Correction to the article
Gorenstein duality for real spectra

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In this erratum we report a mistake in the authors' article [2] and record a corrected statement. Throughout this erratum we will use the notation of [2] and all numbering of lemmas and theorems refers to this source.

Theorem 1.4 is not correct as stated for $n \geq 2$, even on underlying nonequivariant spectra. Indeed, the homotopy groups of $\Gamma_{J_{n-1}} E\mathbb{R}(n)$ are countable in every degree and this is not true for $\mathbb{Z}_{(2)}^{E\mathbb{R}(n)}$. Let us explain where the mistake in the proof lies and then how to correct the statement.

The mistake is in Lemma 5.8. Consider for example $B = BP\mathbb{R}\langle 2 \rangle$, whose underlying homotopy groups are $\mathbb{Z}_{(2)}[v_1, v_2]$. Contrary to the statement of Lemma 5.8, the $\mathbb{Z}[v_2]$–module $\mathbb{Z}_{(2)}[v_1^{\pm 1}, v_2]$ does not have bounded v_2–divisibility. Indeed, for every k the element $v_2^k v_1^{-3k}$ is of degree 0 and divisible by v_2^k. Thus, the proof of Theorem 5.9 breaks down and the theorem is indeed wrong as stated. Note though that Example 5.10 (recovering a result of Ricka) remains unaffected.

What happens indeed is that $\mathbb{Z}_{(2)}^{E\mathbb{R}(n)}$ is a kind of completion of a shift of $\Gamma_{J_{n-1}} E\mathbb{R}(n)$, at least on the level of underlying homotopy groups. Instead of making this precise, we show that Theorems 1.4 and 5.9 are true after a suitable cellularization. For an $M\mathbb{R}$–module K, we say that there is a K–*cellular equivalence* between two $M\mathbb{R}$–modules if there is an $M\mathbb{R}$–linear equivalence between their $\mathbb{R}$–cellularizations with respect to K in the sense of Section 2.B and Proposition 3.8.

Theorem (corrected form of Theorem 5.9) *Let the notation be as in Theorem 5.1 and assume for simplicity that only finitely many m_i are zero and that $m_n = 0$. Let K*

be the $M\mathbb{R}$–module $M\mathbb{R}/(\underline{v}\setminus\bar{v}_n)$, where $\underline{v}\setminus\bar{v}_n$ denotes the sequence of all $\bar{v}_i$ such that $m_i = 0$ and $i \neq n$. Then there is a K–cellular equivalence

$$\mathbb{Z}_{(2)}^{M[\bar{v}_n^{-1}]} \simeq_K \Sigma^{-m'+|\underline{v}|+(k-1)+4-2\rho} M.$$

Proof Modifying the proof of Theorem 5.9, it suffices to show that there is a K–cellular equivalence between $\mathrm{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n}(\Gamma_{\underline{v}\setminus\bar{v}_n} M))$ and $\Sigma^{-1} M[\bar{v}_n^{-1}]$. For this, we claim first that the natural map

$$(1) \qquad \mathrm{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n}(\Gamma_{\underline{v}\setminus\bar{v}_n} M)) \to \mathrm{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n} M)$$

is a K–cellular equivalence. We have indeed a chain of equivalences

$$\mathrm{Hom}_{M\mathbb{R}}\big(K, \mathrm{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n}(\Gamma_{\underline{v}\setminus\bar{v}_n} M))\big) \simeq \mathrm{holim}\,\mathrm{Hom}_{M\mathbb{R}}(K, \Gamma_{\bar{v}_n}(\Gamma_{\underline{v}\setminus\bar{v}_n} M))$$
$$\simeq \mathrm{holim}\,\mathrm{Hom}_{M\mathbb{R}}(K, \Gamma_{\bar{v}_n} M)$$
$$\simeq \mathrm{Hom}_{M\mathbb{R}}(K, \mathrm{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n} M)).$$

Thus, the map in (1) is a K–cellular equivalence. Moreover,

$$\mathrm{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n} M) \simeq \Sigma^{-1} M[\bar{v}_n^{-1}]$$

by Lemma 5.7. Combining this equivalence and the K–cellular equivalence (1) gives the result. $\qquad\square$

In particular, we obtain:

Theorem (corrected form of Theorem 1.4) *For each $n \geq 1$ set*

$$K = M\mathbb{R}/(\bar{v}_1, \ldots, \bar{v}_{n-1}).$$

Then we have a K–cellular equivalence

$$\mathbb{Z}_{(2)}^{E\mathbb{R}(n)} \simeq_K \Sigma^{D_n\rho+(n-1)+2(1-\sigma)} E\mathbb{R}(n).$$

Remark We explain how classical algebra shows that cellularization should be expected in these kinds of examples.

Most basically, let R be a classical local Noetherian k–algebra R with residue field k. The R–module $\mathrm{Hom}_k(R, k)$ plays the role of an Anderson dual, but it is not k–cellular. Its k–cellularization agrees with the injective hull $I(k)$ of k; see Dwyer, Greenlees and Iyengar [1, 7.1].

Dropping the k–algebra assumption, for a commutative Gorenstein local ring R with residue field k and Krull dimension d, we have $\Gamma_{\mathfrak{m}} R \simeq \Sigma^{-d} I(k)$. The statement

corresponding to the corrected form of Theorem 1.4 is $R \simeq_k \Sigma^{-d} I(k)$. The subscript k refers to the fact that the equivalence is only true after (derived) completion at the maximal ideal or after k–cellularization.

References

[1] **W G Dwyer**, **J P C Greenlees**, **S Iyengar**, *Duality in algebra and topology*, Adv. Math. 200 (2006) 357–402 MR

[2] **J P C Greenlees**, **L Meier**, *Gorenstein duality for real spectra*, Algebr. Geom. Topol. 17 (2017) 3547–3619 MR

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