1) \leq n+1$$

hold, and these imply

$$\big\lceil \tfrac{1}{2}(2qx-1)\big\rceil = n - \tfrac{1}{2}(q-1).$$

This equality reduces (3) to

$$\sigma_{T_{2,q}}(x) = -2\big\lceil \tfrac{1}{2}(2qx-1)\big\rceil. \qquad \square$$

Figure 1: A positive crossing (left) and a negative crossing (right)

By Lemma 3.2, we can verify that $\sigma_{J_k}(x) = -2$ for $x \in (1/(2(2k+9)), 3/(2(2k+9)))$ and $\sigma_{J_k}\left(\frac{1}{2}\right) = 2$. Furthermore, since all torus knots are linearly independent in $\mathcal{C}$ [12], the knots J_k are also linearly independent.

Finally we prove Theorem 1.3. To do so, we use the following observation relating kinkiness to ν^+ and τ.

Lemma 3.3 *For any knot K, we have the inequalities*

$$\nu^+(K) \le k^+(K) \quad \textit{and} \quad -k^-(K) \le \tau(K) \le k^+(K).$$

Proof If a knot K_1 is deformed into K_2 by a crossing change from a positive crossing (Figure 1, left) to a negative crossing (Figure 1, right) (resp. from a negative crossing to a positive crossing), then we say that K_1 is deformed into K_2 by a *positive (resp. negative) crossing change*. It is proved in [1, Theorem 1.3] and [17, Corollary 1.5] that if a knot K_+ is deformed into K_- by a positive crossing change, then we have

$$\nu^+(K_-) \le \nu^+(K_+) \le \nu^+(K_-) + 1 \quad \text{and} \quad \tau(K_-) \le \tau(K_+) \le \tau(K_-) + 1.$$

Furthermore, it follows from [14, Proposition 2.1] that for any knot K, there exists a knot J which is concordant to K and which can be deformed into a slice knot L by just $k^+(K)$ positive crossing changes and finitely many negative crossing changes. These imply that

$$\nu^+(K) = \nu^+(J) \le \nu^+(L) + k^+(K) = k^+(K)$$

and

$$\tau(K) = \tau(J) \le \tau(L) + k^+(K) = k^+(K).$$

By applying the same argument to K^*, we have

$$-\tau(K) = \tau(K^*) \le k^+(K^*) = k^-(K).$$ □

Proof of Theorem 1.3 For positive integers k and l, we define $K_{k,l}$ by

$$K_{k,l} := \left(\#_{i=1}^{2k+1} \operatorname{Wh}(T_{2,3})\right) \# ((\operatorname{Wh}(T_{2,3}))_{l+2k+1,1})^*.$$

Since the knots have the trivial Alexander polynomial, $K_{k,l}$ is topologically slice for any $k, l > 0$. We prove that for any $m, n \in \mathbb{Z}_{>0}$, we have that $\{K_{m,l}\}_{l \geq n}$ are mutually distinct, and each of them satisfies $k^+(K_{m,l}) \geq m$ and $k^-(K_{m,l}) \geq n$.

By applying the argument in the proof of Theorem 1.1, we have

$$V_0(K_{k,l}) \geq V_0\big(\#_{i=1}^{2k+1} \mathrm{Wh}(T_{2,3})\big) - V_0((\mathrm{Wh}(T_{2,3}))_{l+2k+1,1}) = k$$

and

$$\tau(K_{k,l}) = 2k + 1 - (l + 2k + 1) = -l.$$

In particular, $K_{k,l}$ is not equal to $K_{k,l'}$ if $l \neq l'$. Furthermore, since $\nu^+(K) = \min\{i \in \mathbb{Z}_{\geq 0} \mid V_i(K) = 0\}$ and $V_{i+1}(K) \geq V_i(K) - 1$, we have $\nu^+(K_{k,l}) \geq k$. Hence Lemma 3.3 proves that $k^+(K_{k,l}) \geq k$ and $k^-(K_{k,l}) \geq l$. This completes the proof. □

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