# Algebra & Number Theory

Volume 5
2011
No. 2

## Correction to a proof in the article Patching and admissibility over two-dimensional complete local domains

Danny Neftin and Elad Paran

msp

ALGEBRA AND NUMBER THEORY 5:2 (2011)
https://doi.org/10.2140/ant.2011.5.287

# Correction to a proof in the article Patching and admissibility over two-dimensional complete local domains

Danny Neftin and Elad Paran

Volume **4**:6 (2010), 743–762

The proof of Lemma 1.8 of the article in the title is incorrect. We supply an alternate argument for Proposition 1.10, whose proof invoked that lemma.

We are grateful to Yong Hu for pointing out to us a gap in the proof of Lemma 1.8 of our article "Patching and admissibility over two-dimensional complete local domains", namely, the isomorphism $R/\mathfrak{p} \cong S/\mathfrak{q}$ implies only that $S = R + \mathfrak{q}$ and not $S = R + \mathfrak{p}S$, as required for this argument.

The lemma is applied for the rings

$$R_0 = D_{J\cup J'}, \quad R_1 = D_J, \quad R_2 = D_{J'}, \quad R = D_\varnothing,$$

where $J \cap J' = \varnothing$, to show that $S := R_1 \cap R_2 = R$. Let us show this assertion directly. In particular, this will trivially imply that for these rings $\mathfrak{q} = \mathfrak{p}S$.

*All references are to the article in question.*

Recall that $I$ is a finite set and that $v$ is the extension of the order function of the ideal $\mathfrak{p} := (x, y) \lhd K[x, y]$ to $K(x, y)$. For $i \in I$, let $z_i = y/(x - c_i y)$ and for a subset $J \subset I$, $D_J$ is defined as the completion of $K[z_j \mid j \in J][x, y]$ with respect to $v$.

**Lemma.** *Let $i, j \in I$ be two distinct indices. Then $D_{\{i\}} \cap D_{\{j\}} = D_\varnothing$.*

*Proof.* By Proposition 1.5, $D_{\{i\}} = K[z_i][[x - c_i y]]$ and hence an element $f \in D_{\{i\}}$ can be written as $\sum_{k=0}^{\infty} f_k(z_i)(x - c_i y)^k$ for some polynomials $f_k$, $k \geq 0$. Assume $f \in D_{\{i,j\}}$ can also be written as $\sum_{k=0}^{\infty} g_k(z_j)(x - c_j y)^k \in D_{\{j\}} = K[z_j][[x - c_j y]]$, where $g_k$ are polynomials for $k \geq 0$.

In particular,

$$f_k(z_i)(x - c_i y)^k = g_k(z_j)(x - c_j y)^k \pmod{\mathfrak{p}^{k+1} D_{\{i,j\}}} \tag{1}$$

*MSC2010:* primary 12F12; secondary 12E30, 12E15, 12F10.
*Keywords:* patching, crossed product, admissible groups, division algebras, complete local domains.

for all $k \geq 0$. We claim that equality (1) in fact holds in $D_{\{i,j\}}$. Indeed, since $x - c_i y = \big(1 + (c_j - c_i)z_j\big)(x - c_j y)$, the difference between the sides of (1) is

$$\big(f_k(z_i)(1 + (c_j - c_i)z_j)^k - g_k(z_j)\big)(x - c_j y)^k \in K[z_i, z_j](x - c_j y)^k. \tag{2}$$

Since $\mathfrak{p}$ is contained in the center of the valuation $v$, Proposition 1.5 implies that the difference (2) is in $\mathfrak{p}^{k+1} D_{\{i,j\}}$ only if it is zero, proving the claim.

By finding a common denominator, one can write an element $f_k(z_i)(x - c_i y)^k$ as $p_k(x, y)/(x - c_i y)^m$ where $m \geq 0$ and $p_k$ is a homogenous polynomial of degree $k + m$ that is prime to $(x - c_i y)^m$. Writing $g_k(z_j)(x - c_j y)^k = q_k(x, y)/(x - c_j y)^l$ for $l \geq 0$ and $q_k$ a homogenous polynomial of degree $k + l$ that is prime to $(x - c_i y)^l$, the equality

$$\frac{p_k(x, y)}{(x - c_i y)^m} = \frac{q_k(x, y)}{(x - c_j y)^l}$$

implies that $m = l = 0$ and hence that $f_k(z_i)(x - c_i y)^k \in K[x, y]$ for all $k \geq 0$. It follows that $f = \sum_{k=0}^{\infty} f_k(z_i)(x - c_i y)^k \in K[\![x, y]\!]$, as required. □

Let us complete the proof of Proposition 1.10:

**Proposition.** *Suppose $J, J' \subseteq I$. Then $D_J \cap D_{J'} = D_{J \cap J'}$.*

*Proof.* Clearly $D_{J \cap J'} \subseteq D_J \cap D_{J'}$. For the converse inclusion, we distinguish between two cases. First suppose that $J \cap J' \neq \varnothing$ and fix $j \in J \cap J'$. Then $D_J = K[z_k \mid k \in J][\![x - c_j y]\!]$, $D_{J'} = K[z_k \mid k \in J'][\![x - c_j y]\!]$ and hence

$$D_J \cap D_{J'} = (K[z_k \mid k \in J] \cap K[z_k \mid k \in J'])[\![x - c_j y]\!].$$

By Lemma 1.9, $K[z_k \mid k \in J] \cap K[z_k \mid k \in J'] = K[z_k \mid k \in J \cap J']$.

Now suppose that $J \cap J' = \varnothing$. If $|J| = |J'| = 1$, then the claim follows from the Lemma. Assume without loss of generality $|J| \geq 2$. For distinct $j_1, j_2 \in J$, we have by the previous case $D_J \cap D_{J' \cup \{j_i\}} = D_{\{j_i\}}$, for $i = 1, 2$. In particular, $D_J \cap D_{J'} \subseteq D_{\{j_1\}} \cap D_{\{j_2\}}$. By the Lemma, $D_{\{j_1\}} \cap D_{\{j_2\}} = D_\varnothing$ implying that $D_J \cap D_{J'} = D_\varnothing$ as required. □

Communicated by Jean-Louis Colliot-Thélène
Received 2011-07-06 Revised 2011-07-13 Accepted 2011-08-10

neftin@umich.edu Department of Mathematics, University of Michigan, Ann Arbor, 530 Church St., Ann Arbor 48109, United States

paranela@post.tau.ac.il School of Mathematical Sciences, Tel Aviv University, Ramat Aviv, 69978 Tel Aviv, Israel
http://www.tau.ac.il/~paranela/

# Algebra & Number Theory

www.jant.org



See inside back cover or www.jant.org for submission instructions.

The subscription price for 2011 is US $150/year for the electronic version, and $210/year (+$35 shipping outside the US) for print and electronic. Subscriptions, requests for back issues from the last three years and changes of subscribers address should be sent to Mathematical Sciences Publishers, Department of Mathematics, University of California, Berkeley, CA 94720-3840, USA.

Algebra & Number Theory (ISSN 1937-0652) at Mathematical Sciences Publishers, Department of Mathematics, University of California, Berkeley, CA 94720-3840 is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

ANT peer review and production are managed by EditFLOW™ from Mathematical Sciences Publishers.

PUBLISHED BY
mathematical sciences publishers
http://msp.org/
A NON-PROFIT CORPORATION
Typeset in LaTeX

Algebra & Number Theory

Volume 5 No. 2 2011



1937-0652(2011)5:2;1-F