Functoriality of the EH class and the LOSS invariant under Lagrangian concordances

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We show that the EH class and the LOSS invariant of Legendrian knots in contact 3–manifolds are functorial under regular Lagrangian concordances in Weinstein cobordisms. This gives computable obstructions to the existence of regular Lagrangian concordances.

1 Introduction

Link Floer homology, defined by Ozsváth and Szabó [26] and denoted by $\widehat{\mathrm{HFL}}$, is an invariant of links well-defined up to isomorphism. In the case of knots, it is also known as knot Floer homology, and denoted by $\widehat{\mathrm{HFK}}$. It becomes natural for *decorated* links, according to the work of Dylan Thurston, Ian Zemke, and the second author [20]. Furthermore, the second author showed [18] that *decorated* link cobordisms induce functorial cobordism maps on $\widehat{\mathrm{HFL}}$. As exhibited by Sarkar [27] and Zemke [30], moving the decorations around the link often induces a nontrivial monodromy of link Floer homology.

Let Λ be a Legendrian knot in a contact 3–manifold (Y, ξ). Then Λ admits a Legendrian tubular neighborhood $N(\Lambda)$, ie a regular neighborhood identified with $S^1 \times \mathbb{R}^2$ such that $\Lambda = S^1 \times \{0\}$, and

$$\xi|_{N(\Lambda)} = \ker(\cos \theta \, dx - \sin \theta \, dy),$$

where θ is the angular coordinate on S^1, and (x, y) are the Euclidean coordinates on $\mathbb{R}^2$; see Geiges [13, Example 2.5.10]. If $M_\Lambda = Y \setminus N(\Lambda)$, then ∂M_Λ is a convex surface with dividing set γ_Λ that consists of two curves, each corresponding to the framing $\mathrm{tb}(\Lambda)$. We call $(M_\Lambda, \gamma_\Lambda)$ the sutured manifold *complementary* to Λ. We denote by $Y_\Lambda = Y_{\mathrm{tb}(\Lambda)}(\Lambda)$ the result of Dehn filling M_Λ with slope γ_Λ, and write Λ' for the core of the Dehn filling. There is a pair of points $P_\Lambda \subset \Lambda'$ such

that $P_\Lambda \times S^1 \subset \Lambda' \times D^2$ is glued to γ_Λ. Furthermore, $R_\pm(P)$ is the component of $\Lambda' \setminus P$ for which $R_\pm(P) \times S^1$ is glued to $R_\pm(\gamma_\Lambda)$. Then $(Y_\Lambda, \Lambda', P_\Lambda)$ is a decorated link in the sense of [18, Definition 4.4]; see Section 2.2.

Stipsicz and Vértesi [28] defined the invariant

$$\mathrm{EH}(\Lambda) := \mathrm{EH}(\xi|_{M_\Lambda}) \in \mathrm{SFH}(-M_\Lambda, -\gamma_\Lambda) \cong \widehat{\mathrm{HFK}}(-Y_\Lambda, \Lambda', P_\Lambda)$$

of Legendrian knots. Note that $\mathrm{EH}(\Lambda)$ is nonvanishing whenever $\mathrm{EH}(\xi) \neq 0$, since the Honda–Kazez–Matić gluing map [15] for the sutured submanifold $(M_\Lambda, \gamma_\Lambda)$ of (Y, ξ) takes $\mathrm{EH}(\Lambda)$ to $\mathrm{EH}(\xi)$.

Definition 1.1 Suppose (X, ω) is a Liouville cobordism from (Y_-, ξ_-) to (Y_+, ξ_+). Let $\Lambda_\pm$ be a Legendrian knot in $(Y_\pm, \xi_\pm)$, and let L be a Lagrangian concordance in (X, ω) from Λ_- to Λ_+. Then X_L is the result of gluing $L \times D^2$ to $X \setminus N(L)$ with framing $\mathrm{tb}(\Lambda_-) = \mathrm{tb}(\Lambda_+)$, and we write L' for $L \times \{0\}$.

Note that (X_L, L') is a concordance from $(Y_{\Lambda_-}, \Lambda'_-)$ to $(Y_{\Lambda_+}, \Lambda'_+)$. If (X, ω) is Weinstein, then Eliashberg, Ganatra, and Lazarev [11, Definition 2.1] say that the Lagrangian cobordism L is *regular* if the Liouville vector field can be chosen to be tangent to L. For example, Conway, Etnyre, and Tosun [7, Lemma 3.4] proved that a Lagrangian cobordism in the symplectization $(\mathbb{R} \times Y, d(e^t \alpha))$ of a contact manifold (Y, ξ) with contact form α is regular if it is decomposable; see Section 2. (Note that we had also observed this lemma, but decided not to include it in this paper once [7] appeared.) We are now ready to state our first main result.

Theorem 1.2 *Suppose that L is a regular Lagrangian concordance from Λ_- to Λ_+ in the Weinstein cobordism (X, ω) from (Y_-, ξ_-) to (Y_+, ξ_+). Choose an arbitrary decoration σ' on $L' \subset X_L$ consisting of two arcs, one of which connects $R_-(P_{\Lambda_-})$ and $R_-(P_{\Lambda_+})$, and the other $R_+(P_{\Lambda_-})$ and $R_+(P_{\Lambda_+})$. We write $\overline{\mathcal{L}'}$ for the reverse of the decorated concordance $\mathcal{L}' = (X_L, L', \sigma')$ from $(-Y_{\Lambda_+}, \Lambda'_+, P_{\Lambda_+})$ to $(-Y_{\Lambda_-}, \Lambda'_-, P_{\Lambda_-})$. Then the knot cobordism map*

$$F_{\overline{\mathcal{L}'}} \colon \widehat{\mathrm{HFK}}(-Y_{\Lambda_+}, \Lambda'_+, P_{\Lambda_+}) \to \widehat{\mathrm{HFK}}(-Y_{\Lambda_-}, \Lambda'_-, P_{\Lambda_-})$$

takes $\mathrm{EH}(\Lambda_+)$ to $\mathrm{EH}(\Lambda_-)$.

It is natural to ask whether Theorem 1.2 holds for exact Lagrangian concordances. It would be true if every exact Lagrangian concordance were regular; see Eliashberg [10, Problem 5.1].

Given a Legendrian knot Λ in a contact 3–manifold (Y, ξ), Lisca, Ozsváth, Stipsicz, and Szabó [21] defined an invariant $\widehat{\mathfrak{L}}(\Lambda) \in \widehat{\mathrm{HFK}}(-Y, \Lambda, \mathfrak{t}_\xi)$, where $\mathfrak{t}_\xi$ is the Spin^c structure defined by the contact structure ξ. This is now commonly known as the LOSS invariant. In light of naturality, this is really an invariant of a Legendrian knot with a decoration P consisting of two points; ie

$$\widehat{\mathfrak{L}}(\Lambda, P) \in \widehat{\mathrm{HFK}}(-Y, \Lambda, P, \mathfrak{t}_\xi).$$

Baldwin, Vela-Vick, and Vértesi [3, page 926] conjectured that the LOSS invariant is functorial under Lagrangian cobordisms. This is supported by a result of Baldwin and Sivek [2, Theorem 1.2], which gives a contravariant map for every Lagrangian concordance that preserves a monopole Floer Legendrian knot invariant. However, they do not show that this map is an invariant of the Lagrangian cylinder. Furthermore, they also proved [1, Theorem 1.5] that there is a (non-natural) isomorphism between knot Floer homology and monopole knot homology that sends the LOSS invariant to their monopole Legendrian invariant. It is also worth mentioning that it is currently not known whether the cobordism maps in the Heegaard Floer setting agree with the maps in the monopole setting.

Stipsicz and Vértesi [28] showed that gluing a suitable basic slice to ∂M_Λ induces a map

$$F\colon \mathrm{SFH}(-M_\Lambda, -\gamma_\Lambda) \to \widehat{\mathrm{HFK}}(-Y, \Lambda, P)$$

that takes $\mathrm{EH}(\Lambda)$ to $\widehat{\mathfrak{L}}(\Lambda, P)$. As F is not always invertible, $\widehat{\mathfrak{L}}$ carries less information than EH. However, unlike EH, the LOSS invariant can vanish even when $\mathrm{EH}(\xi) \neq 0$, which can be used to obstruct the existence of regular Lagrangian concordances. Our second result is that $\widehat{\mathfrak{L}}$ is functorial under regular Lagrangian concordances.

Theorem 1.3 *Let L be a regular Lagrangian concordance from Λ_- to Λ_+ in the Weinstein cobordism (X, ω) from (Y_-, ξ_-) to (Y_+, ξ_+). Choose arbitrary decorations $P_\pm$ on $\Lambda_\pm$ consisting of two points, and a decoration σ on L consisting of two arcs, one of which connects $R_-(P_-)$ and $R_-(P_+)$, and the other $R_+(P_-)$ and $R_+(P_+)$. Then $\mathcal{L} := (X, L, \sigma)$ is a decorated concordance from (Y_-, Λ_-, P_-) to (Y_+, Λ_+, P_+) such that*

$$F_{\overline{\mathcal{L}}}(\widehat{\mathfrak{L}}(\Lambda_+, P_+)) = \widehat{\mathfrak{L}}(\Lambda_-, P_-).$$

This implies the following, which also follows from the work of Baldwin and Sivek [2].

Corollary 1.4 *Let $\Lambda_\pm$ be a Legendrian knot in $(Y_\pm, \xi_\pm)$. If $\widehat{\mathfrak{L}}(\Lambda_+) = 0$, but $\widehat{\mathfrak{L}}(\Lambda_-) \neq 0$, then there is no regular Lagrangian concordance from Λ_- to Λ_+.*

Marengon and the second author [19, Theorem 1.2] showed that, given a decorated knot concordance $\mathcal{C}$ in $I \times S^3$ from (S^3, K_0, P_0) to (S^3, K_1, P_1) and admissible diagrams $(\Sigma_i, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, w_i, z_i)$ of (S^3, K_i, P_i) for $i \in \{0, 1\}$, there is a filtered chain map

$$f_{\mathcal{C}} \colon \widehat{\mathrm{CF}}(\Sigma_0, \boldsymbol{\alpha}_0, \boldsymbol{\beta}_0, w_0) \to \widehat{\mathrm{CF}}(\Sigma_1, \boldsymbol{\alpha}_1, \boldsymbol{\beta}_1, w_1)$$

of homological degree zero such that the induced morphism of spectral sequences agrees with $F_{\mathcal{C}}$ on the E^1 page and with $\mathrm{Id}_{\mathbb{F}_2}$ on the total homology and on the E^∞ page. Consider the maps

$$\delta_k \colon \widehat{\mathrm{HFK}}_d(S^3, K, s) \to \widehat{\mathrm{HFK}}_{d-1}(S^3, K, s - k)$$

for $k \geq 1$. Using these, we have the following refinement of Corollary 1.4.

Corollary 1.5 *Let* Λ_+ *and* Λ_- *be Legendrian knots in* $(S^3, \xi_{\mathrm{std}})$. *If* $\delta_k(\widehat{\mathfrak{L}}(\Lambda_+)) = 0$, *but* $\delta_k(\widehat{\mathfrak{L}}(\Lambda_-)) \neq 0$, *then there is no decomposable Lagrangian concordance from* Λ_- *to* Λ_+.

Note that $\widehat{\mathfrak{L}}$ is unchanged by positive stabilization, and hence gives rise to an invariant $\widehat{\mathfrak{T}}$ of transverse knots. Baldwin, Vela-Vick, and Vértesi [3, Theorem 1.1] showed that $\widehat{\mathfrak{T}}$ agrees with $\widehat{\theta}$ defined using grid diagrams, which is algorithmically computable. Ng, Ozsváth, and Thurston [22, Theorem 1.1 and 1.2] proved that the mirrors of 10_{132} and $12n_{200}$ have Legendrian representatives Λ_1 and Λ_2, both with tb $= -1$ and $r = 0$, for which $\widehat{\theta}(\Lambda_1) = 0$ and $\widehat{\theta}(\Lambda_2) \neq 0$. Furthermore, the pretzel knots $P(-4, -3, 3)$ and $P(-6, -3, 3)$ have pairs of Legendrian representatives Λ_1 and Λ_2, both with tb $= -1$ and $r = 0$, for which $\widehat{\theta}(\Lambda_1) \neq 0$ and $\widehat{\theta}(\Lambda_2) \neq 0$, but $\delta_1 \circ \widehat{\theta}(\Lambda_1) = 0$ while $\delta_1 \circ \widehat{\theta}(\Lambda_2) \neq 0$. Hence, we obtain the following.

Proposition 1.6 *Let*

$$K \in \{-10_{132}, -12n_{200}, P(-4, -3, 3), P(-6, -3, 3)\}.$$

Then there exist Legendrian knots Λ_1 *and* Λ_2 *in* $(S^3, \xi_{\mathrm{std}})$ *of topological type* K *with the same rotation and Thurston–Bennequin numbers such that there is no decomposable Lagrangian concordance from* Λ_2 *to* Λ_1.

The knot $P(-4, -3, 3)$ is $\overline{10_{140}}$ and $P(-6, -3, 3)$ is $12n_{582}$; see [8, Remark 4.5]. Furthermore, the pairs of Legendrian representatives Λ_1 and Λ_2 shown in [22, Figures 4 and 5] are Lagrangian slice; ie there are Lagrangian concordances from the unknot to Λ_1 and to Λ_2. In fact, these are both decomposable. It follows from Proposition 1.6 that there is no decomposable Lagrangian concordance from Λ_2 to the unknot. Indeed,

otherwise, composing this with the decomposable Lagrangian concordance from the unknot to Λ_1, we would get a decomposable Lagrangian concordance from Λ_2 to Λ_1. More generally, Cornwell, Ng, and Sivek [8, Theorem 3.2] showed that there is no decomposable Lagrangian concordance from a Legendrian knot to any Legendrian unknot.

Let K be a framed knot in Y with longitude ℓ and meridian m. Then let $M = Y \setminus N(K)$ be the knot complement, and suppose that γ_n is a dividing set on ∂M consisting of two parallel curves of slope $\ell + n \cdot m$. Gluing a basic slice to $\partial M \times I$ with dividing set γ_n on $\partial M \times \{0\}$ and γ_{n+1} on $\partial M \times \{1\}$ induces a map

$$\sigma_+ \colon \mathrm{SFH}(-M, -\gamma_n) \to \mathrm{SFH}(-M, -\gamma_{n+1}).$$

We will prove the following result in Section 4.

Corollary 1.7 *Let $\Lambda_\pm$ be a Legendrian knot in $(Y_\pm, \xi_\pm)$. If there is a regular Lagrangian concordance from Λ_- to Λ_+ and $\widehat{\mathfrak{L}}(\Lambda_+) \in \mathrm{im}(\sigma_+^k)$ for some $k \in \mathbb{N}$, then $\widehat{\mathfrak{L}}(\Lambda_-) \in \mathrm{im}(\sigma_+^k)$.*

Let K be a framed null-homologous knot in Y. The first author [14] and Etnyre, Vela-Vick, and Zarev [12] showed that the limit of $\mathrm{SFH}(M, \gamma_n)$ along the maps σ_+ is $\mathrm{HFK}^-(-Y, K)$. Furthermore, if K is Legendrian, then $\mathrm{EH}(K)$ limits to the LOSS invariant $\mathfrak{L}^-(K)$. Then Proposition 3.7 implies that every parametrized concordance (see Section 3) induces a map on the limit. This construction is due to Tovstopyat-Nelip [29] when $Y = S^3$. It is an interesting question whether this coincides with the cobordism map defined by Zemke on HFK^-.

Proposition 1.8 *Suppose that $K_\pm$ is a null-homologous knot in $Y_\pm$, and let C be a parametrized concordance from K_- to K_+. Then its reverse induces a map*

$$F_{\overline{C}}^- \colon \mathrm{HFK}^-(-Y_+, K_+) \to \mathrm{HFK}^-(-Y_-, K_-).$$

Furthermore, if $\xi_\pm$ is a contact structure on $Y_\pm$, and $\Lambda_\pm$ is a Legendrian representative of $K_\pm$, while C is a regular Lagrangian concordance, then

$$F_{\overline{C}}^-(\mathfrak{L}^-(\Lambda_+)) = \mathfrak{L}^-(\Lambda_-).$$

Note that Sarkar [27] and Zemke [30] proved that the basepoint-moving map is trivial on HFK^-. The analogue of Corollary 1.7 is the following:

Corollary 1.9 *Let $\Lambda_\pm$ be a null-homologous Legendrian knot in $(Y_\pm, \xi_\pm)$. If there is a regular Lagrangian concordance from Λ_- to Λ_+ and $\mathfrak{L}^-(\Lambda_+) \in \mathrm{im}(U^k)$ for some $k \in \mathbb{N}$, then $\mathfrak{L}^-(\Lambda_-) \in \mathrm{im}(U^k)$.*

Acknowledgements

2 Background

2.1 Lagrangian cobordisms

An oriented link Λ in the contact 3–manifold (Y, ξ) is *Legendrian* if it is everywhere tangent to ξ. Let (Y_-, ξ_-) and (Y_+, ξ_+) be contact 3–manifolds. Then a *Weinstein cobordism* from (Y_-, ξ_-) to (Y_+, ξ_+) is a quadruple (X, ω, v, ϕ), where

 (1) X is a cobordism from Y_- to Y_+,

 (2) ω is a symplectic form on X,

 (3) v is a Liouville vector field on X (ie $L_v \omega = \omega$) that points into X along Y_- and points out of X along Y_+,

 (4) $\lambda = \iota_v \omega$ induces $\xi_\pm$ on $Y_\pm$, and

 (5) $\phi \colon X \to \mathbb{R}$ is a Morse function such that $d\phi(v) \geq c\|v\|^2$ for some $c \in \mathbb{R}_+$ and some Riemannian metric on X.

Weinstein cobordisms are precisely the ones that can be obtained by attaching so-called Weinstein handles of indices 0, 1, and 2 to Y_-, where the 2–handles are attached along Legendrian knots Λ with framing $\mathrm{tb}(\Lambda) - 1$.

Suppose now that $\Lambda_\pm$ is a Legendrian link in $(Y_\pm, \xi_\pm)$. According to Eliashberg, Ganatra, and Lazarev [11], an *exact Lagrangian cobordism* from Λ_- to Λ_+ consists of a Weinstein cobordism (X, ω, v, ϕ) from (Y_-, ξ_-) to (Y_+, ξ_+), together with a Lagrangian $L \subset X$ such that

 (1) L is exact (ie $\lambda|_L$ is exact),

 (2) v is tangent to L near ∂L, and

 (3) $\partial L = -\Lambda_- \cup \Lambda_+$.

The exact Lagrangian cobordism L is *regular* if it is tangent to v; see [11, Definition 2.1]. In this paper, we say that L is a *Lagrangian concordance* if it is a cylinder (this

notion is more general than what is usually understood by concordance, in that we do not require the ambient manifold to be the symplectization of a contact manifold).

Let $\alpha_0 = dz - y\,dx$ be the standard contact form and $\xi_0 = \ker(\alpha_0)$ the standard contact structure on $\mathbb{R}^3$. Note that S^3 with the standard contact structure $\xi_{std} = \ker(\alpha_{std})$ is the one-point compactification of $(\mathbb{R}^3, \xi_0)$. Chantraine [4] originally introduced the notion of *Lagrangian cobordism* in the symplectization $(\mathbb{R} \times \mathbb{R}^3, d(e^t \alpha_0))$ between Legendrian links in $(\mathbb{R}^3, \xi_0)$. In the rest of this subsection, we focus on this special case, as it admits a nice class of regular Lagrangian cobordisms.

Definition 2.1 Let Λ_- and Λ_+ be Legendrian links in $(\mathbb{R}^3, \xi_0)$. Then a Lagrangian cobordism from Λ_- to Λ_+ consists of an embedded Lagrangian submanifold L of the symplectization $(\mathbb{R} \times \mathbb{R}^3, d(e^t \alpha_0))$ such that, for some $T \in \mathbb{R}_+$,

$$\mathcal{E}_+(L, T) := L \cap ((T, \infty) \times \mathbb{R}^3) = (T, \infty) \times \Lambda_+,$$
$$\mathcal{E}_-(L, T) := L \cap ((-\infty, -T) \times \mathbb{R}^3) = (-\infty, -T) \times \Lambda_-,$$

and

$$L_T := L \setminus (\mathcal{E}_+(L, T) \cup \mathcal{E}_-(L, T))$$

is compact with boundary $\Lambda_+ \times \{T\} \sqcup -\Lambda_- \times \{-T\}$. The Lagrangian cobordism L is *exact* if there is a function $f \in C^\infty(L, \mathbb{R})$ such that $df = e^t \alpha_0|_L$, and f is constant on $\mathcal{E}_+(L)$ and $\mathcal{E}_-(L)$; see Ekholm, Honda, and Kálmán [9, Definition 1.1].

According to Chantraine [4, Theorem 1.3], if Λ_- and Λ_+ are knots and L is a Lagrangian cobordism from Λ_- to Λ_+, then $r(\Lambda_-) = r(\Lambda_+)$ and $tb(\Lambda_+) - tb(\Lambda_-) = -\chi(L)$, where r is the rotation number and tb is the Thurston–Bennequin number.

If two Legendrian knots are Legendrian isotopic, then they are Lagrangian concordant. Every Lagrangian concordance is automatically exact. Chantraine [6] showed that Lagrangian concordance is *not* a symmetric relation.

Chantraine [5, Definition 1.4] and Ekholm, Honda, and Kálmán [9, Section 6] defined the class of *decomposable* Lagrangian cobordisms. These are products of certain elementary cobordisms, namely Legendrian Reidemeister moves, saddles, and births of unknot components; see Figure 1. Each decomposable Lagrangian cobordism is exact, but it is not known whether the converse holds; see [5, Question 1.5; 9, Question 8.10; 10, Problem 5.1]. For a potential example of a nondecomposable Lagrangian concordance, see Cornwell, Ng, and Sivek [8, Conjecture 3.3]. Conway, Etnyre, and Tosun [7, Section 3.2] extended the notion of decomposable Lagrangian cobordism to
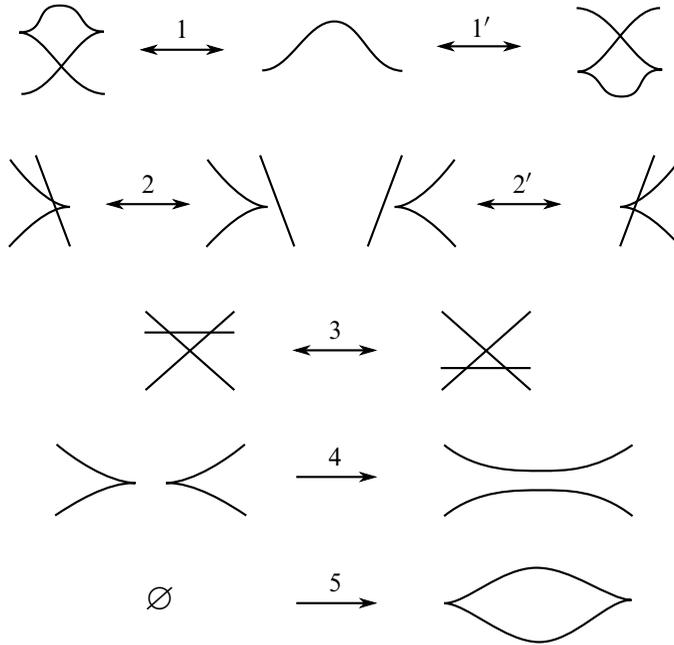
Figure 1: Front projections (ie projections to the (x, z) plane) of elementary Lagrangian cobordisms. Moves 1, 1', 2, 2', and 3 are the Legendrian Reidemeister moves, 4 is a saddle move, and 5 is a birth. A Lagrangian cobordism is decomposable if it is a composition of elementary ones.

symplectizations of arbitrary contact 3–manifolds, and they proved [7, Lemma 3.4] that the decomposable Lagrangian cobordisms in a symplectization are all regular.

2.2 Sutured Floer homology

Sutured Floer homology, defined by the second author [17], assigns a finite-dimensional $\mathbb{F}_2$–vector space to a balanced sutured manifold. It is an extension of the hat version of Heegaard Floer homology, due to Ozsváth and Szabó [23; 24], to 3–manifolds with boundary. If ξ is a contact structure on M such that ∂M is convex with dividing set γ, then Honda, Kazez, and Matić [16] assign to ξ an element $\mathrm{EH}(\xi) \in \mathrm{SFH}(-M, -\gamma)$.

The second author [18, Definitions 2.3 and 2.4] introduced the notion of sutured manifold cobordism.

Definition 2.2 Let (M_-, γ_-) and (M_+, γ_+) be balanced sutured manifolds. A sutured manifold cobordism from (M_-, γ_-) to (M_+, γ_+) is a triple $\mathcal{W} = (W, Z, [\xi])$ such that

(1) W is a compact oriented 4–manifold with boundary and corners along ∂Z,

(2) Z is a compact codimension-zero submanifold of ∂W with $\partial W \setminus \text{Int}(Z) = -M_- \sqcup M_+$, and

(3) $[\xi]$ is the equivalence class of a positive contact structure on Z such that $\partial M_\pm$ is a convex surface with dividing set $\gamma_\pm$.

We say that $\mathcal{W}$ is *special* if $Z = I \times (\partial M_-)$ and ξ is an I–invariant contact structure such that $\{t\} \times \partial M_-$ is a convex surface with dividing set $\{t\} \times \gamma_-$ for every $t \in I$ with respect to the contact vector field $\partial/\partial t$.

In [18], the second author also constructed functorial maps for sutured manifold cobordisms. In particular, every sutured manifold cobordism $\mathcal{W}$ from (M_-, γ_-) to (M_+, γ_+) induces a linear map

$$F_{\mathcal{W}}\colon \text{SFH}(M_-, \gamma_-) \to \text{SFH}(M_+, \gamma_+).$$

This is a composition $F_{\mathcal{W}^s} \circ \Phi_{-\xi}$, where $-\xi$ is ξ with its co-orientation reversed, and

$$\Phi_{-\xi}\colon \text{SFH}(M_-, \gamma_-) \to \text{SFH}(M_- \cup -Z, \gamma_+)$$

is the Honda–Kazez–Matić gluing map [15] induced by $-\xi$ and the sutured submanifold $(-M_-, -\gamma_-)$ of $(-M_- \cup Z, -\gamma_+)$. We can now view W as a cobordism $\mathcal{W}^s$ from $(M_- \cup -Z, \gamma_+)$ to (M_+, γ_+) that is a product along the boundary, ie as a special cobordism. It induces a map $F_{\mathcal{W}^s}$ by composing maps assigned to 4–dimensional handle attachments, defined analogously to the Ozsváth–Szabó handle maps [25].

A decorated knot is a triple (Y, K, P), where K is a knot in the oriented 3–manifold Y, and P is a pair of points on K that divide it into arcs $R_-(P)$ and $R_+(P)$. A decorated concordance $\mathcal{X} = (X, C, \sigma)$ from (Y_0, K_0, P_0) to (Y_1, K_1, P_1) consists of a cobordism X from Y_0 to Y_1, a properly embedded annulus C in X with $\partial C = -K_0 \cup K_1$, and a pair of arcs $\sigma \subset C$ such that one arc connects $R_+(P_0)$ with $R_+(P_1)$, and the other $R_-(P_0)$ with $R_-(P_1)$; see [18, Definition 4.5]. The sutured manifold cobordism $\mathcal{W}(\mathcal{X}) = (W, Z, [\xi])$ complementary to the concordance $\mathcal{X}$ is defined by taking $W = X \setminus N(C)$, and taking Z to be the unit normal circle bundle of C, oriented as ∂W, together with the S^1–invariant contact structure ξ that induces the dividing set σ on C. We define the map $F_{\mathcal{X}}$ induced by the decorated concordance $\mathcal{X}$ to be $F_{\mathcal{W}(\mathcal{X})}$.

If $\mathcal{W}$ is a sutured manifold cobordism from (M_-, γ_-) to (M_+, γ_+), then we can also view it as a cobordism $\overline{\mathcal{W}} = (W, Z, [-\xi])$ from $(-M_+, -\gamma_+)$ to $(-M_-, -\gamma_-)$. In

this paper, we call $\overline{\mathcal{W}}$ the *reverse* of $\mathcal{W}$, as we think of our cobordisms going from left to right. The second author [18, Theorem 11.8] showed that $F_{\overline{\mathcal{W}}} = F_{\mathcal{W}}^*$ when $\mathcal{W}$ is a special cobordism, where the dual $F_{\mathcal{W}}^*$ is taken with respect to the natural pairing

$$\langle \ , \ \rangle \colon \mathrm{SFH}(M, \gamma) \otimes \mathrm{SFH}(-M, -\gamma) \to \mathbb{F}_2,$$

obtained by choosing a diagram $(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta})$ of (M, γ) and $(-\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta})$ of $(-M, -\gamma)$, and defining $\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \delta_{\boldsymbol{x}, \boldsymbol{y}}$ for $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta$. The sutured manifold cobordism $\mathcal{W}(\mathcal{X})$ complementary to a concordance is equivalent to a special cobordism, hence $F_{\overline{\mathcal{X}}} = F_{\mathcal{X}}^*$. This applies to the maps featuring in Theorems 1.2 and 1.3.

3 Parametrized concordances

Definition 3.1 Let K_i be a knot in the oriented 3–manifold Y_i. A *parametrized concordance* from K_0 to K_1 is a properly embedded annulus C in a cobordism X from Y_0 to Y_1, together with a normal framing and an identification $C \approx I \times S^1$ such that $\{i\} \times S^1 = \{i\} \times K_i$ for $i \in \{0, 1\}$.

Remark 3.2 By framing, we mean a concrete identification of a regular neighborhood $N(C)$ of C with $C \times D^2$. The normal framing of C restricts to framings of K_0 and K_1.

Given a parametrized concordance (X, C) from (Y_0, K_0) to (Y_1, K_1), let

$$\begin{aligned} W_C &= X \setminus N(C), \\ M_i &= Y_i \setminus N(C) \quad \text{for } i \in \{0, 1\}, \\ Z_C &= C \times S^1 \qquad \text{oriented as } \partial W_C. \end{aligned}$$

If γ is a dividing set on $S^1 \times S^1$ consisting of two parallel curves, then let ξ_γ be the I–invariant contact structure on $Z \approx I \times S^1 \times S^1$ such that $\{t\} \times S^1 \times S^1$ is convex with dividing set $\gamma_t = \{t\} \times \gamma$ for every $t \in I$.

Example 3.3 Suppose that (X, C, σ) is a decorated concordance from (Y_0, K_0, P_0) to (Y_1, K_1, P_1). If we choose the identification $C \cong S^1 \times I$ such that

$$\begin{aligned} \sigma &= (I \times \{x\}) \cup (I \times \{y\}) \quad \text{for some } x, y \in S^1, \\ P_i &= \{i\} \times P \qquad\qquad\qquad \text{for } i \in \{0, 1\}, \end{aligned}$$

then the S^1–invariant contact structure ξ on $Z \approx C \times S^1$ with dividing set σ on C is I–invariant as well. Hence $\xi = \xi_\gamma$ for $\gamma = P \times S^1$.

As usual, we identify $\mathbb{Q}P^1$ with $\mathbb{Q} \cup \{\infty\}$ by mapping $[p:q]$ to q/p if $p \neq 0$, and to ∞ otherwise. For $r = [p:q] \in \mathbb{Q}P^1$ and $p, q \in \mathbb{Z}$ relatively prime, let

$$\ell_r := \{(e^{2\pi ipt}, e^{2\pi iqt}) : t \in I\} \subset S^1 \times S^1,$$

and write

$$\gamma_r = \ell_r \cup -(e^{\pi i/q}, 1)\ell_r$$

when $q \neq 0$, and $\gamma_r = \ell_r \cup -(1, -1)\ell_r$ otherwise. Then γ_r is a dividing set on $T^2 = S^1 \times S^1$.

Let K be a framed knot in an oriented 3–manifold Y; ie we are given an identification $N(K) \approx S^1 \times D^2$. Then we can talk about the sutured manifold

$$Y(K, r) := \left(Y \setminus \text{Int}(N(K)), \gamma_r\right)$$

for any $r \in \mathbb{Q}P^1$. For example, if (K, P) is a decorated knot, $|P| = 2$, and the framing $N(K) \approx S^1 \times D^2$ is chosen such that $P = \{(1, 0), (-1, 0)\}$, then $Y(K, P) = Y(K, \infty)$, as γ_0 consists of two meridional sutures over P.

If (X, C) is a parametrized concordance from (Y_0, K_0) to (Y_1, K_1) and $r \in \mathbb{Q}P^1$, then we denote the contact structure ξ_{γ_r} by ξ_r, and we let

$$F_{C,r} := F_{(W_C, Z_C, [\xi_r])} \colon \text{SFH}(Y_0(K_0, r)) \to \text{SFH}(Y_1(K_1, r)),$$

where the framings of K_0 and K_1 are the restrictions of the normal framing of C.

Definition 3.4 Suppose that (X, C) is a parametrized concordance from (Y_0, K_0) to (Y_1, K_1) and $r \in \mathbb{Q}P^1$. We denote by $(X_r(C), C_r)$ the parametrized concordance where $X_r(C)$ is obtained by gluing $N(C)$ to $X \setminus N(C)$ along a map that sends each meridian $\{t\} \times \gamma_\infty$ to $\{t\} \times \gamma_r$ for $t \in I$, and $C_r = C \times \{0\} \subset C \times D^2 \approx N(C)$. This carries a natural parametrization. We call $(X_r(C), C_r)$ the result of *r–surgery along the concordance C*.

Note that Definition 1.1 is a special instance of the above construction. In particular, $(X_L, L') = (X_n(L), L_n)$, where n is $\text{tb}(\Lambda_\pm)$ measured with respect to the normal framing of L.

Lemma 3.5 *Let (X, C) be a parametrized concordance from (Y_0, K_0) to (Y_1, K_1) and $r \in \mathbb{Q}P^1$. If we choose a decoration σ_r of the surgered concordance $(X_r(C), C_r)$ as in Example 3.3, and the complementary sutured manifold cobordism*

$$\mathcal{W}(X_r(C), C_r, \sigma_r) = (W, Z, [\xi]),$$

then $[\xi] = [\xi_r]$.

Proof This is a straightforward consequence of the definitions. $\qquad \square$

For $(r, r') \in \mathbb{Q}P^1 \times \mathbb{Q}P^1$, let

$$\gamma_{r,r'} := \gamma_r \times \{0\} \cup \gamma_{r'} \times \{1\} \subset \partial(T^2 \times I).$$

Then $(T^2 \times I, \gamma_{r,r'})$ is a balanced sutured manifold. If ζ is a contact structure on $(T^2 \times I, \gamma_{r,r'})$, then it induces a gluing map

$$\Phi_{K,\zeta} \colon \mathrm{SFH}(-Y(K, r)) \to \mathrm{SFH}(-Y(K, r')).$$

Let $\overline{C}$ be the reverse of the concordance C.

Proposition 3.6 *Suppose that (X, C) is a parametrized concordance from (Y_0, K_0) to (Y_1, K_1). If $(r, r') \in \mathbb{Q}P^1 \times \mathbb{Q}P^1$ and ζ is a contact structure on $(T^2 \times I, \gamma_{r,r'})$, then the following diagram is commutative:*

$$
\begin{array}{ccc}
\mathrm{SFH}(-Y_1(K_1, r)) & \xrightarrow{\ \Phi_{K_1,\zeta}\ } & \mathrm{SFH}(-Y_1(K_1, r')) \\
\Big\downarrow{\scriptstyle F_{\overline{C},r}} & & \Big\downarrow{\scriptstyle F_{\overline{C},r'}} \\
\mathrm{SFH}(-Y_0(K_0, r)) & \xrightarrow{\ \Phi_{K_0,\zeta}\ } & \mathrm{SFH}(-Y_0(K_0, r'))
\end{array}
$$

Proof The map $F_{\overline{C},r}$ is defined by gluing $(Z_{\overline{C}}, \xi_r)$ to $Y_1(K_1, r)$, and then attaching 4–dimensional handles. As $Z_{\overline{C}}$ is just a collar of the boundary of the result of the gluing, we can assume all the handles are attached away from $Z_{\overline{C}}$. Then $\Phi_{K_0,\zeta}$ is induced by gluing $(T^2 \times I, \zeta)$ to $Y_0(K_0, r)$. By [18, Proposition 11.5], disjoint gluing and handle maps commute. Hence, we also obtain $\Phi_{K_0,\zeta} \circ F_{\overline{C},r}$ if we first glue $\xi_r \cup \zeta$ to $Y_1(K_1, r)$, followed by the handle attachments along $Y_1(K_1, r)$. Since ξ_r and $\xi_{r'}$ are I–invariant, $\xi_r \cup \zeta$ is isotopic to $\zeta \cup \xi_{r'}$. Gluing ζ to $Y_1(K_1)$ induces $\Phi_{K_1,\zeta}$, and then gluing $\xi_{r'}$ to $Y_1(K_1, r')$ and attaching the handles induce $F_{\overline{C},r'}$. The result follows. $\qquad \square$

Proposition 3.7 *Suppose that (X, C) is a parametrized concordance from (Y_-, K_-) to (Y_+, K_+), and let $n \in \mathbb{Z}$. Then*

$$\sigma_+ \circ F_{\overline{C},n} = F_{\overline{C},n+1} \circ \sigma_+.$$

Proof For $n \in \mathbb{Z}$, we denote by ζ_n the basic slice on $(T^2 \times I, \gamma_{n,n+1})$ corresponding to positive stabilization. Given a knot K in Y, we let

$$\sigma_+ \colon \mathrm{SFH}(-Y(K, n)) \to \mathrm{SFH}(-Y(K, n+1))$$

be the map induced by gluing ζ_n. Then this commutes with parametrized concordance maps by Proposition 3.6. $\qquad \square$

Let Λ be a Legendrian knot in (Y, ξ). Using the above notation, the map

$$F\colon \mathrm{SFH}(-M_\Lambda, -\gamma_\Lambda) \to \widehat{\mathrm{HFK}}(-Y, \Lambda)$$

of Stipsicz and Vértesi [28, Theorem 4.2] that takes $\mathrm{EH}(\Lambda)$ to $\widehat{\mathfrak{L}}(\Lambda)$ is defined the following way. Choose an identification $N(\Lambda) \approx S^1 \times D^2$, and let $n \in \mathbb{Q}P^1$ be $\mathrm{tb}(\Lambda)$ with respect to this framing. The framing determines the decoration $P = \{(1, 0), (-1, 0)\}$ of Λ. Let ζ be the basic slice on $(T^2 \times I, \gamma_{n,0})$ determined by a bypass arc oriented coherently with the meridian of Λ (this is given by the orientation of Λ). Then

$$F = \Phi_{\Lambda, \zeta}\colon \mathrm{SFH}(-Y(\Lambda, n)) \to \mathrm{SFH}(-Y(\Lambda, 0)) \cong \widehat{\mathrm{HFK}}(-Y, \Lambda, P).$$

We denote the image of $\mathrm{EH}(\Lambda)$ by $\widehat{\mathfrak{L}}(\Lambda, P)$.

4 Proofs

Definition 4.1 Let $\Lambda_\pm$ be a Legendrian knot in $(Y_\pm, \xi_\pm)$, and write

$$(M_\pm, \gamma_\pm) := (M_{\Lambda_\pm}, \gamma_{\Lambda_\pm}).$$

Given a parametrized Lagrangian concordance L in the Weinstein cobordism (X, ω) from Λ_- to Λ_+, we can associate to it a special cobordism $\mathcal{W}(L) = (W_L, Z_L, [\xi_L])$ from (M_-, γ_-) to (M_+, γ_+), as follows. Let $N(L)$ be a Lagrangian tubular neighborhood of L. Then $N(L)$ is identified with the unit cotangent disk bundle DT^*L. We denote by $\partial_h N(L)$ the part of $\partial N(L)$ identified with the unit cotangent circle bundle ST^*L. This carries the canonical contact structure ξ_{can}, which has dividing set $\gamma_\pm$ on $\partial M_\pm$. Then the parametrization of L induces a contactomorphism

$$\varphi\colon (M_-, \gamma_-, \xi_-|_{M_-}) \to (M_- \cup \partial_h N(L), \gamma_-, \xi_-|_{M_-} \cup \xi_{\mathrm{can}}).$$

We then set

$$W_L = (X \setminus N(L)) \cup (I \times M_+),$$

and $Z_L = I \times \partial M_+$, together with the $\mathbb{R}$–invariant contact structure ξ_L such that $\{t\} \times \partial M_+$ is a convex surface with dividing set $\{t\} \times \gamma_+$ for every $t \in I$. We identify the incoming end of $\mathcal{W}(L)$ with (M_-, γ_-) using φ. We call $\mathcal{W}(L)$ the sutured manifold cobordism *complementary* to the parametrized Lagrangian cobordism L.

Remark 4.2 It is important to note that ξ_{can} is a positive contact structure if we orient $\partial_h N(L)$ as the boundary of $N(L)$, which is the opposite of the boundary orientation of $X \setminus N(L)$. Hence, the contact structure ξ_{can} is not isotopic to any S^1–invariant contact structure arising from a decoration of the $\mathrm{tb}(\Lambda_\pm)$–surgered concordance L'.

A sutured manifold cobordism $\mathcal{W} = (W, Z, [\xi])$ from the contact manifold with convex boundary (M_-, γ_-, ζ_-) to (M_+, γ_+, ζ_+) is Weinstein essentially if its special part from (M_-, ζ_-) to $(M_+ \cup Z, \zeta_+ \cup \xi)$ can be obtained by attaching Weinstein 1– and 2–handles; see [18, Remark 11.23].

Proof of Theorem 1.2 Choose a parametrization of L' compatible with the decoration σ' as in Example 3.3. We endow L with the parametrization induced by the $\text{tb}(\Lambda_\pm)$–framed surgery. Let $\mathcal{W}(L) = (W_L, Z_L, [\xi_L])$ be the special cobordism complementary to L, as in Definition 4.1. By Lemma 3.5, the sutured manifold cobordisms $\mathcal{W}(\mathcal{L}')$ and $\mathcal{W}(L)$ from (M_-, γ_-) to (M_+, γ_+) are equivalent. Hence $F_{\overline{\mathcal{L}'}} = F_{\overline{\mathcal{W}(\mathcal{L}')}} = F_{\overline{\mathcal{W}(L)}}$.

Using Lemma 2.2 and the preceding discussion of Eliashberg, Ganatra, and Lazarev [11], we obtain that W_L can be built by attaching Weinstein handles to

$$(M_- \cup \partial_h N(L), \gamma_+, \xi_-|_{M_-} \cup \xi_{\text{can}}).$$

Hence, by [18, Remark 11.23], the sutured manifold cobordism $\mathcal{W}(L)$ is Weinstein in the sense of [18, Definition 11.22]. The result now follows from a result of the second author [18, Theorem 11.24] that the reverse of a Weinstein sutured manifold cobordism induces a map that preserve the EH class. $\qquad\square$

Proof of Theorem 1.3 Let L be a regular Lagrangian concordance from Λ_- to Λ_+. Choose a parametrization of L compatible with the decoration σ as in Example 3.3, and let r be $\text{tb}(\Lambda_-) = \text{tb}(\Lambda_+)$ measured with respect to this normal framing. If we apply Proposition 3.6 with $C = L$, $K_0 = \Lambda_-$, $K_1 = \Lambda_+$, $r' = 0$, and ζ the basic slice on $(T^2 \times I, \gamma_{r,0})$ used in the definition of the Stipsicz–Vértesi map, then we obtain the commutative diagram

$$
\begin{array}{ccc}
\text{SFH}(-Y_+(\Lambda_+, r)) & \xrightarrow{\Phi_{\Lambda_+, \xi}} & \text{SFH}(-Y_+(\Lambda_+, 0)) \\
\Big\downarrow{\scriptstyle F_{\bar{L}, r}} & & \Big\downarrow{\scriptstyle F_{\bar{L}, 0}} \\
\text{SFH}(-Y_-(\Lambda_-, r)) & \xrightarrow{\Phi_{\Lambda_-, \xi}} & \text{SFH}(-Y_-(\Lambda_-, 0))
\end{array}
$$

Note that $\text{SFH}(-Y_\pm(\Lambda_\pm, 0)) \cong \widehat{\text{HFK}}(-Y_\pm, \Lambda_\pm, P_\pm)$ tautologically. If we endow L' with the parametrization induced by r–framed surgery along L and denote by $\mathcal{L}'$ the corresponding decorated concordance, then $F_{\bar{L}, r} = F_{\overline{\mathcal{L}'}}$ by Lemma 3.5. Furthermore,

$F_{\overline{L},0} = F_{\overline{\mathcal{L}}}$. Hence, we can rewrite the above diagram as

(4-1)
$$
\begin{array}{ccc}
\mathrm{SFH}(-Y_+(\Lambda_+, r)) & \xrightarrow{\Phi_{\Lambda_+,\xi}} & \widehat{\mathrm{HFK}}(-Y_+, \Lambda_+, P_+) \\
\downarrow{\scriptstyle F_{\overline{\mathcal{L}'}}} & & \downarrow{\scriptstyle F_{\overline{\mathcal{L}}}} \\
\mathrm{SFH}(-Y_-(\Lambda_-, r)) & \xrightarrow{\Phi_{\Lambda_-,\xi}} & \widehat{\mathrm{HFK}}(-Y_-, \Lambda_-, P_-)
\end{array}
$$

Now consider $\mathrm{EH}(\Lambda_+) \in \mathrm{SFH}(-Y_+(\Lambda_+, r))$. By Theorem 1.2, we have

$$
F_{\overline{\mathcal{L}'}}(\mathrm{EH}(\Lambda_+)) = \mathrm{EH}(\Lambda_-).
$$

Because $\Phi_{\Lambda_\pm,\xi}(\mathrm{EH}(\Lambda_\pm)) = \widehat{\mathfrak{L}}(\Lambda_\pm, P_\pm)$, the commutativity of diagram (4-1) amounts to $F_{L,\sigma}(\widehat{\mathfrak{L}}(\Lambda_+, P_+)) = \widehat{\mathfrak{L}}(\Lambda_-, P_-)$. $\qquad\square$

Proof of Corollary 1.7 Let L be a Lagrangian concordance from Λ_- to Λ_+. Choose a parametrization of L, and let n be $\mathrm{tb}(\Lambda_+) = \mathrm{tb}(\Lambda_-)$ with respect to the normal framing of L. Then $F_{\overline{L},n} = F_{\overline{\mathcal{L}'}}$ by Lemma 3.5. The result now follows from Theorem 1.3 and Proposition 3.7. $\qquad\square$

References

[1] **J A Baldwin**, **S Sivek**, *On the equivalence of contact invariants in sutured Floer homology theories*, preprint (2016) arXiv

[2] **J A Baldwin**, **S Sivek**, *Invariants of Legendrian and transverse knots in monopole knot homology*, J. Symplectic Geom. 16 (2018) 959–1000 MR

[3] **J A Baldwin**, **D S Vela-Vick**, **V Vértesi**, *On the equivalence of Legendrian and transverse invariants in knot Floer homology*, Geom. Topol. 17 (2013) 925–974 MR

[4] **B Chantraine**, *Lagrangian concordance of Legendrian knots*, Algebr. Geom. Topol. 10 (2010) 63–85 MR

[5] **B Chantraine**, *Some non-collarable slices of Lagrangian surfaces*, Bull. Lond. Math. Soc. 44 (2012) 981–987 MR

[6] **B Chantraine**, *Lagrangian concordance is not a symmetric relation*, Quantum Topol. 6 (2015) 451–474 MR

[7] **J Conway**, **J B Etnyre**, **B Tosun**, *Symplectic fillings, contact surgeries, and Lagrangian disks*, preprint (2017) arXiv

[8] **C Cornwell**, **L Ng**, **S Sivek**, *Obstructions to Lagrangian concordance*, Algebr. Geom. Topol. 16 (2016) 797–824 MR

[9] **T Ekholm**, **K Honda**, **T Kálmán**, *Legendrian knots and exact Lagrangian cobordisms*, J. Eur. Math. Soc. 18 (2016) 2627–2689 MR

[10] **Y Eliashberg**, *Weinstein manifolds revisited*, from "Modern geometry: a celebration of the work of Simon Donaldson" (V Muñoz, I Smith, R P Thomas, editors), Proc. Sympos. Pure Math. 99, Amer. Math. Soc., Providence, RI (2018) 59–82 MR

[11] **Y Eliashberg**, **S Ganatra**, **O Lazarev**, *Flexible Lagrangians*, preprint (2015) arXiv

[12] **J B Etnyre**, **D S Vela-Vick**, **R Zarev**, *Sutured Floer homology and invariants of Legendrian and transverse knots*, Geom. Topol. 21 (2017) 1469–1582 MR

[13] **H Geiges**, *An introduction to contact topology*, Cambridge Studies in Advanced Mathematics 109, Cambridge Univ. Press (2008) MR

[14] **M Golla**, *Comparing invariants of Legendrian knots*, Quantum Topol. 6 (2015) 365–402 MR

[15] **K Honda**, **W H Kazez**, **G Matić**, *Contact structures, sutured Floer homology and TQFT*, preprint (2008) arXiv

[16] **K Honda**, **W H Kazez**, **G Matić**, *The contact invariant in sutured Floer homology*, Invent. Math. 176 (2009) 637–676 MR

[17] **A Juhász**, *Holomorphic discs and sutured manifolds*, Algebr. Geom. Topol. 6 (2006) 1429–1457 MR

[18] **A Juhász**, *Cobordisms of sutured manifolds and the functoriality of link Floer homology*, Adv. Math. 299 (2016) 940–1038 MR

[19] **A Juhász**, **M Marengon**, *Concordance maps in knot Floer homology*, Geom. Topol. 20 (2016) 3623–3673 MR

[20] **A Juhász**, **D P Thurston**, **I Zemke**, *Naturality and mapping class groups in Heegaard Floer homology*, preprint (2012) arXiv

[21] **P Lisca**, **P Ozsváth**, **A I Stipsicz**, **Z Szabó**, *Heegaard Floer invariants of Legendrian knots in contact three-manifolds*, J. Eur. Math. Soc. 11 (2009) 1307–1363 MR

[22] **L Ng**, **P Ozsváth**, **D Thurston**, *Transverse knots distinguished by knot Floer homology*, J. Symplectic Geom. 6 (2008) 461–490 MR

[23] **P Ozsváth**, **Z Szabó**, *Holomorphic disks and three-manifold invariants: properties and applications*, Ann. of Math. 159 (2004) 1159–1245 MR

[24] **P Ozsváth**, **Z Szabó**, *Holomorphic disks and topological invariants for closed three-manifolds*, Ann. of Math. 159 (2004) 1027–1158 MR

[25] **P Ozsváth**, **Z Szabó**, *Holomorphic triangles and invariants for smooth four-manifolds*, Adv. Math. 202 (2006) 326–400 MR

[26] **P Ozsváth**, **Z Szabó**, *Holomorphic disks, link invariants and the multi-variable Alexander polynomial*, Algebr. Geom. Topol. 8 (2008) 615–692 MR

[27] **S Sarkar**, *Moving basepoints and the induced automorphisms of link Floer homology*, Algebr. Geom. Topol. 15 (2015) 2479–2515 MR

[28]   **A I Stipsicz**, **V Vértesi**, *On invariants for Legendrian knots*, Pacific J. Math. 239 (2009) 157–177 MR

[29]   **L Tovstopyat-Nelip**, *Concordance maps in HFK^-*, preprint (2016) arXiv

[30]   **I Zemke**, *Quasistabilization and basepoint moving maps in link Floer homology*, Algebr. Geom. Topol. 17 (2017) 3461–3518 MR

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