Algebraic & Geometric Topology

Volume 24 (2024)

Branched covers and rational homology balls

CHARLES LIVINGSTON

msp

Algebraic & Geometric Topology 24:1 (2024) 587–594
DOI: 10.2140/agt.2024.24.587
Published: 18 March 2024

Branched covers and rational homology balls

CHARLES LIVINGSTON

The concordance group of knots in S^3 contains a subgroup isomorphic to $(\mathbb{Z}_2)^\infty$, each element of which is represented by a knot K with the property that, for every $n > 0$, the n–fold cyclic cover of S^3 branched over K bounds a rational homology ball. This implies that the kernel of the canonical homomorphism from the knot concordance group to the infinite direct sum of rational homology cobordism groups (defined via prime-power branched covers) contains an infinitely generated two-torsion subgroup.

57K10, 57M12

1 Introduction

There is a homomorphism

$$\varphi\colon \mathcal{C} \to \prod_{q\in\mathcal{Q}} \Theta^3_{\mathbb{Q}},$$

where $\mathcal{C}$ is the smooth concordance group of knots in S^3, $\mathcal{Q}$ is the set of prime power integers, and $\Theta^3_{\mathbb{Q}}$ is the rational homology cobordism group. For a knot K and $q \in \mathcal{Q}$, the q–component of $\varphi(K)$ is the class of $M_q(K)$, the q–fold cyclic cover of S^3 branched over K.

In [1], Aceto, Meier, A Miller, M Miller, Park, and Stipsicz proved that $\ker\varphi$ contains a subgroup isomorphic to $(\mathbb{Z}_2)^5$. Here we will prove that $\ker\varphi$ contains a subgroup isomorphic to $(\mathbb{Z}_2)^\infty$. Our examples are of the form $K\#-K^r$, where $-K$ denotes the concordance inverse of K (the mirror image of K with string orientation reversed), and K^r is formed from K by reversing its string orientation. Such knots are easily seen to be in the kernel of φ; the more difficult work is to find nontrivial examples of order two.

The first known example of a nontrivial element in $\ker\varphi$ was represented by the knot $K_1 = 8_{17}\,\#-8^r_{17}$, which is of order two in $\mathcal{C}$. That K_1 is of order at most two is elementary; that K_1 is nontrivial in $\mathcal{C}$ is one of the main results of Kirk and Livingston in [9], proved using twisted Alexander polynomials.

The results of Kim and Livingston [7] provide an infinitely generated free subgroup of $\ker\varphi$. Conjecturally, $\mathcal{C} \cong \mathbb{Z}^\infty \oplus (\mathbb{Z}_2)^\infty$; if true, then $\ker\varphi \cong \mathbb{Z}^\infty \oplus (\mathbb{Z}_2)^\infty$.

1.1 Main result

Figure 1 illustrates a knot P_n in a solid torus, where J_n represents the braid illustrated on the right in the case of $n = 5$; n will always be odd. We let K_n denote the satellite of 8_{17} built from P_n. In standard

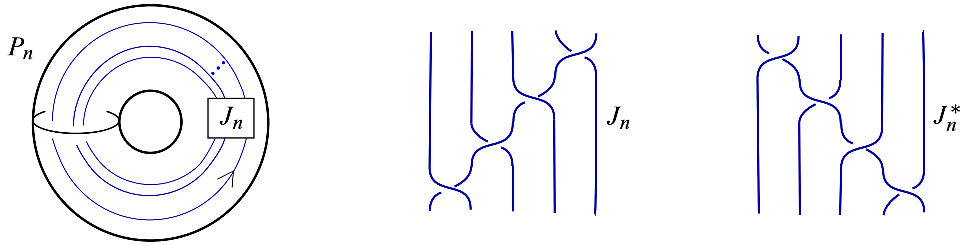

Figure 1: The knot $P_n \subset S^1 \times B^2$, J_n, and J_n^*.

notation, $K_n = P_n(8_{17})$. For future reference, we illustrate the braid J_n^* formed by rotating J_n around the vertical axis.

Theorem 1 *Let $K_n = P_n(8_{17})$. For all odd n, the knot $L_n = K_n \# -K_n^r$ satisfies $2L_n = 0 \in \mathcal{C}$ and $L_n \in \ker \varphi$. There is an infinite set of prime integers $\mathcal{P}$ for which $L_\alpha \neq L_\beta \in \mathcal{C}$ for all $\alpha \neq \beta$ in $\mathcal{P}$. In particular, the set of knots $\{L_n\}_{n \in \mathcal{P}}$ generates a subgroup of $\ker \varphi$ that is isomorphic to $(\mathbb{Z}_2)^\infty$.*

The rest of the paper presents a proof of this theorem. The first two claims are easily dealt with in Sections 2 and 3. The more difficult step of the proof calls on an analysis of twisted Alexander polynomials and their relevance to knot slicing; a review of twisted polynomials is included in Section 4. The proof of Theorem 1 is completed in Section 5, with the exception of a number-theoretic result that is described Appendix A.

Acknowledgements Thanks to Darrell Haile for assisting me in the proof of the number-theoretic result in Appendix A. Allison Miller provided valuable feedback about an early draft of this paper. This work was supported by a grant from the National Science Foundation, NSF-DMS-1505586.

2 Proof that $2L_n = 0 \in \mathcal{C}$

Let $P_n^* \subset S^1 \times B^2$ denote the knot formed using the braid J_n^* in Figure 2. For any knot K, let $P_n^*(K)$ denote the satellite of K built using P_n^*. It should be clear that P_n and P_n^* are orientation-preserving isotopic, and thus for all knots K, $P_n(K) = P_n^*(K)$.

Figure 2 illustrates, for an arbitrary knot K, the connected sum $P_n(K) \# P_n^*(K) = P_n(K) \# P_n(K)$ in the case of $n = 5$. Performing $n-1$ band moves in the evident way yields the $(0, n)$–cable of $K \# K$. Thus, if $K \# K = 0 \in \mathcal{C}$, then the n components of this link can be capped off with parallel copies of the slice disk for $K \# K$, implying that $P_n(K) \# P_n(K) = 0 \in \mathcal{C}$. In particular, $2K_n = 0 \in \mathcal{C}$ and $2K_n^r = 0 \in \mathcal{C}$.

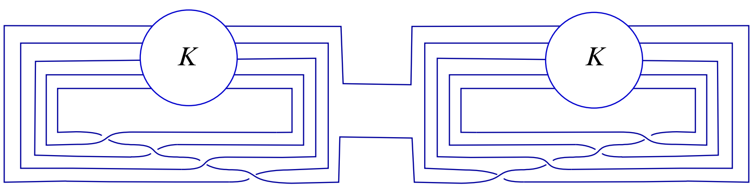

Figure 2: $P_5(K) \# P_5(K)$.

3 Proof that $L_n \in \ker \varphi$

We prove a stronger statement: *for all odd n, and for all positive integers q, $M_q(L_n)$ is a rational homology sphere that represents $0 \in \Theta^3_{\mathbb{Q}}$.*

The q–fold cyclic cover of S^3 branched over $K_n \# -K_n^r$ is the same space as the q–fold cyclic cover of S^3 branched over $K_n \# -K_n$. A slice disk for $K_n \# -K_n$ is built from $(S^3 \times I, K_n \times I)$ by removing a copy of $B^3 \times I$. Taking the q–fold branched cover shows that the q–fold cyclic cover of B^4 branched over that slice disk is diffeomorphic to $M_q(K_n)^* \times I$, where $M_q(K_n)^*$ denotes a punctured copy of $M_q(K_n)$. It remains to show that $M_q(K_n)$ is a rational homology 3–sphere.

A formula of Fox [5] and Goeritz [6] states that the order of the first homology of $M_q(K_n)$ is given by the product of values $\Delta_{K_n}(\omega_q^i)$, where $\Delta_{K_n}(t)$ denotes the Alexander polynomial, ω_q is a primitive q–root of unity, and i runs from 1 to $q-1$.

A result of Seifert [11] shows that $\Delta_{K_n}(t) = \Delta_{8_{17}}(t^n)\Delta_{P_n(U)}$, where U is the unknot. We have that $P_n(U) = U$. The Alexander polynomial for 8_{17} is

$$\Delta_{8_{17}}(t) = 1 - 4t + 8t^2 - 11t^3 + 8t^4 - 4t^5 + t^6.$$

A numeric computation confirms that this polynomial does not have roots on the unit complex circle, and hence $\Delta_{8_{17}}(t^n)$ has no roots on the unit complex circle. From this is follows that $\Delta_{K_n}(\omega_q^i) \neq 0$ for all i; thus the order of the homology of $M_q(K_n)$ is finite.

4 Review of twisted polynomials and 8_{17}

In this section we review twisted Alexander polynomials and their application in [8; 9] showing that $8_{17} \# -8_{17}^r \neq 0 \in \mathcal{C}$.

Let $(X, B) \to (S^3, K)$ be the q–fold cyclic branched cover of a knot K with q a prime power. In particular, X is a rational homology sphere. There is a canonical surjection $\epsilon\colon H_1(X - B) \to \mathbb{Z}$. Suppose that $\rho\colon H_1(X) \to \mathbb{Z}_p$ is a homomorphism for some prime p. Then there is an associated twisted polynomial $\Delta_{K,\epsilon,\rho}(t) \in \mathbb{Q}(\omega_p)[t]$. It is well-defined, up to factors of the form at^k, where $a \neq 0 \in \mathbb{Q}(\omega_p)$. These polynomials are discriminants of Casson–Gordon invariants, first defined in [3].

In the case of $K = 8_{17}$ and $q = 3$, we have $H_1(X) \cong \mathbb{Z}_{13} \oplus \mathbb{Z}_{13}$, and as a $\mathbb{Z}_{13}$–vector space this splits as a direct sum of a 3–eigenspace and a 9–eigenspace under the order three action of the deck transformation. Both eigenspaces are 1–dimensional. We denote this splitting by $E_3 \oplus E_9$. There are corresponding characters ρ_3 and ρ_9 of $H_1(X)$ onto $\mathbb{Z}_{13}$; these are defined as the quotient maps onto $H_1(X)/E_3$ and onto $H_1(X)/E_9$. We let ρ_0 denote the trivial $\mathbb{Z}_{13}$–valued character.

The values of $\Delta_{8_{17},\epsilon,\rho_i}(t)$ are given in [9], duplicated here in Appendix B. For $i = 0$ it is polynomial in $\mathbb{Q}[t]$. For $i = 3$ and $i = 9$ it is in $\mathbb{Q}(\omega_{13})[t]$ and is not in $\mathbb{Q}[t]$. An essential observation is that, for 8_{17}^r, the roles of ρ_3 and ρ_9 are reversed. All three of the polynomials are irreducible in their respective polynomial rings, once any factors of $(1-t)$ and t are removed.

In [9] the proof that $8_{17}\,\#\,{-8_{17}^r}$ is not slice comes down to the observation that no product of the form

$$\sigma_\delta(\Delta_{8_{17},\epsilon,\rho_3}(t))\sigma_\gamma(\Delta_{8_{17},\epsilon,\rho_i}(t)) \quad \text{or} \quad \sigma_\delta(\Delta_{8_{17},\epsilon,\rho_9}(t))\sigma_\gamma(\Delta_{8_{17},\epsilon,\rho_j}(t))$$

is of the form $af(t)\overline{f(t^{-1})}(1-t)^j$ for some $f(t)\in\mathbb{Q}(\omega_{13})[t]$. (That is, these products are not *norms* in the polynomial ring $\mathbb{Q}(\omega_{13})[t,t^{-1}]$, modulo powers of $(1-t)$ and t.) Here $i=0$ or $i=9$, and $j=0$ or $j=3$. The number a is in $\mathbb{Q}(\omega)$ and the σ_ν are Galois automorphisms of $\mathbb{Q}(\omega_p)$ (which acts by sending ω_p to ω_p^ν).

Showing that the product of the polynomials does not factor in this way is elementary once it is established that $\Delta_{8_{17},\epsilon,\rho_3}(t)$ and $\Delta_{8_{17},\epsilon,\rho_9}(t)$ are irreducible and not Galois conjugate.

5 Main proof

Using the fact that $-P_n(8_{17})^r = P_n(8_{17})^r$, the knot $L_\alpha\,\#\,L_\beta$ can be expanded as

$$P_\alpha(8_{17})\,\#\,P_\alpha(8_{17})^r\,\#\,P_\beta(8_{17})\,\#\,P_\beta(8_{17})^r.$$

We begin by analyzing the 3–fold cover of S^3 branched over $P_n(8_{17})$, and assume that 3 does not divide n. This cover is $M_3(P_n(8_{17}))$ and we denote the branch set in the cover by $\widetilde{B}$.

There is the obvious separating torus T in $S^3\setminus P_n(8_{17})$. Since 3 does not divide n, T has a connected separating lift $\widetilde{T}\subset M_3(P_n(8_{17}))$. One sees that $\widetilde{T}$ splits $M_3(P_n(8_{17}))$ into two components: X, the 3–fold cyclic cover of $S^3\setminus 8_{17}$, and Y, the 3–fold cyclic branched cover of $S^1\times B^2$, branched over P_n. A simple exercise shows that, since $P_n(U)$ is unknotted, Y is the complement of some knot $\tilde{J}_n\subset S^3$.

A Mayer–Vietoris argument shows that $H_1(M_3(P_n(8_{17})))\cong\mathbb{Z}_{13}\oplus\mathbb{Z}_{13}$ and the two canonical representations ρ_3 and ρ_9 that are defined on X extend trivially on Y, and so to $M_3(P_n(8_{17}))$. We denote these extension by ρ_3' and ρ_9'. Let ϵ' be the canonical surjective homomorphism $\epsilon'\colon H_1(M_3(P_n(8_{17}))\setminus\widetilde{B})\to\mathbb{Z}$. Restricted to X we have $\epsilon'(x)=\epsilon(nx)$, where ϵ was the canonical representation to $\mathbb{Z}$ defined for the cover of $S^3\setminus 8_{17}$.

In [8, Theorem 3.7] there is a discussion of twisted Alexander polynomials of satellite knots in S^3, working in the greater generality of homomorphisms to the unitary group $U(m)$. (A map to $\mathbb{Z}_p$ can be viewed as a representation to $U(1)$.) The proof of that theorem, which relies on the multiplicativity of Reidemeister torsion, applies in the current setting, yielding the following lemma:

Lemma 2 $$\Delta_{P_n(8_{17}),\epsilon',\rho_3'}(t)=\Delta_{8_{17},\epsilon,\rho_3}(t^n)\Delta_{\tilde{J}_n}(t).$$

Similar results hold for the knot $P_n(8_{17})^r$ and for the character ρ_9.

As described in [8; 9], Casson–Gordon theory implies that, if $L_\alpha \# L_\beta$ is slice, then for some 3–eigenvector or for some 9–eigenvector the corresponding twisted Alexander polynomial is a norm; that is, it factors as $at^k f(t)\overline{f(t^{-1})}$, modulo multiples of $(1-t)$. If it is a 3–eigenvector, the relevant polynomial is of the form

$$(1) \quad \Delta(t) = \sigma_a(\Delta_{8_{17},\epsilon,\rho_3}(t^\alpha))^x \sigma_b(\Delta_{8_{17},\epsilon,\rho_9}(t^\alpha))^y \sigma_c(\Delta_{8_{17},\epsilon,\rho_3}(t^\beta))^z \sigma_d(\Delta_{8_{17},\epsilon,\rho_9}(t^\beta))^w (\Delta_{\tilde{J}_\alpha}(t)\Delta_{\tilde{J}_\beta}(t))^2,$$

where one of x, y, z, or w is equal to 1, and each of the others are either 1 or 0.

The four $\mathbb{Q}(\omega_{13})[t]$–polynomials that appear here,

$$\Delta_{8_{17},\epsilon,\rho_3}(t^\alpha), \quad \Delta_{8_{17},\epsilon,\rho_9}(t^\alpha), \quad \Delta_{8_{17},\epsilon,\rho_3}(t^\beta), \quad \text{and} \quad \Delta_{8_{17},\epsilon,\rho_9}(t^\beta),$$

and all their Galois conjugates are easily seen to be distinct for any pair $\alpha \neq \beta$. The following number-theoretic result implies that there is an infinite set of primes $\mathcal{P}$ such that, if $\alpha \in \mathcal{P}$ and $\beta \in \mathcal{P}$, then no product as given in (1) can be a norm in $\mathbb{Q}(\omega_{13})[t]$, proving that the connected sum $L_\alpha \# L_\beta$ is not slice. We will present a proof in Appendix A.

Lemma 3 *Let $f(t) \in \mathbb{Z}(\omega_p)[t]$ be an irreducible monic polynomial. If there exists $\zeta \in \mathbb{C}$ such that $f(\zeta) = 0$ and $\zeta^n \neq 1$ for all $n > 0$, then the set of primes p for which $f(t^p)$ is reducible is finite.*

Proof of Theorem 1 The last factor in (1) involving the $\tilde{J}_n$ is a norm, so it can be ignored in determining if the product is a norm.

A numeric computation shows that the twisted polynomials $\Delta_{8_{17},\epsilon,\rho_i}(t)$ for $i = 3$ and $i = 9$ do not have roots on the unit circle, so Lemma 3 can be applied with $\mathbb{F} = \mathbb{Q}(\omega_{13})$. Let $\mathcal{P}$ be the infinite set of primes with the property that if $p \in \mathcal{P}$, then $\Delta_{8_{17},\epsilon,\rho_3}(t^p)$ and $\Delta_{8_{17},\epsilon,\rho_9}(t^p)$ are irreducible. Consider the case of $x = 1$ in (1). Then, assuming that $\alpha \in \mathcal{P}$ and $\beta \in \mathcal{P}$, the term $\sigma_a(\Delta_{8_{17},\epsilon,\rho_3})(t^\alpha)$ that appears in (1) is relatively prime to the remaining factors, and all the factors are irreducible, modulo powers of t and $1-t$. Hence, the product cannot be of the form $t^k(1-t)^j f(t)f(t^{-1})$ for any $f(t) \in \mathbb{Q}(\omega_{13})[t]$. The cases of y, z, or $w = 1$ are the same. □

Appendix A Factoring $f(t^p)$

In this appendix we prove Lemma 3, stated in somewhat more generality as Lemma 4 below. We first summarize some background material. Further details can be found in any graduate textbook on algebraic number theory.

- $\mathbb{A} \subset \mathbb{C}$ denotes the ring of algebraic integers. This is the ring consisting of all roots of monic polynomials in $\mathbb{Z}[t]$.
- For an extension field $\mathbb{F}/\mathbb{Q}$, the ring of algebraic integers in $\mathbb{F}$ is defined by $\mathcal{O}_\mathbb{F} = \mathbb{F} \cap \mathbb{A}$.
- The property of *transitivity* states that, if $f(t) \in \mathcal{O}_\mathbb{F}[t]$ is monic and $f(\zeta) = 0$, then $\zeta \in \mathbb{A}$.

- $\mathcal{O}_{\mathbb{F}}^{\times}$ is defined to be the set of units in $\mathcal{O}_{\mathbb{F}}$.
- The *norm* of an element $x \in \mathcal{O}_{\mathbb{F}}$ is defined as $N(x) = \prod x_i \in \mathbb{Z}$, where the x_i are the complex Galois conjugates of x. This map satisfies $N(xy) = N(x)N(y)$ for all $x, y \in \mathcal{O}_{\mathbb{F}}$. An element $x \in \mathcal{O}_{\mathbb{F}}$ is in $\mathcal{O}_{\mathbb{F}}^{\times}$ if and only if $N(x) = \pm 1$.
- The *Dirichlet unit theorem* states that, for a finite extension $\mathbb{F}/\mathbb{Q}$, the abelian group $\mathcal{O}_{\mathbb{F}}^{\times}$ is finitely generated and isomorphic to $G \oplus \mathbb{Z}^{r+s-1}$, where G is finite cyclic, r is the number of embeddings of $\mathbb{F}$ in $\mathbb{R}$, and $2s$ is the number of nonreal embeddings of $\mathbb{F}$ in $\mathbb{C}$.

Lemma 4 *Let $\mathbb{F}$ be a finite extension of $\mathbb{Q}$, and let $f(t) \in \mathcal{O}_{\mathbb{F}}[t]$ be an irreducible monic polynomial. If there exists $\zeta \in \mathbb{C}$ such that $f(\zeta) = 0$ and $\zeta^n \neq 1$ for all $n > 0$, then the set of primes p for which $f(t^p)$ is reducible is finite.*

Proof **Step 1** *If $f(\zeta) = 0$, then $\zeta \in \mathcal{O}_{F(\zeta)}$.*

This follows immediately from the assumption that $f(t)$ is monic.

Step 2 *Suppose that $f(t) \in \mathbb{F}[t]$ is irreducible and $f(\zeta) = 0$. If, for some prime p, $f(t^p)$ is reducible over $\mathbb{F}$, then $\zeta = \eta^p$ for some $\eta \in \mathcal{O}_{\mathbb{F}(\zeta)}$.*

Let $\xi \in \mathbb{C}$ satisfy $\xi^p = \zeta$. Since $f(t)$ is irreducible of degree n and $f(t^p)$ is reducible, we have the degrees of extensions satisfying $[\mathbb{F}(\zeta) : \mathbb{F}] = n$ and $[\mathbb{F}(\xi) : \mathbb{F}] < np$. It follows from the multiplicity of degrees of extensions that $[\mathbb{F}(\xi) : \mathbb{F}(\zeta)] < p$.

The polynomial $t^p - \zeta \in \mathbb{F}(\zeta)[t]$ has ξ as a root. For all i, $\omega_p^i \xi$ is also a root, so $t^p - \zeta$ factors completely in $\mathbb{C}[t]$ as

$$t^p - \zeta = (t - \xi)(t - \omega_p \xi) \cdots (t - \omega_p^{p-1} \xi).$$

By the degree calculation just given, $t^p - \zeta$ has an irreducible factor $g(t) \in \mathbb{F}(\zeta)[t]$ of degree $l < p$. We can write $g(t) = \prod (t - \omega_p^i \xi)$, where the product is over some proper subset of $\{0, \dots, p-1\}$. Multiplying this out, one finds that the constant term is of the form $\omega_p^j \xi^l \in \mathbb{F}(\zeta)$ for some j and $l < p$. Since l and p are relatively prime, there are integers u and v such that $ul + vp = 1$. Thus, $(\omega_p^j \xi^l)^u (\xi^p)^v = \omega_p^s \xi$ for some s. In particular, for some s, we have $\omega_p^s \xi \in \mathbb{F}(\zeta)$. We let $\eta = \omega_p^s \xi$ and find that $\eta^p = (\omega_p^s)^p \xi^p = \zeta$. Finally, η satisfies the monic polynomial $f(t^p)$ and thus is in $\mathcal{O}_{\mathbb{F}(\zeta)}$.

Step 3 *The set of primes p such that $\zeta = \eta^p$ for some $\eta \in \mathcal{O}_{\mathbb{F}(\zeta)}$ is finite.*

If $\zeta = \eta^p$, then $N(\zeta) = N(\eta)^p$. If $N(\zeta) \neq \pm 1$, then the set of p for which $N(\zeta) = a^p$ for some integer a is finite.

If $N(\zeta) = \pm 1$, then $\zeta \in \mathcal{O}_{\mathbb{F}(\zeta)}^{\times}$. Hence ζ represents a nontorsion element in a finitely generated abelian group, and thus it has a finite number of roots. □

Comments The argument just given is based on a summary of the proof for the case $\mathbb{F} = \mathbb{Q}$ presented on MathOverflow by Dimitrov [4]. Step 2 is a special case of the *Vahlen–Capelli theorem*, proved in the case of $\mathbb{F} = \mathbb{Q}$ by Vahlen and for fields of characteristic 0 by Capelli [2]. A proof for fields of finite characteristic is given by Rédei [10].

Appendix B Twisted polynomials of 8_{17}

Here are the three needed polynomials. We write ω for ω_{13}.

$$\Delta_{8_{17},\epsilon,\rho_0}(t) = 1-t-34t^2-101t^3-34t^4-t^5+t^6,$$

$$\begin{aligned}\Delta_{8_{17},\epsilon,\rho_3}(t)/(1-t)&\\ = 1&+t(2\omega+2\omega^2+2\omega^3+4\omega^4+2\omega^5+2\omega^6+\omega^7+\omega^8+2\omega^9+4\omega^{10}+\omega^{11}+4\omega^{12})\\ &+t^2(-15\omega-10\omega^2-15\omega^3-15\omega^4-10\omega^5-10\omega^6-10\omega^7-10\omega^8-15\omega^9-15\omega^{10}-10\omega^{11}-15\omega^{12})\\ &+t^3(4\omega+\omega^2+4\omega^3+2\omega^4+\omega^5+\omega^6+2\omega^7+2\omega^8+4\omega^9+2\omega^{10}+2\omega^{11}+2\omega^{12})+t^4,\end{aligned}$$

$$\begin{aligned}\Delta_{8_{17},\epsilon,\rho_9}(t)/(1-t)&\\ = 1&+t(6\omega+5\omega^2+6\omega^3+6\omega^4+5\omega^5+5\omega^6+5\omega^7+5\omega^8+6\omega^9+6\omega^{10}+5\omega^{11}+6\omega^{12})\\ &+t^2(-13\omega-12\omega^2-13\omega^3-13\omega^4-12\omega^5-12\omega^6-12\omega^7-12\omega^8-13\omega^9-13\omega^{10}-12\omega^{11}-13\omega^{12})\\ &+t^3(6\omega+5\omega^2+6\omega^3+6\omega^4+5\omega^5+5\omega^6+5\omega^7+5\omega^8+6\omega^9+6\omega^{10}+5\omega^{11}+6\omega^{12})+t^4.\end{aligned}$$

References

[1] **P Aceto**, **J Meier**, **A N Miller**, **M Miller**, **J Park**, **A I Stipsicz**, *Branched covers bounding rational homology balls*, Algebr. Geom. Topol. 21 (2021) 3569–3599 MR Zbl

[2] **A Capelli**, *Sulla riduttibilità delle equazioni algebriche*, Napoli Rend. 3 (1897) 243–252 Zbl

[3] **A J Casson**, **C M Gordon**, *Cobordism of classical knots*, from "À la recherche de la topologie perdue" (L Guillou, A Marin, editors), Progr. Math. 62, Birkhäuser, Boston (1986) 181–199 MR Zbl

[4] **V Dimitrov**, *$f(x)$ is irreducible but $f(x^n)$ is reducible*, MathOverflow thread (2014) Available at `https://mathoverflow.net/q/179387`

[5] **R H Fox**, *Free differential calculus, III: Subgroups*, Ann. of Math. 64 (1956) 407–419 MR Zbl

[6] **L Goeritz**, *Die Betti'schen Zahlen Der Zyklischen Uberlagerungsraume Der Knotenaussenraume*, Amer. J. Math. 56 (1934) 194–198 MR Zbl

[7] **T Kim**, **C Livingston**, *Knot reversal acts non-trivially on the concordance group of topologically slice knots*, Selecta Math. 28 (2022) art. id. 38 MR Zbl

[8] **P Kirk**, **C Livingston**, *Twisted Alexander invariants, Reidemeister torsion, and Casson–Gordon invariants*, Topology 38 (1999) 635–661 MR Zbl

[9] **P Kirk**, **C Livingston**, *Twisted knot polynomials: inversion, mutation and concordance*, Topology 38 (1999) 663–671 MR Zbl

[10] **L Rédei**, *Algebra*, *I*, Pergamon, Oxford (1967) MR Zbl

[11] **H Seifert**, *On the homology invariants of knots*, Quart. J. Math. Oxford Ser. 1 (1950) 23–32 MR Zbl

Department of Mathematics, Indiana University
Bloomington, IN, United States

livingst@indiana.edu

Received: 17 May 2022 Revised: 3 August 2022

ALGEBRAIC & GEOMETRIC TOPOLOGY

msp.org/agt

See inside back cover or msp.org/agt for submission instructions.

The subscription price for 2024 is US $705/year for the electronic version, and $1040/year (+$70, if shipping outside the US) for print and electronic. Subscriptions, requests for back issues and changes of subscriber address should be sent to MSP. Algebraic & Geometric Topology is indexed by Mathematical Reviews, Zentralblatt MATH, Current Mathematical Publications and the Science Citation Index.

Algebraic & Geometric Topology (ISSN 1472-2747 printed, 1472-2739 electronic) is published 9 times per year and continuously online, by Mathematical Sciences Publishers, c/o Department of Mathematics, University of California, 798 Evans Hall #3840, Berkeley, CA 94720-3840. Periodical rate postage paid at Oakland, CA 94615-9651, and additional mailing offices. POSTMASTER: send address changes to Mathematical Sciences Publishers, c/o Department of Mathematics, University of California, 798 Evans Hall #3840, Berkeley, CA 94720-3840.

AGT peer review and production are managed by EditFlow® from MSP.

PUBLISHED BY

mathematical sciences publishers

nonprofit scientific publishing

https://msp.org/

ALGEBRAIC & GEOMETRIC TOPOLOGY

Volume 24 Issue 1 (pages 1–594) 2024

Chow–Witt rings of Grassmannians 1
MATTHIAS WENDT

Higher chromatic Thom spectra via unstable homotopy theory 49
SANATH K DEVALAPURKAR

The deformation space of nonorientable hyperbolic 3–manifolds 109
JUAN LUIS DURÁN BATALLA and JOAN PORTI

Realization of Lie algebras and classifying spaces of crossed modules 141
YVES FÉLIX and DANIEL TANRÉ

Knot Floer homology, link Floer homology and link detection 159
FRASER BINNS and GAGE MARTIN

Models for knot spaces and Atiyah duality 183
SYUNJI MORIYA

Automorphismes du groupe des automorphismes d'un groupe de Coxeter universel 251
YASSINE GUERCH

The $RO(C_4)$ cohomology of the infinite real projective space 277
NICK GEORGAKOPOULOS

Annular Khovanov homology and augmented links 325
HONGJIAN YANG

Smith ideals of operadic algebras in monoidal model categories 341
DAVID WHITE and DONALD YAU

The persistent topology of optimal transport based metric thickenings 393
HENRY ADAMS, FACUNDO MÉMOLI, MICHAEL MOY and QINGSONG WANG

A generalization of moment-angle manifolds with noncontractible orbit spaces 449
LI YU

Equivariant Seiberg–Witten–Floer cohomology 493
DAVID BARAGLIA and PEDRAM HEKMATI

Constructions stemming from nonseparating planar graphs and their Colin de Verdière invariant 555
ANDREI PAVELESCU and ELENA PAVELESCU

Census L–space knots are braid positive, except for one that is not 569
KENNETH L BAKER and MARC KEGEL

Branched covers and rational homology balls 587
CHARLES LIVINGSTON