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The guts of nearly fibered knots

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The guts of a knot is an invariant defined for the knot complement by Agol–Zhang. Nearly fibered knots, which are defined as knots whose Floer homology has dimension two in the top Alexander grading, were introduced by Baldwin–Sivek. We provide three models for the guts of nearly fibered knots in the 3-sphere. As a corollary, the nearly fibered condition can be purely topologically characterized and is independent of the specific version of Floer theory.

1 Introduction

By work of Ghiggini [5], Ni [14] and Kronheimer–Mrowka [9], a knot $K \subset S^3$ is fibered if and only if its knot homology in any branch of Floer theory is 1-dimensional in the top Alexander grading. Hence it is natural to ask what happens if the top grading summand of the knot homology is 2-dimensional. Recently, Baldwin–Sivek in [4] introduced the following definition.

Definition 1.1 A knot $K \subset S^3$ is said to be *nearly fibered* (in the Heegaard Floer sense) if

$$\widehat{\text{HFK}}(S^3, K, g(K); \mathbb{Q}) \cong \mathbb{Q}^2.$$

Their definition is stated with Heegaard Floer theory, but we can also define nearly fibered knots in the instanton sense by requiring

$$\text{KHI}(S^3, K, g(K)) \cong \mathbb{C}^2,$$

where KHI denotes the instanton knot homology [9] of $K \subset S^3$.

In this note, we show that the nearly fibered condition has a purely topological characterization, and is independent of the branches of Floer theory. To better describe this criterion, we use the notion of guts of knots recently introduced by Agol–Zhang [1].

Given a knot $K \subset S^3$, we can view its complement $S^3 \setminus N(K)$ as a sutured manifold with its whole boundary being the suture. We can pick a maximal collection of pairwise disjoint and pairwise nonparallel minimal-genus Seifert surfaces S of K , and perform a sutured manifold decomposition

$$(1-1) \quad S^3 \setminus N(K) \xrightarrow{S} (M', \gamma').$$

We can then pick a maximal collection of pairwise disjoint and pairwise nonparallel nontrivial product annuli A inside (M', γ') and perform a second sutured manifold decomposition¹

$$(1-2) \quad (M', \gamma') \xrightarrow{A} (M, \gamma) \sqcup (M_1, \gamma_1).$$

Here (M_1, γ_1) is a product sutured manifold, and no component of (M, γ) is a product.

Definition 1.2 [1] The guts of a knot $K \subset S^3$ is defined to be the sutured manifold (M, γ) .

Theorem 1.3 [1, Theorem 1.1] *The guts of a knot $K \subset S^3$ is well defined, i.e., independent of the choices of maximal collections of Seifert surfaces and product annuli in the construction.*

In this note, we prove that a knot $K \subset S^3$ is nearly fibered if and only if its guts falls into one of the three basic models described below. We only state and prove the theorem in instanton theory, but a similar argument applies to Heegaard Floer theory as well.

Theorem 1.4 *Suppose $K \subset S^3$ is a knot of genus g . Let (M, γ) be its guts. Then we have*

$$\text{KHI}(S^3, K, g) \cong \mathbb{C}^2$$

if and only if its guts (M, γ) falls into one of the following three models up to orientation reversal of the ambient 3-manifold:

- (M1) M is a solid torus and γ consists of four longitudes.
- (M2) M is a solid torus and γ consists of two curves of slope 2.
- (M3) M is the complement of the right-handed trefoil and γ consists of two curves of slope 2.

We have the following corollary.

Corollary 1.5 *A knot is nearly fibered in the instanton sense if and only if it is nearly fibered in the Heegaard Floer sense.*

2 Proofs and comments

Proof of Theorem 1.4 We first prove the necessary condition. Suppose $K \subset S^3$ is a genus- g nearly fibered knot. Let (M, γ) be its guts. We first study the sutured manifold decomposition in (1-1).

Claim 1 Any maximal collection of pairwise disjoint and pairwise nonparallel minimal-genus Seifert surfaces contains only one Seifert surface.

¹Here, note that in Agol and Zhang's paper [1], they also require to decompose along nontrivial product disks to obtain the guts. In this paper we drop the step of decomposing along possible product disks because of [7, Lemma 2.13]: the only two taut balanced sutured manifolds that admit no nontrivial product annulus but admit nontrivial product disks are both product sutured manifolds, and hence are actually the components to be dropped when obtaining guts.

Proof of Claim 1 Suppose S is a minimal-genus Seifert surface of K . We can perform a sutured manifold decomposition of $S^3 \setminus N(K)$ along S :

$$S^3 \setminus N(K) \xrightarrow{S} (S^3 \setminus [-1, 1] \times S, \{0\} \times \partial S).$$

By the proof of [9, Proposition 7.16], we know that there is an isomorphism

$$(2-1) \quad \text{SHI}(S^3 \setminus [-1, 1] \times S, \{0\} \times \partial S) \cong \text{KHI}(S^3, K, g) \cong \mathbb{C}^2.$$

If there is another minimal-genus Seifert surface S' that is disjoint from S and is not parallel to S , then S' also induces a nonboundary parallel surface in $(S^3 \setminus [-1, 1] \times S, \{0\} \times \partial S)$, which implies the sutured manifold is not horizontally prime. From the instanton version of [9, Propositions 6.5 and 6.6], we know that one of the two pieces obtained from $(S^3 \setminus [-1, 1] \times S, \{0\} \times \partial S)$ by cutting along S' must have 1-dimensional sutured homology, because 2 is a prime number. From [9, Theorem 7.18], that piece is a product sutured manifold, which contradicts the assumption that S' is not parallel to S . \square

Now Claim 1 above and Theorem 1.3 imply that the sutured manifold (M', γ') in (1-1) can be taken to be simply the complement of S :

$$(M', \gamma') = (S^3 \setminus [-1, 1] \times S, \{0\} \times \partial S).$$

Next, we study the sutured manifold decomposition (1-2). Note that by construction (M_1, γ_1) is a product sutured manifold, so from [9, Theorem 7.18] and the instanton version of [9, Propositions 6.5 and 6.7], we know that

$$\text{SHI}(M, \gamma) \cong \text{SHI}(S^3 \setminus [-1, 1] \times S, \{0\} \times \partial S) \cong \mathbb{C}^2.$$

Also, the same argument as above shows that (M, γ) is horizontally prime. Thus we conclude that (M, γ) is reduced in the sense of [7, Definition 2.12]. Then [6, Corollary 1.16] applies and we conclude that

$$b_1(M) = b^1(M) \leq 2 - 1 = 1.$$

We claim that $g(\partial M) = b_1(M)$. Indeed, we know that (M, γ) is obtained from the knot complement by decomposition. So [7, Lemma 5.1] implies that $H_2(M) = 0$. As a result, by the universal coefficient theorem and the Poincaré duality, we have

$$H_1(M, \partial M; \mathbb{Q}) \cong H^2(M; \mathbb{Q}) \cong H_2(M; \mathbb{Q}) = 0.$$

Hence the long exact sequence of the pair $(M, \partial M)$ implies that the map

$$i_* : H_1(\partial M; \mathbb{Q}) \rightarrow H_1(M; \mathbb{Q})$$

is surjective. Hence the “half lives and half dies” theorem in 3-dimensional topology implies that

$$g(\partial M) = \frac{1}{2}b_1(\partial M) = b_1(M).$$

Now $g(\partial M) = b_1(M) \leq 1$. If $g(\partial M) = 0$, since M is irreducible, we know $M = B^3$. Then for any possible γ on ∂M , we cannot have $\text{SHI}(M, \gamma) \cong \mathbb{C}^2$. Hence we must have $\partial M \cong T^2$. It is well known that any (smooth) torus in S^3 bounds a solid torus. Hence we have two cases.

Case 1 (the manifold M is a solid torus) The instanton Floer homology of any sutured solid torus can be found in [11, Section 4.3]. So the only two models are the ones as stated in (M1) and (M2).

Case 2 (the manifold $S^3 \setminus M$ is a solid torus, i.e., there is a knot $J \subset S^3$ so that $M \cong S^3 \setminus N(J)$) Suppose γ has $2n$ components. Let γ_2 be the union of two adjacent components of γ , which are necessarily oppositely oriented. Next, we make the following claim.

Claim 2 Suppose (M, γ) is a balanced sutured manifold and assume that three components of γ are parallel disregarding the orientation. Write γ_3 to be the disjoint union of these three copies. Note that two components of γ_3 are coherently oriented and are opposite to the third. Let γ_1 be either of the two coherently oriented components and write $\gamma' = (\gamma \setminus \gamma_3) \cup \gamma_1$. Then we have

$$\text{SHI}(M, \gamma) = \text{SHI}(M, \gamma') \otimes \mathbb{C}^2.$$

Proof of Claim 2 The proof essentially follows from the proof of [8, Theorem 3.1]. There exists an embedded annulus $A \subset \partial M$ such that A contains γ_3 and each component of γ_3 is a core of A . Push the interior of A into the interior of M to produce a properly embedded annulus. Fix any orientation of A . Then there is a product annulus decomposition

$$(M, \gamma) \xrightarrow{A} (V, \gamma^4) \sqcup (M, \gamma'),$$

where V is a solid torus and γ^4 consists of four longitudes (there is a unique way, up to isotopy, to make (V, γ^4) a balanced sutured manifold). Now an instanton version of [9, Proposition 6.7] implies that

$$\text{SHI}(M, \gamma) = \text{SHI}(M, \gamma') \otimes \text{SHI}(V, \gamma^4).$$

In the proof of [8, Theorem 3.1], Kronheimer and Mrowka already computed that

$$\text{SHI}(V, \gamma^4) \cong \mathbb{C}^2. \quad \square$$

Applying Claim 2 repetitively, we conclude that

$$\text{SHI}(M, \gamma) \cong \mathbb{C}^{2^{n-1}} \otimes \text{SHI}(M, \gamma_2).$$

Since

$$\text{SHI}(M, \gamma) \cong \mathbb{C}^2,$$

either $n = 2$ and $\text{SHI}(M, \gamma_2) \cong \mathbb{C}$, or $n = 1$. For the former case, from [9, Theorem 7.18] we know M must also be a solid torus which reduces to Case 1. For the latter case, we further divide it into two subcases.

Case 2.1 (each component of γ represents a generator of $\ker i_* \subset H_1(\partial M) \cong \mathbb{Z}^2$, where

$$i_* : H_1(\partial M) \rightarrow H_1(M)$$

is the map induced by the natural inclusion

$$i : \partial M \hookrightarrow M$$

In this case, first recall that M is a knot complement $S^3 \setminus N(J)$ and hence $H_2(M, \partial M)$ is generated by a minimal-genus Seifert surface T of the knot J . The assumption of Case 2.1 is equivalent to that γ is parallel to $\partial T \subset \partial M$. We can assume that $\partial T \cap \gamma = \emptyset$. If T is a disk, then $R(\gamma)$ is compressible and $\text{SHI}(M, \gamma) = 0$ by the adjunction inequality (see [9, Proposition 7.5]). From now on we assume that T has genus at least 1. We know from [6, Lemma 6.2] that we have two taut decompositions

$$(M, \gamma) \overset{\pm T}{\rightsquigarrow} (M_{\pm}, \gamma_{\pm}).$$

We make the following claim.

Claim 3 We have an inclusion

$$\text{SHI}(M_+, \gamma_+) \oplus \text{SHI}(M_-, \gamma_-) \hookrightarrow \text{SHI}(M, \gamma).$$

Proof of Claim 3 We adopt the idea in [11, Section 3]. We isotope T to T^{\pm} such that the decomposition of (M, γ) along T^+ is (M_+, γ_+) and the decomposition of (M, γ) along $-T^-$ is (M_-, γ_-) . T^{\pm} are called positive and negative stabilizations of T as in [11, Definition 3.1], and we know that $-(T^-) = T^+$. By [11, Theorem 3.4], each T^{\pm} induces a \mathbb{Z} -grading on $\text{SHI}(M, \gamma)$. Then [11, Lemma 4.2] implies that

$$\text{SHI}(M, \gamma, T^+, g(T)) \cong \text{SHI}(M_+, \gamma_+)$$

and

$$\begin{aligned} \text{SHI}(M, \gamma, T^-, -g(T)) &= \text{SHI}(M, \gamma, -T^-, g(T)) \\ &\cong \text{SHI}(M_-, \gamma_-). \end{aligned}$$

Proposition 4.1 of [11] implies that²

$$\text{SHI}(M, \gamma, T^-, -g(T)) \subset \text{SHI}(M, \gamma, T^+, 1 - g(T)).$$

Hence we are done since $g(T) \neq 1 - g(T)$. □

Observe that both γ_+ and γ_- contain at least three components that are parallel to each other. Let γ'_{\pm} be the suture obtained from γ_{\pm} by replacing three copies with one copy. Applying Claim 2, we know that

$$\text{SHI}(M_{\pm}, \gamma_{\pm}) \cong \text{SHI}(M_{\pm}, \gamma'_{\pm}) \otimes \mathbb{C}^2.$$

Tautness together with [9, Theorem 7.12] then implies

$$\dim \text{SHI}(M_{\pm}, \gamma_{\pm}) \geq 2.$$

²When reversing the orientation of the manifold and the suture, positive and negative stabilizations of T are also switched.

As a result, we have

$$\dim \text{SHI}(M, \gamma) \geq 4,$$

which leads to a contradiction in this case.

Case 2.2 (components of γ do not represent generators of $\ker i_*$) Let Y be the Dehn filling of M along a component of γ . We make the following claim.

Claim 4 We have $\dim I^\sharp(Y) = 2$.

Proof of Claim 4 In order to prove Claim 4, we need the following three facts.

- (1) We have $\dim I^\sharp(Y) \neq 0$.
- (2) We have $\dim I^\sharp(Y) = \dim \text{SHI}(Y(1)) \leq \dim \text{SHI}(M, \gamma) = 2$.
- (3) We have $\dim I^\sharp(Y) \equiv \dim \text{SHI}(M, \gamma) \pmod{2}$.

To show (1), note that the fact $[\gamma] \notin \ker i_*$ implies that Y is a rational homology sphere. Hence by [15, Corollary 1.4], we know $\dim I^\sharp(Y) \neq 0$.

To show (2), recall that $M = S^3 \setminus N(J)$ is the knot complement and γ has two components. Let $\gamma_0 \subset \gamma$ be any component. We can attach a 3-dimensional 2-handle along γ_0 . The resulting manifold is $Y \setminus B^3$. Hence we have a balanced sutured manifold

$$Y(1) = (Y \setminus B^3, \gamma \setminus \gamma_0).$$

Now let T be the cocore arc of the 2-handle. This arc T is a vertical tangle inside $Y(1)$ as in [16, Definition 1.1]. Now observe that (M, γ) can be obtained from $Y(1)$ by removing T , i.e.,

$$Y(1)_T = (Y(1) \setminus N(T), (\gamma \setminus \gamma_0) \cup \mu_T) \cong (M, \gamma),$$

where μ_T is a meridian of T , and the assumption of Case 2.2 implies $[T] = 0 \in H_1(Y \setminus B^3, \partial(Y \setminus B^3); \mathbb{Q})$. Then by [12, Proposition 1.4] we conclude that

$$\dim I^\sharp(Y) = \dim \text{SHI}(Y(1)) \leq \dim \text{SHI}(M, \gamma) = 2.$$

To show (3), we need to unpack the proof of [12, Proposition 1.4], which is ultimately the proof of [12, Proposition 3.14]. We view [12, Proposition 1.4] as a special case of [12, Proposition 3.14] when $T_0 = \emptyset$. Equation (3.2) of [12] implies that there are sutures Γ_{n-1} and Γ_n such that we have an exact triangle

$$(2-2) \quad \begin{array}{ccc} \text{SHI}(-M, -\Gamma_{n-1}) & \xrightarrow{\quad\quad\quad} & \text{SHI}(-M, -\Gamma_n) \\ & \swarrow \quad \quad \quad \searrow & \\ & \text{SHI}(-M, -\gamma) & \end{array}$$

And [12, Lemma 3.21] can be rewritten (by replacing n in the original equation by $n - 1$) as

$$(2-3) \quad \begin{array}{ccc} \text{SHI}(-M, -\Gamma_{n-1}) & \xrightarrow{\hspace{10em}} & \text{SHI}(-M, -\Gamma_n) \\ & \swarrow \hspace{2em} \searrow & \\ & I^\sharp(-Y) \cong \text{SHI}(-Y(1)) & \end{array}$$

Hence some basic linear algebra together with (2-2) and (2-3) implies that

$$\dim I^\sharp(-Y) \equiv \dim \text{SHI}(-M, -\gamma) \pmod{2}.$$

As in [10, Theorem 1.2], we know $\text{SHI}(M, \gamma)$ and $\text{SHI}(-M, \gamma)$ are naturally dual to each other. Since $\partial M \cong T^2$, we know γ and $-\gamma$ are isotopic, we conclude that

$$\dim \text{SHI}(-M, -\gamma) = \dim \text{SHI}(M, \gamma).$$

A similar argument applies to $\text{SHI}(-Y(1))$. □

Recall $M = S^3 \setminus N(J)$ is a knot complement and Y is obtained from M by filling along a component of γ , and hence Y can be viewed as a Dehn surgery along J . The assumption of Case 2.2 implies that the surgery slope is nonzero. By passing to the mirror of J , which corresponds to reversing the orientation of M , we can assume that the surgery slope is positive. By Claim 4, we know that

$$\dim I^\sharp(Y) = 2 = |H_1(Y)|.$$

Note that by [3, Theorem 1.15], the unknot and the right-handed trefoil are the only two knots on which the positive Dehn surgeries induce instanton L-spaces Y with $|H_1(Y)| = 2$. (According to the theorem, such a knot must be fibered and has genus at most 1 and thus must be either the unknot, the trefoil, or the figure eight. Note the last knot is not strongly quasipositive.) The case of unknot still reduces to Case 1. The case of the right-handed trefoil is a new one. By [2, Theorem 1.1, Table 1], the surgery slope must be 2. Hence γ consists of curves of slope 2, which concludes the proof of the necessary condition.

Finally, the sufficient condition follows immediately from the first isomorphism in (2-1) and the fact that gluing a product sutured manifold other than a 3-ball to an arbitrary sutured manifold via identification of a suture does not change the sutured instanton Floer homology (see [9, Proposition 6.7]). □

Remark 2.1 We have the following comments which strengthen the description of the guts in Theorem 1.4.

(1) We can compute the Euler characteristic in each of the three models. From [8], for the first model,

$$\chi(\text{KHI}(S^3, K, g(K))) = \chi(\text{SHI}(M, \gamma)) = 0.$$

As a result, we know that the symmetrized Alexander polynomial $\Delta_K(t)$ of K has degree at most $g(K) - 1$. On the other hand, if (M, γ) is one of the other two models, we can compute as in [13] that

$$\chi(\text{SHI}(M, \gamma)) = \pm 2.$$

As a result, we know that $\Delta_K(t)$ has degree $g(K)$ and the top nonzero coefficients are ± 2 .

(2) Let S be a minimal genus Seifert surface of the knot $K \subset S^3$. Recall as in (1-1) and (1-2), we have a decomposition

$$S^3 \setminus N(K) \xrightarrow{S} (M', \gamma') \xrightarrow{A} (M, \gamma) \sqcup (M_1, \gamma_1),$$

where (M_1, γ_1) is a product sutured manifold and (M, γ) is the guts. We write

$$(M_1, \gamma_1) = ([-1, 1] \times F, \{0\} \times \partial F).$$

The proof of [4, Lemma 3.4] implies that the Seifert surface complement $(S^3 \setminus [-1, 1] \times S, \{0\} \times \partial S)$ admits no product annuli whose boundary has a component that is parallel to the suture $\{0\} \times \partial S$ on $\partial(S^3 \setminus [-1, 1] \times S)$. As a result, we can further conclude that ∂F must have one more component than γ , and all but one components of ∂F are glued to all of γ . This actually rules out one model in the case $g(K) = 1$ as in the following example.

Example 2.2 We keep the notation as in Remark 2.1. When $g(K) = 1$, we know that

$$S \cong (R_+(\gamma) \cup \{1\} \times F).$$

Since in all three models we have $\chi(R_+(\gamma)) = 0$, we know that

$$\chi(F) = -1.$$

From part (2) of the Remark 2.1, we know that ∂F has one more component than γ . Then $\chi(F)$ rules out the model in which γ has four components. As a result, we only have two models:

- M is the complement of the unknot and γ consists of two curves of slope 2.
- M is the complement of the right-handed trefoil and γ consists of two curves of slope 2.

Furthermore, in this case, the surface F must be a pair of pants. Yet gluing such a thickened pair of pants to (M, γ) along two of the three boundary components is equivalent to gluing a product 1-handle to (M, γ) . Turning this around, we know that (M, γ) being one of the above two models is obtained from the complement of the Seifert surface by a disk decomposition. This coincides with the discussion in [4, Section 1.2] right above [4, Theorem 5.1]. Note that these two models do exist: for example, they give rise to the knot 5_2 in Rolfsen's table and the 2-twisted Whitehead double of the right-handed trefoil with positive clasp.

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References

- [1] **I Agol, Y Zhang**, *Guts in sutured decompositions and the Thurston norm* (2022) arXiv 2203.12095
- [2] **J A Baldwin, S Sivek**, *Framed instanton homology and concordance*, *J. Topol.* 14:4 (2021) 1113–1175 MR
- [3] **J A Baldwin, S Sivek**, *Instantons and L-space surgeries*, *J. Eur. Math. Soc.* 25:10 (2023) 4033–4122 MR

- [4] **J A Baldwin, S Sivek**, *Floer homology and non-fibered knot detection*, *Forum Math. Pi* 13 (2025) art. id. e1 MR
- [5] **P Ghiggini**, *Knot Floer homology detects genus-one fibred knots*, *Amer. J. Math.* 130:5 (2008) 1151–1169 MR
- [6] **S Ghosh, Z Li**, *Decomposing sutured monopole and instanton Floer homologies*, *Selecta Math. (N.S.)* 29:3 (2023) art. id. 40 MR
- [7] **A Juhász**, *The sutured Floer homology polytope*, *Geom. Topol.* 14:3 (2010) 1303–1354 MR
- [8] **P Kronheimer, T Mrowka**, *Instanton Floer homology and the Alexander polynomial*, *Algebr. Geom. Topol.* 10:3 (2010) 1715–1738 MR
- [9] **P Kronheimer, T Mrowka**, *Knots, sutures, and excision*, *J. Differential Geom.* 84:2 (2010) 301–364 MR
- [10] **Z Li**, *Gluing maps and cobordism maps in sutured monopole and instanton Floer theories*, *Algebr. Geom. Topol.* 21:6 (2021) 3019–3071 MR
- [11] **Z Li**, *Knot homologies in monopole and instanton theories via sutures*, *J. Symplectic Geom.* 19:6 (2021) 1339–1420 MR
- [12] **Z Li, F Ye**, *Instanton Floer homology, sutures, and Heegaard diagrams*, *J. Topol.* 15:1 (2022) 39–107 MR
- [13] **Z Li, F Ye**, *Instanton Floer homology, sutures, and Euler characteristics*, *Quantum Topol.* 14:2 (2023) 201–284 MR
- [14] **Y Ni**, *Knot Floer homology detects fibred knots*, *Invent. Math.* 170:3 (2007) 577–608 MR
- [15] **C W Scaduto**, *Instantons and odd Khovanov homology*, *J. Topol.* 8:3 (2015) 744–810 MR
- [16] **Y Xie, B Zhang**, *Instanton Floer homology for sutured manifolds with tangles*, *J. Differential Geom.* 130:3 (2025) 701–769 MR

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