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Branched covers and rational homology balls

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The concordance group of knots in S^3 contains a subgroup isomorphic to $(\mathbb{Z}_2)^\infty$, each element of which is represented by a knot K with the property that, for every $n > 0$, the n–fold cyclic cover of S^3 branched over K bounds a rational homology ball. This implies that the kernel of the canonical homomorphism from the knot concordance group to the infinite direct sum of rational homology cobordism groups (defined via prime-power branched covers) contains an infinitely generated two-torsion subgroup.

57K10, 57M12

1 Introduction

There is a homomorphism

$$\varphi\colon \mathcal{C} \to \prod_{q\in\mathcal{Q}} \Theta^3_{\mathbb{Q}},$$

where $\mathcal{C}$ is the smooth concordance group of knots in S^3, $\mathcal{Q}$ is the set of prime power integers, and $\Theta^3_{\mathbb{Q}}$ is the rational homology cobordism group. For a knot K and $q \in \mathcal{Q}$, the q–component of $\varphi(K)$ is the class of $M_q(K)$, the q–fold cyclic cover of S^3 branched over K.

In [1], Aceto, Meier, A Miller, M Miller, Park, and Stipsicz proved that $\ker\varphi$ contains a subgroup isomorphic to $(\mathbb{Z}_2)^5$. Here we will prove that $\ker\varphi$ contains a subgroup isomorphic to $(\mathbb{Z}_2)^\infty$. Our examples are of the form $K\#{-K^r}$, where $-K$ denotes the concordance inverse of K (the mirror image of K with string orientation reversed), and K^r is formed from K by reversing its string orientation. Such knots are easily seen to be in the kernel of φ; the more difficult work is to find nontrivial examples of order two.

The first known example of a nontrivial element in $\ker\varphi$ was represented by the knot $K_1 = 8_{17}\,\#{-8^r_{17}}$, which is of order two in $\mathcal{C}$. That K_1 is of order at most two is elementary; that K_1 is nontrivial in $\mathcal{C}$ is one of the main results of Kirk and Livingston in [9], proved using twisted Alexander polynomials.

The results of Kim and Livingston [7] provide an infinitely generated free subgroup of $\ker\varphi$. Conjecturally, $\mathcal{C} \cong \mathbb{Z}^\infty \oplus (\mathbb{Z}_2)^\infty$; if true, then $\ker\varphi \cong \mathbb{Z}^\infty \oplus (\mathbb{Z}_2)^\infty$.

1.1 Main result

Figure 1 illustrates a knot P_n in a solid torus, where J_n represents the braid illustrated on the right in the case of $n = 5$; n will always be odd. We let K_n denote the satellite of 8_{17} built from P_n. In standard

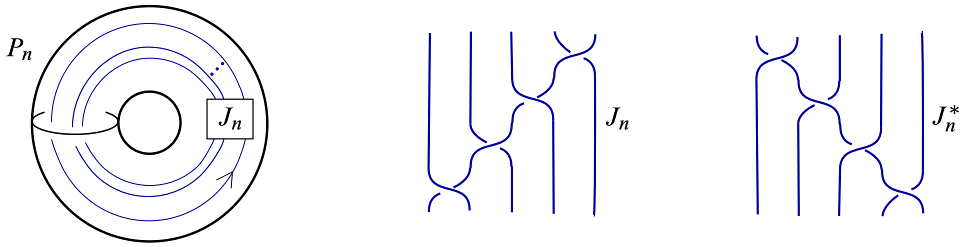

Figure 1: The knot $P_n \subset S^1 \times B^2$, J_n, and J_n^*.

notation, $K_n = P_n(8_{17})$. For future reference, we illustrate the braid J_n^* formed by rotating J_n around the vertical axis.

Theorem 1 *Let $K_n = P_n(8_{17})$. For all odd n, the knot $L_n = K_n \mathbin{\#} -K_n^r$ satisfies $2L_n = 0 \in \mathcal{C}$ and $L_n \in \ker\varphi$. There is an infinite set of prime integers $\mathcal{P}$ for which $L_\alpha \neq L_\beta \in \mathcal{C}$ for all $\alpha \neq \beta$ in $\mathcal{P}$. In particular, the set of knots $\{L_n\}_{n\in\mathcal{P}}$ generates a subgroup of $\ker\varphi$ that is isomorphic to $(\mathbb{Z}_2)^\infty$.*

The rest of the paper presents a proof of this theorem. The first two claims are easily dealt with in Sections 2 and 3. The more difficult step of the proof calls on an analysis of twisted Alexander polynomials and their relevance to knot slicing; a review of twisted polynomials is included in Section 4. The proof of Theorem 1 is completed in Section 5, with the exception of a number-theoretic result that is described Appendix A.

Acknowledgements Thanks to Darrell Haile for assisting me in the proof of the number-theoretic result in Appendix A. Allison Miller provided valuable feedback about an early draft of this paper. This work was supported by a grant from the National Science Foundation, NSF-DMS-1505586.

2 Proof that $2L_n = 0 \in \mathcal{C}$

Let $P_n^* \subset S^1 \times B^2$ denote the knot formed using the braid J_n^* in Figure 2. For any knot K, let $P_n^*(K)$ denote the satellite of K built using P_n^*. It should be clear that P_n and P_n^* are orientation-preserving isotopic, and thus for all knots K, $P_n(K) = P_n^*(K)$.

Figure 2 illustrates, for an arbitrary knot K, the connected sum $P_n(K) \mathbin{\#} P_n^*(K) = P_n(K) \mathbin{\#} P_n(K)$ in the case of $n = 5$. Performing $n-1$ band moves in the evident way yields the $(0,n)$–cable of $K \mathbin{\#} K$. Thus, if $K \mathbin{\#} K = 0 \in \mathcal{C}$, then the n components of this link can be capped off with parallel copies of the slice disk for $K \mathbin{\#} K$, implying that $P_n(K) \mathbin{\#} P_n(K) = 0 \in \mathcal{C}$. In particular, $2K_n = 0 \in \mathcal{C}$ and $2K_n^r = 0 \in \mathcal{C}$.

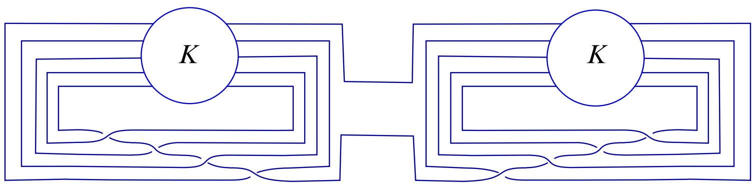

Figure 2: $P_5(K) \mathbin{\#} P_5(K)$.

3 Proof that $L_n \in \ker \varphi$

We prove a stronger statement: *for all odd n, and for all positive integers q, $M_q(L_n)$ is a rational homology sphere that represents* $0 \in \Theta^3_{\mathbb{Q}}$.

The q–fold cyclic cover of S^3 branched over $K_n \# -K_n^r$ is the same space as the q–fold cyclic cover of S^3 branched over $K_n \# -K_n$. A slice disk for $K_n \# -K_n$ is built from $(S^3 \times I, K_n \times I)$ by removing a copy of $B^3 \times I$. Taking the q–fold branched cover shows that the q–fold cyclic cover of B^4 branched over that slice disk is diffeomorphic to $M_q(K_n)^* \times I$, where $M_q(K_n)^*$ denotes a punctured copy of $M_q(K_n)$. It remains to show that $M_q(K_n)$ is a rational homology 3–sphere.

A formula of Fox [5] and Goeritz [6] states that the order of the first homology of $M_q(K_n)$ is given by the product of values $\Delta_{K_n}(\omega_q^i)$, where $\Delta_{K_n}(t)$ denotes the Alexander polynomial, ω_q is a primitive q–root of unity, and i runs from 1 to $q-1$.

A result of Seifert [11] shows that $\Delta_{K_n}(t) = \Delta_{8_{17}}(t^n)\Delta_{P_n(U)}$, where U is the unknot. We have that $P_n(U) = U$. The Alexander polynomial for 8_{17} is

$$\Delta_{8_{17}}(t) = 1 - 4t + 8t^2 - 11t^3 + 8t^4 - 4t^5 + t^6.$$

A numeric computation confirms that this polynomial does not have roots on the unit complex circle, and hence $\Delta_{8_{17}}(t^n)$ has no roots on the unit complex circle. From this is follows that $\Delta_{K_n}(\omega_q^i) \neq 0$ for all i; thus the order of the homology of $M_q(K_n)$ is finite.

4 Review of twisted polynomials and 8_{17}

In this section we review twisted Alexander polynomials and their application in [8; 9] showing that $8_{17} \# -8_{17}^r \neq 0 \in \mathcal{C}$.

Let $(X, B) \to (S^3, K)$ be the q–fold cyclic branched cover of a knot K with q a prime power. In particular, X is a rational homology sphere. There is a canonical surjection $\epsilon \colon H_1(X - B) \to \mathbb{Z}$. Suppose that $\rho \colon H_1(X) \to \mathbb{Z}_p$ is a homomorphism for some prime p. Then there is an associated twisted polynomial $\Delta_{K,\epsilon,\rho}(t) \in \mathbb{Q}(\omega_p)[t]$. It is well-defined, up to factors of the form at^k, where $a \neq 0 \in \mathbb{Q}(\omega_p)$. These polynomials are discriminants of Casson–Gordon invariants, first defined in [3].

In the case of $K = 8_{17}$ and $q = 3$, we have $H_1(X) \cong \mathbb{Z}_{13} \oplus \mathbb{Z}_{13}$, and as a $\mathbb{Z}_{13}$–vector space this splits as a direct sum of a 3–eigenspace and a 9–eigenspace under the order three action of the deck transformation. Both eigenspaces are 1–dimensional. We denote this splitting by $E_3 \oplus E_9$. There are corresponding characters ρ_3 and ρ_9 of $H_1(X)$ onto $\mathbb{Z}_{13}$; these are defined as the quotient maps onto $H_1(X)/E_3$ and onto $H_1(X)/E_9$. We let ρ_0 denote the trivial $\mathbb{Z}_{13}$–valued character.

The values of $\Delta_{8_{17},\epsilon,\rho_i}(t)$ are given in [9], duplicated here in Appendix B. For $i = 0$ it is polynomial in $\mathbb{Q}[t]$. For $i = 3$ and $i = 9$ it is in $\mathbb{Q}(\omega_{13})[t]$ and is not in $\mathbb{Q}[t]$. An essential observation is that, for 8_{17}^r, the roles of ρ_3 and ρ_9 are reversed. All three of the polynomials are irreducible in their respective polynomial rings, once any factors of $(1-t)$ and t are removed.

In [9] the proof that $8_{17} \# -8_{17}^r$ is not slice comes down to the observation that no product of the form

$$\sigma_\delta(\Delta_{8_{17},\epsilon,\rho_3}(t))\sigma_\gamma(\Delta_{8_{17},\epsilon,\rho_i}(t)) \quad \text{or} \quad \sigma_\delta(\Delta_{8_{17},\epsilon,\rho_9}(t))\sigma_\gamma(\Delta_{8_{17},\epsilon,\rho_j}(t))$$

is of the form $af(t)\overline{f(t^{-1})}(1-t)^j$ for some $f(t) \in \mathbb{Q}(\omega_{13})[t]$. (That is, these products are not *norms* in the polynomial ring $\mathbb{Q}(\omega_{13})[t, t^{-1}]$, modulo powers of $(1-t)$ and t.) Here $i = 0$ or $i = 9$, and $j = 0$ or $j = 3$. The number a is in $\mathbb{Q}(\omega)$ and the σ_ν are Galois automorphisms of $\mathbb{Q}(\omega_p)$ (which acts by sending ω_p to ω_p^ν).

Showing that the product of the polynomials does not factor in this way is elementary once it is established that $\Delta_{8_{17},\epsilon,\rho_3}(t)$ and $\Delta_{8_{17},\epsilon,\rho_9}(t)$ are irreducible and not Galois conjugate.

5 Main proof

Using the fact that $-P_n(8_{17})^r = P_n(8_{17})^r$, the knot $L_\alpha \# L_\beta$ can be expanded as

$$P_\alpha(8_{17}) \# P_\alpha(8_{17})^r \# P_\beta(8_{17}) \# P_\beta(8_{17})^r.$$

We begin by analyzing the 3–fold cover of S^3 branched over $P_n(8_{17})$, and assume that 3 does not divide n. This cover is $M_3(P_n(8_{17}))$ and we denote the branch set in the cover by $\widetilde{B}$.

There is the obvious separating torus T in $S^3 \setminus P_n(8_{17})$. Since 3 does not divide n, T has a connected separating lift $\widetilde{T} \subset M_3(P_n(8_{17}))$. One sees that $\widetilde{T}$ splits $M_3(P_n(8_{17}))$ into two components: X, the 3–fold cyclic cover of $S^3 \setminus 8_{17}$, and Y, the 3–fold cyclic branched cover of $S^1 \times B^2$, branched over P_n. A simple exercise shows that, since $P_n(U)$ is unknotted, Y is the complement of some knot $\tilde{J}_n \subset S^3$.

A Mayer–Vietoris argument shows that $H_1(M_3(P_n(8_{17}))) \cong \mathbb{Z}_{13} \oplus \mathbb{Z}_{13}$ and the two canonical representations ρ_3 and ρ_9 that are defined on X extend trivially on Y, and so to $M_3(P_n(8_{17}))$. We denote these extension by ρ_3' and ρ_9'. Let ϵ' be the canonical surjective homomorphism $\epsilon' \colon H_1(M_3(P_n(8_{17}))) \setminus \widetilde{B}) \to \mathbb{Z}$. Restricted to X we have $\epsilon'(x) = \epsilon(nx)$, where ϵ was the canonical representation to $\mathbb{Z}$ defined for the cover of $S^3 \setminus 8_{17}$.

In [8, Theorem 3.7] there is a discussion of twisted Alexander polynomials of satellite knots in S^3, working in the greater generality of homomorphisms to the unitary group $U(m)$. (A map to $\mathbb{Z}_p$ can be viewed as a representation to $U(1)$.) The proof of that theorem, which relies on the multiplicativity of Reidemeister torsion, applies in the current setting, yielding the following lemma:

Lemma 2 $$\Delta_{P_n(8_{17}),\epsilon',\rho_3'}(t) = \Delta_{8_{17},\epsilon,\rho_3}(t^n)\Delta_{\tilde{J}_n}(t).$$

Similar results hold for the knot $P_n(8_{17})^r$ and for the character ρ_9.

As described in [8; 9], Casson–Gordon theory implies that, if $L_\alpha \# L_\beta$ is slice, then for some 3–eigenvector or for some 9–eigenvector the corresponding twisted Alexander polynomial is a norm; that is, it factors as $at^k f(t)\overline{f(t^{-1})}$, modulo multiples of $(1-t)$. If it is a 3–eigenvector, the relevant polynomial is of the form

$$(1)\quad \Delta(t) = \sigma_a(\Delta_{8_{17},\epsilon,\rho_3}(t^\alpha))^x \sigma_b(\Delta_{8_{17},\epsilon,\rho_9}(t^\alpha))^y \sigma_c(\Delta_{8_{17},\epsilon,\rho_3}(t^\beta))^z \sigma_d(\Delta_{8_{17},\epsilon,\rho_9}(t^\beta))^w (\Delta_{\tilde{J}_\alpha}(t)\Delta_{\tilde{J}_\beta}(t))^2,$$

where one of x, y, z, or w is equal to 1, and each of the others are either 1 or 0.

The four $\mathbb{Q}(\omega_{13})[t]$–polynomials that appear here,

$$\Delta_{8_{17},\epsilon,\rho_3}(t^\alpha), \quad \Delta_{8_{17},\epsilon,\rho_9}(t^\alpha), \quad \Delta_{8_{17},\epsilon,\rho_3}(t^\beta), \quad \text{and} \quad \Delta_{8_{17},\epsilon,\rho_9}(t^\beta),$$

and all their Galois conjugates are easily seen to be distinct for any pair $\alpha \neq \beta$. The following number-theoretic result implies that there is an infinite set of primes $\mathcal{P}$ such that, if $\alpha \in \mathcal{P}$ and $\beta \in \mathcal{P}$, then no product as given in (1) can be a norm in $\mathbb{Q}(\omega_{13})[t]$, proving that the connected sum $L_\alpha \# L_\beta$ is not slice. We will present a proof in Appendix A.

Lemma 3 *Let $f(t) \in \mathbb{Z}(\omega_p)[t]$ be an irreducible monic polynomial. If there exists $\zeta \in \mathbb{C}$ such that $f(\zeta) = 0$ and $\zeta^n \neq 1$ for all $n > 0$, then the set of primes p for which $f(t^p)$ is reducible is finite.*

Proof of Theorem 1 The last factor in (1) involving the $\tilde{J}_n$ is a norm, so it can be ignored in determining if the product is a norm.

A numeric computation shows that the twisted polynomials $\Delta_{8_{17},\epsilon,\rho_i}(t)$ for $i = 3$ and $i = 9$ do not have roots on the unit circle, so Lemma 3 can be applied with $\mathbb{F} = \mathbb{Q}(\omega_{13})$. Let $\mathcal{P}$ be the infinite set of primes with the property that if $p \in \mathcal{P}$, then $\Delta_{8_{17},\epsilon,\rho_3}(t^p)$ and $\Delta_{8_{17},\epsilon,\rho_9}(t^p)$ are irreducible. Consider the case of $x = 1$ in (1). Then, assuming that $\alpha \in \mathcal{P}$ and $\beta \in \mathcal{P}$, the term $\sigma_a(\Delta_{8_{17},\epsilon,\rho_3})(t^\alpha)$ that appears in (1) is relatively prime to the remaining factors, and all the factors are irreducible, modulo powers of t and $1-t$. Hence, the product cannot be of the form $t^k(1-t)^j f(t)\overline{f(t^{-1})}$ for any $f(t) \in \mathbb{Q}(\omega_{13})[t]$. The cases of y, z, or $w = 1$ are the same. □

Appendix A Factoring $f(t^p)$

In this appendix we prove Lemma 3, stated in somewhat more generality as Lemma 4 below. We first summarize some background material. Further details can be found in any graduate textbook on algebraic number theory.

- $\mathbb{A} \subset \mathbb{C}$ denotes the ring of algebraic integers. This is the ring consisting of all roots of monic polynomials in $\mathbb{Z}[t]$.
- For an extension field $\mathbb{F}/\mathbb{Q}$, the ring of algebraic integers in $\mathbb{F}$ is defined by $\mathcal{O}_\mathbb{F} = \mathbb{F} \cap \mathbb{A}$.
- The property of *transitivity* states that, if $f(t) \in \mathcal{O}_\mathbb{F}[t]$ is monic and $f(\zeta) = 0$, then $\zeta \in \mathbb{A}$.

- $\mathcal{O}_{\mathbb{F}}^{\times}$ is defined to be the set of units in $\mathcal{O}_{\mathbb{F}}$.
- The *norm* of an element $x \in \mathcal{O}_{\mathbb{F}}$ is defined as $N(x) = \prod x_i \in \mathbb{Z}$, where the x_i are the complex Galois conjugates of x. This map satisfies $N(xy) = N(x)N(y)$ for all $x, y \in \mathcal{O}_{\mathbb{F}}$. An element $x \in \mathcal{O}_{\mathbb{F}}$ is in $\mathcal{O}_{\mathbb{F}}^{\times}$ if and only if $N(x) = \pm 1$.
- The *Dirichlet unit theorem* states that, for a finite extension $\mathbb{F}/\mathbb{Q}$, the abelian group $\mathcal{O}_{\mathbb{F}}^{\times}$ is finitely generated and isomorphic to $G \oplus \mathbb{Z}^{r+s-1}$, where G is finite cyclic, r is the number of embeddings of $\mathbb{F}$ in $\mathbb{R}$, and $2s$ is the number of nonreal embeddings of $\mathbb{F}$ in $\mathbb{C}$.

Lemma 4 *Let $\mathbb{F}$ be a finite extension of $\mathbb{Q}$, and let $f(t) \in \mathcal{O}_{\mathbb{F}}[t]$ be an irreducible monic polynomial. If there exists $\zeta \in \mathbb{C}$ such that $f(\zeta) = 0$ and $\zeta^n \neq 1$ for all $n > 0$, then the set of primes p for which $f(t^p)$ is reducible is finite.*

Proof **Step 1** *If $f(\zeta) = 0$, then $\zeta \in \mathcal{O}_{F(\zeta)}$.*

This follows immediately from the assumption that $f(t)$ is monic.

Step 2 *Suppose that $f(t) \in \mathbb{F}[t]$ is irreducible and $f(\zeta) = 0$. If, for some prime p, $f(t^p)$ is reducible over $\mathbb{F}$, then $\zeta = \eta^p$ for some $\eta \in \mathcal{O}_{\mathbb{F}(\zeta)}$.*

Let $\xi \in \mathbb{C}$ satisfy $\xi^p = \zeta$. Since $f(t)$ is irreducible of degree n and $f(t^p)$ is reducible, we have the degrees of extensions satisfying $[\mathbb{F}(\zeta) : \mathbb{F}] = n$ and $[\mathbb{F}(\xi) : \mathbb{F}] < np$. It follows from the multiplicity of degrees of extensions that $[\mathbb{F}(\xi) : \mathbb{F}(\zeta)] < p$.

The polynomial $t^p - \zeta \in \mathbb{F}(\zeta)[t]$ has ξ as a root. For all i, $\omega_p^i \xi$ is also a root, so $t^p - \zeta$ factors completely in $\mathbb{C}[t]$ as

$$t^p - \zeta = (t - \xi)(t - \omega_p \xi) \cdots (t - \omega_p^{p-1} \xi).$$

By the degree calculation just given, $t^p - \zeta$ has an irreducible factor $g(t) \in \mathbb{F}(\zeta)[t]$ of degree $l < p$. We can write $g(t) = \prod (t - \omega_p^i \xi)$, where the product is over some proper subset of $\{0, \dots, p-1\}$. Multiplying this out, one finds that the constant term is of the form $\omega_p^j \xi^l \in \mathbb{F}(\zeta)$ for some j and $l < p$. Since l and p are relatively prime, there are integers u and v such that $ul + vp = 1$. Thus, $(\omega_p^j \xi^l)^u (\xi^p)^v = \omega_p^s \xi$ for some s. In particular, for some s, we have $\omega_p^s \xi \in \mathbb{F}(\zeta)$. We let $\eta = \omega_p^s \xi$ and find that $\eta^p = (\omega_p^s)^p \xi^p = \zeta$. Finally, η satisfies the monic polynomial $f(t^p)$ and thus is in $\mathcal{O}_{\mathbb{F}(\zeta)}$.

Step 3 *The set of primes p such that $\zeta = \eta^p$ for some $\eta \in \mathcal{O}_{\mathbb{F}(\zeta)}$ is finite.*

If $\zeta = \eta^p$, then $N(\zeta) = N(\eta)^p$. If $N(\zeta) \neq \pm 1$, then the set of p for which $N(\zeta) = a^p$ for some integer a is finite.

If $N(\zeta) = \pm 1$, then $\zeta \in \mathcal{O}_{\mathbb{F}(\zeta)}^{\times}$. Hence ζ represents a nontorsion element in a finitely generated abelian group, and thus it has a finite number of roots. □

Comments The argument just given is based on a summary of the proof for the case $\mathbb{F} = \mathbb{Q}$ presented on MathOverflow by Dimitrov [4]. Step 2 is a special case of the *Vahlen–Capelli theorem*, proved in the case of $\mathbb{F} = \mathbb{Q}$ by Vahlen and for fields of characteristic 0 by Capelli [2]. A proof for fields of finite characteristic is given by Rédei [10].

Appendix B Twisted polynomials of 8_{17}

Here are the three needed polynomials. We write ω for ω_{13}.

$$\Delta_{8_{17},\epsilon,\rho_0}(t) = 1-t-34t^2-101t^3-34t^4-t^5+t^6,$$

$$\begin{aligned}\Delta_{8_{17},\epsilon,\rho_3}(t)/(1-t)&\\ = 1&+t(2\omega+2\omega^2+2\omega^3+4\omega^4+2\omega^5+2\omega^6+\omega^7+\omega^8+2\omega^9+4\omega^{10}+\omega^{11}+4\omega^{12})\\ &+t^2(-15\omega-10\omega^2-15\omega^3-15\omega^4-10\omega^5-10\omega^6-10\omega^7-10\omega^8-15\omega^9-15\omega^{10}-10\omega^{11}-15\omega^{12})\\ &+t^3(4\omega+\omega^2+4\omega^3+2\omega^4+\omega^5+\omega^6+2\omega^7+2\omega^8+4\omega^9+2\omega^{10}+2\omega^{11}+2\omega^{12})+t^4,\end{aligned}$$

$$\begin{aligned}\Delta_{8_{17},\epsilon,\rho_9}(t)/(1-t)&\\ = 1&+t(6\omega+5\omega^2+6\omega^3+6\omega^4+5\omega^5+5\omega^6+5\omega^7+5\omega^8+6\omega^9+6\omega^{10}+5\omega^{11}+6\omega^{12})\\ &+t^2(-13\omega-12\omega^2-13\omega^3-13\omega^4-12\omega^5-12\omega^6-12\omega^7-12\omega^8-13\omega^9-13\omega^{10}-12\omega^{11}-13\omega^{12})\\ &+t^3(6\omega+5\omega^2+6\omega^3+6\omega^4+5\omega^5+5\omega^6+5\omega^7+5\omega^8+6\omega^9+6\omega^{10}+5\omega^{11}+6\omega^{12})+t^4.\end{aligned}$$

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