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Euler characteristics and actions of automorphism groups of free groups

SHENGKUI YE

Let M^r be a connected orientable manifold with the Euler characteristic $\chi(M) \not\equiv 0 \bmod 6$. Denote by $\mathrm{SAut}(F_n)$ the unique subgroup of index two in the automorphism group of a free group. Then any group action of $\mathrm{SAut}(F_n)$ (and thus the special linear group $\mathrm{SL}_n(\mathbb{Z})$) with $n \geq r+2$ on M^r by homeomorphisms is trivial. This confirms a conjecture related to Zimmer's program for these manifolds.

57S20; 57S17

1 Introduction

Let $\mathrm{SL}_n(\mathbb{Z})$ be the special linear group over integers. There is an action of $\mathrm{SL}_n(\mathbb{Z})$ on the sphere S^{n-1} induced by the linear action on the Euclidean space $\mathbb{R}^n$. It is believed that this action is minimal in the following sense.

Conjecture 1.1 Any action of $\mathrm{SL}_n(\mathbb{Z})$ with $n \geq 3$ on a compact connected r–manifold by homeomorphisms factors through a finite group action if $r < n-1$.

The smooth version of this conjecture was formulated by Farb and Shalen [8], and is related to the Zimmer program concerning group actions of lattices in Lie groups on manifolds (see the survey articles Fisher [9] and Zimmer and Morris [17] for more details). When $r = 1$, Conjecture 1.1 is already proved by Witte [15]. Weinberger [14] confirms the conjecture when $M = T^r$ is a torus. Bridson and Vogtmann [5] confirm the conjecture when $M = S^r$ is a sphere. Ye [16] confirms the conjecture for all flat manifolds. For $C^{1+\beta}$ group actions of a finite-index subgroup in $\mathrm{SL}_n(\mathbb{Z})$, one of the results proved by Brown, Rodriguez-Hertz and Wang [7] confirms Conjecture 1.1 for surfaces. For C^2 group actions of cocompact lattices, Brown, Fisher and Hurtado [6] confirms Conjecture 1.1. Note that the C^0 actions could be very different from smooth actions. It seems that very few other cases have been confirmed (for group actions preserving extra structures, many results have been obtained; see [9; 17]).

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Let $\mathrm{SAut}(F_n)$ denote the unique subgroup of index two in the automorphism group $\mathrm{Aut}(F_n)$ of the free group F_n. Note that there is a surjection $\phi\colon \mathrm{SAut}(F_n) \to \mathrm{SL}_n(\mathbb{Z})$ given by the abelianization of F_n. In this note, we obtain the following general result on topological actions.

Theorem 1.2 *Let M^r be a connected (resp. orientable) manifold with the Euler characteristic $\chi(M) \not\equiv 0 \bmod 3$ (resp. $\chi(M) \not\equiv 0 \bmod 6$). Then any group action of $\mathrm{SAut}(F_n)$ with $n > r+1$ on M^r by homeomorphisms is trivial.*

Since any group action of $\mathrm{SL}_n(\mathbb{Z})$ could be lifted to an action of $\mathrm{SAut}(F_n)$, Theorem 1.2 confirms Conjecture 1.1 for orientable manifolds with nonvanishing Euler characteristic modulo 6.

Remarks 1.3 (i) The bound of n cannot be improved, since $\mathrm{SAut}(F_n)$ acts through $\mathrm{SL}_n(\mathbb{Z})$ nontrivially on S^{n-1}.

(ii) Belolipetsky and Lubotzky [1] prove that for any finite group G and any dimension $r \geq 2$, there exists a hyperbolic manifold M^r such that $\mathrm{Isom}(M) \cong G$. Therefore, $\mathrm{SL}_n(\mathbb{Z})$ and thus $\mathrm{SAut}(F_n)$ could act nontrivially through a finite quotient group on such a hyperbolic manifold. This implies that the condition on the Euler characteristic could not be dropped.

(iii) To satisfy the assumption on the Euler characteristic, the dimension r has to be even. There are however no further restrictions on r, as the following example shows. Let $\{g_i\}$ be a sequence of nonnegative integers with $g_i \not\equiv 1 \bmod 3$ and Σ_{g_i} an orientable surface of genus g_i. For any even number r,

$$M^r = \Sigma_{g_1} \times \Sigma_{g_2} \times \cdots \times \Sigma_{g_{r/2}}$$

has nonzero (mod 6) Euler characteristic and thus satisfies the condition of Theorem 1.2.

Our proof of Theorem 1.2 relies on torsion and so will not be applicable to finite-index subgroups. Actually, Theorem 1.2 does not hold for general finite-index subgroups. For example, let $q < \mathbb{Z}$ be a nontrivial ideal and C a nontrivial cyclic subgroup of $\mathrm{SL}_n(\mathbb{Z}/q)$. Let $f\colon \mathrm{SAut}(F_n) \xrightarrow{\phi} \mathrm{SL}_n(\mathbb{Z}) \to \mathrm{SL}_n(\mathbb{Z}/q)$ be the group homomorphism induced by quotient ring homomorphism. The group $f^{-1}(C)$ could act nontrivially on S^2 by rotations through C. By a profound result of Grunewald and Lubotzky [11, Corollary 1.2], there is a group homomorphism ρ from a finite-index subgroup G of $\mathrm{SAut}(F_n)$ with $n \geq 3$ to $\mathrm{SL}_{n-1}(\mathbb{Z})$ such that $\mathrm{Im}\, f$ is of finite index. Therefore, the group G could act through $\mathrm{SL}_{n-1}(\mathbb{Z})$ on S^{n-2}, which is an infinite-group action.

2 Proofs

The cohomology n–manifold mod p (a prime) considered in this article will be as in Borel [2]. Roughly speaking, a cohomology n–manifold mod p is a locally compact Hausdorff space which has a local cohomology structure (with coefficient group $\mathbb{Z}/p$) resembling that of Euclidean n–space. Let L be $\mathbb{Z}$ or $\mathbb{Z}/p$. All homology groups in this section are Borel–Moore homology groups with compact supports and coefficients in a sheaf $\mathcal{A}$ of modules over L. The homology groups of X are denoted by $H^c_*(X;\mathcal{A})$ and the Alexander–Spanier cohomology groups (with coefficients in L and compact supports) are denoted by $H_c^*(X;L)$. We define the cohomology dimension $\dim_L X = \min\{n : H_c^{n+1}(U;L) = 0 \text{ for all open } U \subset X\}$. If $L = \mathbb{Z}/p$, we write $\dim_p X$ for $\dim_L X$. For integer $k \geq 0$, let $\mathcal{O}_k$ denote the sheaf associated to the presheaf $U \mapsto H^c_k(X, X\backslash U; L)$. An n–dimensional homology manifold over L (denoted by n–hm_L) is a locally compact Hausdorff space X with $\dim_L X < +\infty$, and $\mathcal{O}_k(X;L) = 0$ for $p \neq n$ and $\mathcal{O}_n(X;L)$ is locally constant with stalks isomorphic to L. The sheaf $\mathcal{O}_n$ is called the orientation sheaf. There is a similar notion of cohomology manifold over L, denoted by n–cm_L (see [4, page 373].

Topological manifolds are (co)homology manifolds over L.

In order to prove Theorem 1.2, we need several lemmas.

Lemma 2.1 (Borel [2, Theorem 4.3, page 182]) *Let G be an elementary p–group operating on a first-countable cohomology n–manifold X mod p. Let $x \in X$ be a fixed point of G on X and let $n(H)$ be the cohomology dimension mod p of the component of x in the fixed-point set of a subgroup H of G. If $r = n(G)$, we have*

$$n - r = \sum_H (n(H) - r),$$

where H runs through the subgroups of G of index p.

Lemma 2.2 (Mann and Su [13, Theorem 2.2]) *Let G be an elementary p–group of rank k operating effectively on a first-countable connected cohomology r–manifold X mod p. Suppose $\dim_p F(G) = r_0 \geq 0$, where $F(G)$ is the fixed-point set of G on X. Then $k \leq \frac{1}{2}(r - r_0)$ if $p \neq 2$ and $k \leq r - r_0$ if $p = 2$.*

Let X be an oriented cohomology r–manifold X over $\mathbb{Z}$ (in the sense of Bredon [3]). A homeomorphism $f \colon X \to X$ is orientation-preserving if the orientation is preserved. In the following lemma, we consider the case of orientation-preserving actions.

Lemma 2.3 *Let G be a nontrivial elementary 2–group of rank k operating effectively on a first-countable connected oriented cohomology r–manifold X over $\mathbb{Z}$ by orientation-preserving homeomorphisms. Suppose* $\dim_2 F(G) = r_0 \geq 0$, *where $F(G)$ is the fixed-point set of G on X. Then $k \leq r - 1 - r_0$.*

Proof Note that the manifold X is also a cohomology r–manifold over $\mathbb{Z}/2$ and the fixed-point set $\mathrm{Fix}(g)$ is a cohomology manifold over $\mathbb{Z}/2$ by Smith theory (see [2, Theorem 2.2 and the bottom of page 78]). If there is a nontrivial element $g \in G$ such that the dimension of the fixed-point set $\mathrm{Fix}(g)$ is r, the element g acts trivially by invariance of domain (see Bredon [4, Corollary 16.19, page 383]). This is a contradiction to the assumption that G acts effectively. Therefore, we could assume that $\mathrm{Fix}(g)$ is of nontrivial even codimension by Bredon [2, Theorem 2.5, page 79]. (We use the assumption that X is a cohomology manifold over $\mathbb{Z}$.) Now the lemma becomes obvious for $r = 1$. When $r = 2$, the dimension of $\mathrm{Fix}(H)$ is zero for any nontrivial subgroup $H < G$. If $r = 2$ and $k = 1$, the fixed-point set $\mathrm{Fix}(G)$ is of dimension 0 and the statement holds. If $r = 2$ and $k \geq 2$, this would be impossible by Borel's formula in Lemma 2.1.

Choose a nontrivial element $g \in G$ such that the fixed-point set $\mathrm{Fix}(g)$ is of the maximal dimension among all nontrivial elements in G. Fix a connected component M of $\mathrm{Fix}(g)$ containing a connected component of $F(G)$ with the largest dimension. Choose a decomposition $G = \langle g \rangle \bigoplus G_0$ for some subgroup $G_0 < G$. The action of the complement G_0 leaves M invariant. If some nontrivial element $h \in G_0$ acts trivially on M, let $H = \langle g, h \rangle \cong (\mathbb{Z}/2)^2$. By the assumption that the fixed-point set $\mathrm{Fix}(g)$ is of maximal dimension, each nontrivial element in H has its fixed-point set of dimension $\dim_2 \mathrm{Fix}(g)$. This is impossible by Borel's formula in Lemma 2.1. Therefore, the action of G_0 on M is effective. Note that $\mathrm{Fix}(g)$ is a cohomology manifold over $\mathbb{Z}/2$ (by Smith theory) of dimension at most $r-2$. Thus the rank of G_0 is at most $r-2-r_0$ by Lemma 2.2. Therefore,

$$k = \mathrm{rank}(G_0) + 1 \leq r - 1 - r_0. \qquad \square$$

The inequality in Lemma 2.3 is sharp, by considering the linear action of the diagonal subgroup $(\mathbb{Z}/2)^{n-1} < \mathrm{SL}_n(\mathbb{Z})$ on $\mathbb{R}^n$.

Let X be a locally compact Hausdorff space and a finite group $G = (\mathbb{Z}/p)^n$ acting on X by homeomorphisms. In the remaining part of this article, we suppose that the Euler characteristic $\sum_i (-1)^i \dim H^i(X; \mathbb{Z}/p) =: \chi(X; \mathbb{Z}/p)$ is defined. The following results are well known from Smith theory (see [2, Theorem 3.2 on page 40 and Theorem 4.4 on pages 42–43]).

Lemma 2.4 *We have the following.*

(i) *Suppose that the cyclic group $G = \mathbb{Z}/p$ operates freely on X, with $\dim_{\mathbb{Z}} X < \infty$ and $H^*(X;\mathbb{Z}/p)$ finite-dimensional. Then $H^*(X/G;\mathbb{Z}/p)$ is finite-dimensional and*

$$\chi(X;\mathbb{Z}/p) = p\chi(X/G;\mathbb{Z}/p).$$

(ii) *Suppose that the cyclic group $\mathbb{Z}/p$ operates on X, with $\dim_{\mathbb{Z}/p} X < \infty$ and $H^*(X;\mathbb{Z}/p)$ finite-dimensional. Let F be the fixed-point set. Then $H^*(F;\mathbb{Z}/p)$ and $H^*(X-F;\mathbb{Z}/p)$ are finite-dimensional and*

$$\chi(X;\mathbb{Z}/p) = \chi(X-F;\mathbb{Z}/p) + \chi(F;\mathbb{Z}/p).$$

Denote by G_x the stabilizer of $x \in X$. Suppose that $X = \bigcup_{i=0}^{n} X_i$ is the union of subspaces $X_i = \{x \in X : \mathrm{order}(G_x) = p^i\}$. It is clear that each X_i is G–invariant and $X_n = \mathrm{Fix}(G)$.

Theorem 2.5 *Suppose that G is a* (not necessarily abelian) *p–group of order p^n acting on X. Then*

$$\chi(X;\mathbb{Z}/p) = \sum_{i=0}^{n} \chi(X_i;\mathbb{Z}/p) = \sum_{i=0}^{n} p^{n-i} a_i$$

for some integers a_i. Actually, we have $\chi(X_i;\mathbb{Z}/p) = p^{n-i} a_i$.

Proof We prove the theorem by induction on n. When $n = 0$, the statement is trivial by the assumption that the Euler characteristic $\chi(X;\mathbb{Z}/p)$ is defined. When $n = 1$, this is Lemma 2.4 by noting that $F = X_1$ and $X_0 = X - F$. Choose a to be an order-p element in the center of G. Let $F = \mathrm{Fix}(a)$ and $X_0 = X - F$. The quotient group $G/\langle a\rangle$ acts on the quotient space $X_0/\langle a\rangle$ and F. Let

$$\begin{aligned} Y_i &= \bigl\{x \in (X-F)/\langle a\rangle : |(G/\langle a\rangle)_x| = p^i\bigr\}, \\ Z_i &= \bigl\{x \in F : |(G/\langle a\rangle)_x| = p^i\bigr\}. \end{aligned}$$

We will denote $\chi(X;\mathbb{Z}_p)$ by $\chi(X)$ for short. By the induction step, we have that

$$\chi((X-F)/\langle a\rangle) = \sum_{i=0}^{n-1} \chi(Y_i) = \sum_{i=0}^{n-1} p^{n-1-i} a_i',$$

and

$$\chi(F)=\sum_{i=0}^{n-1}\chi(Z_i)=\sum_{i=0}^{n-1}p^{n-1-i}b_i.$$

The first equality in the statement of the theorem is proved by noting that $X_i = q^{-1}(Y_i)\cup Z_{i-1}$ with the convention that $Z_{-1}=\varnothing$, where $q\colon (X-F)\to(X-F)/\langle a\rangle$ is the projection. Therefore, we have

$$\begin{aligned}\chi(X)&=\chi(X-F)+\chi(F)\\&=p\chi((X-F)/\langle a\rangle)+\chi(F)\\&=p^na_0'+\sum_{i=1}^{n-1}p^{n-i}(a_i'+b_{i-1})+b_{n-1}.\end{aligned}$$

The proof is finished by choosing $a_0=a_0'$, $a_i = a_i'+b_{i-1}$ for $1\le i\le n-1$ and $a_n=b_{n-1}$. The last statement — that $\chi(X_i;\mathbb{Z}/p)=p^{n-i}a_i$ — could be proved by noting $X_i=q^{-1}(Y_i)\cup Z_{i-1}$ and a similar induction argument. □

For a group G and a prime p, let the p–rank be $\mathrm{rk}_p(G)=\sup\{k:(\mathbb{Z}/p)^k\hookrightarrow G\}$. It is possible that $\mathrm{rk}_p(G)=+\infty$.

Theorem 2.6 *Let M^r be a first-countable connected cohomology r–manifold over $\mathbb{Z}/p$ and* $\mathrm{Homeo}(M)$ *the group of self-homeomorphisms.* (We adopt the convention that $p^n=1$ when $n<0$.) *Then the p–rank satisfies*

$$p^{\mathrm{rk}_p(\mathrm{Homeo}(M))-[r/2]}\mid\chi(M;\mathbb{Z}/p)$$

when p is odd and

$$2^{\mathrm{rk}_2(\mathrm{Homeo}(M))-r}\mid\chi(M;\mathbb{Z}/2)$$

when $p=2$. If M^r with $r\ge 1$ is an oriented connected cohomology r–manifold over $\mathbb{Z}$ and $\mathrm{Homeo}_+(M)$ *is the group of orientation-preserving self-homeomorphisms, we have*

$$2^{\mathrm{rk}_2(\mathrm{Homeo}(M))-r+1}\mid\chi(M;\mathbb{Z}/2).$$

Proof Suppose that an elementary p–group $G=(\mathbb{Z}/p)^{n_p}$ acts effectively on M for $n_p=\mathrm{rk}_p(\mathrm{Homeo}(M))$. If the group action is free, we have $p^{n_p}\mid\chi(M)$ by Theorem 2.5 and the statements are obvious. In the following, we suppose that the group action is not free. We let $X=M$ and X_i as in Theorem 2.5 for the sake of sticking to the notation of Theorem 2.5. Denote by G_x the stabilizer of $x\in X_i$ for nonempty X_i. By Lemma 2.2

we have $i := \mathrm{rank}(G_x) \leq \frac{r}{2}$ if $p \neq 2$ and $\mathrm{rank}(G_x) \leq r$ if $p = 2$. Therefore, we have $p^{n_p - i} \geq p^{n_p - r/2}$ when $p \neq 2$ (or $p^{n_p - i} \geq p^{n_p - r}$ when $p = 2$). This implies that $p^{n_p - [r/2]} \mid \chi(M; \mathbb{Z}/p)$ (or $2^{n_2 - r} \mid \chi(M; \mathbb{Z}/2)$ when $p = 2$) considering Theorem 2.5. A similar argument proves the orientation-preserving case using Lemma 2.3, by noting that the subgroup G_x acts on M orientation-preservingly if G does. □

Remark 2.7 When M is a surface, Theorem 2.6 was already known to Kulkarni [12].

Fixing a basis $\{a_1, \ldots, a_n\}$ for the free group F_n, we define several elements in $\mathrm{Aut}(F_n)$ as the following. The inversions are defined as

$$e_i\colon \begin{cases} a_i \mapsto a_i^{-1}, \\ a_j \mapsto a_j & \text{if } j \neq i, \end{cases}$$

while the permutations are

$$(ij)\colon \begin{cases} a_i \mapsto a_j, \\ a_j \mapsto a_i, \\ a_k \mapsto a_k & \text{if } k \neq i, j. \end{cases}$$

The subgroup $N < \mathrm{Aut}(F_n)$ generated by all e_i for $i = 1, \ldots, n$ is isomorphic to $(\mathbb{Z}/2)^n$. The subgroup $W_n < \mathrm{Aut}(F_n)$ is generated by N and all (ij) for $1 \leq i \neq j \leq n$. Denote $SW_n = W_n \cap \mathrm{SAut}(F_n)$ and $SN = N \cap \mathrm{SAut}(F_n)$. The element $\Delta = e_1 e_2 \cdots e_n$ is central in W_n and lies in $\mathrm{SAut}(F_n)$ precisely when n is even.

The following result is Proposition 3.1 of [5].

Lemma 2.8 *Suppose $n \geq 3$ and let f be a homomorphism from $\mathrm{SAut}(F_n)$ to a group G. If $f|_{SW_n}$ has nontrivial kernel K, then one of the following holds:*

1. *n is even, $K = \langle \Delta \rangle$ and f factors through $\mathrm{PSL}(n, \mathbb{Z})$,*
2. *$K = SN$ and the image of f is isomorphic to $\mathrm{SL}(n, \mathbb{Z}/2)$, or*
3. *f is the trivial map.*

When $n = 2m$ is even, for each $1 \leq i \leq m$ define $R_i\colon F_n \to F_n$ by $a_{2i-1} \mapsto a_{2i}^{-1}$, $a_{2i} \mapsto a_{2i}^{-1} a_{2i-1}$ and $a_j \mapsto a_j$ for $j \neq 2i, 2i-1$. Let $T < \mathrm{SAut}(F_n)$ be the subgroup generated by all R_i for $i = 1, \ldots, m$. By Lemma 3.2 of Bridson and Vogtmann [5], T is isomorphic to $(\mathbb{Z}/3)^m$. The following result is Proposition 3.4 of [5].

Lemma 2.9 *For $m \geq 2$ and any group G, let $\phi\colon \mathrm{SAut}(F_{2m}) \to G$ be a homomorphism. If $\phi|_T$ is not injective, then ϕ is trivial.*

Proof of Theorem 1.2 Let $f\colon \mathrm{SAut}(F_n) \to \mathrm{Homeo}(M)$ be a group homomorphism. Since the Euler characteristics satisfy $\chi(M;\mathbb{Z}/2) = \chi(M;\mathbb{Z}/3)$ (see [3, Theorem 5.2 and Corollary 5.7]), they will be simply denoted by $\chi(M)$. Since any action of $\mathrm{SAut}(F_n)$ on a nonorientable manifold M can be uniquely lifted to be an action on the orientable double covering $\overline{M}$ (see [4, Corollary 9.4, page 67]), we may assume that M is oriented and the group action is orientation-preserving by noting that $\mathrm{SAut}(F_n)$ is perfect (see [10]). When M is nonorientable and $\chi(M) \not\equiv 0 \bmod 3$, we would still have $\chi(\overline{M}) \not\equiv 0 \bmod 3$.

When $n = 3$, the manifold M is of dimension 1. This case is already proved by Bridson and Vogtmann [5]. Suppose that $n \geq 4$. Choose $m = \left[\frac{n}{2}\right]$ (the integer part) and $T \cong (\mathbb{Z}/3)^m$. Let $\mathrm{SAut}(F_{2m})$ be the subgroup of $\mathrm{SAut}(F_n)$ fixing a_n if n is odd. Note that $\mathrm{SAut}(F_n)$ is normally generated by a Nielsen automorphism in $\mathrm{SAut}(F_{2m})$ (see [10]). If f is not trivial, the restriction $f|_{\mathrm{SAut}(F_{2m})}$ is not trivial and thus the map $f|_T$ is injective by Lemma 2.9. Theorem 2.6 implies that $3 \mid \chi(M)$, by noting that $n - r \geq 2$. This is a contradiction in the nonorientable case. If $\mathrm{Im}\, f$ contains a copy of $(\mathbb{Z}/2)^{n-2}$, Theorem 2.6 would imply that $2 \mid \chi(M)$. This would be a contradiction to the assumption that $\chi(M) \not\equiv 0 \bmod 6$ for the orientable manifold M. Therefore, the restriction $f|_{SN}$ is not injective and case (1) in Lemma 2.8 cannot happen, since $SN \cong (\mathbb{Z}/2)^{n-1}$. If case (2) happens, the image satisfies $\mathrm{Im}\, f = \mathrm{SL}(n,\mathbb{Z}/2)$. Let $x_{1i}(1)$ denote the matrix with ones along the diagonal, 1 in the $(1,i)^{\text{th}}$ position and zeros elsewhere. Since the subgroup $\langle x_{12}(1), x_{13}(1), \ldots, x_{1n}(1)\rangle \cong (\mathbb{Z}/2)^{n-1}$, we still have $2 \mid \chi(M)$. This is a contradiction, which implies that f has to be trivial. □

From the above proof, we see that Theorem 1.2 also holds for cohomology manifolds over $\mathbb{Z}$.

Remark 2.10 For a specific n, the conditions of Theorem 1.2 may be improved. For example, when n is odd, Case (1) in Lemma 2.8 cannot happen. A similar proof as that of Theorem 1.2 shows that any action of $\mathrm{SAut}(F_{2k+1})$ with $k \geq 1$ on an orientable manifold M^r with $\chi(M) \not\equiv 0 \bmod 12$ (resp. $\chi(M) \not\equiv 0 \bmod 2$) by homeomorphisms is trivial when $2k > r$ (resp. $2k \geq r$).

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Department of Mathematical Sciences, Xi'an Jiaotong-Liverpool University
Jiangsu, China

shengkui.ye@xjtlu.edu.cn

https://yeshengkui.wordpress.com/

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