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Correction to a proof in the article Patching and admissibility over two-dimensional complete local domains

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The proof of Lemma 1.8 of the article in the title is incorrect. We supply an alternate argument for Proposition 1.10, whose proof invoked that lemma.

We are grateful to Yong Hu for pointing out to us a gap in the proof of Lemma 1.8 of our article "Patching and admissibility over two-dimensional complete local domains", namely, the isomorphism $R/\mathfrak{p} \cong S/\mathfrak{q}$ implies only that $S = R + \mathfrak{q}$ and not $S = R + \mathfrak{p}S$, as required for this argument.

The lemma is applied for the rings

$$R_0 = D_{J \cup J'}, \quad R_1 = D_J, \quad R_2 = D_{J'}, \quad R = D_\varnothing,$$

where $J \cap J' = \varnothing$, to show that $S := R_1 \cap R_2 = R$. Let us show this assertion directly. In particular, this will trivially imply that for these rings $\mathfrak{q} = \mathfrak{p}S$.

All references are to the article in question.

Recall that I is a finite set and that v is the extension of the order function of the ideal $\mathfrak{p} := (x, y) \lhd K[x, y]$ to $K(x, y)$. For $i \in I$, let $z_i = y/(x - c_i y)$ and for a subset $J \subset I$, D_J is defined as the completion of $K[z_j \mid j \in J][x, y]$ with respect to v.

Lemma. *Let $i, j \in I$ be two distinct indices. Then $D_{\{i\}} \cap D_{\{j\}} = D_\varnothing$.*

Proof. By Proposition 1.5, $D_{\{i\}} = K[z_i][[x - c_i y]]$ and hence an element $f \in D_{\{i\}}$ can be written as $\sum_{k=0}^{\infty} f_k(z_i)(x - c_i y)^k$ for some polynomials f_k, $k \geq 0$. Assume $f \in D_{\{i,j\}}$ can also be written as $\sum_{k=0}^{\infty} g_k(z_j)(x - c_j y)^k \in D_{\{j\}} = K[z_j][[x - c_j y]]$, where g_k are polynomials for $k \geq 0$.

In particular,

$$f_k(z_i)(x - c_i y)^k = g_k(z_j)(x - c_j y)^k \pmod{\mathfrak{p}^{k+1} D_{\{i,j\}}} \tag{1}$$

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for all $k \geq 0$. We claim that equality (1) in fact holds in $D_{\{i,j\}}$. Indeed, since $x - c_i y = \big(1 + (c_j - c_i)z_j\big)(x - c_j y)$, the difference between the sides of (1) is

$$\big(f_k(z_i)(1 + (c_j - c_i)z_j)^k - g_k(z_j)\big)(x - c_j y)^k \in K[z_i, z_j](x - c_j y)^k. \quad (2)$$

Since $\mathfrak{p}$ is contained in the center of the valuation v, Proposition 1.5 implies that the difference (2) is in $\mathfrak{p}^{k+1} D_{\{i,j\}}$ only if it is zero, proving the claim.

By finding a common denominator, one can write an element $f_k(z_i)(x - c_i y)^k$ as $p_k(x, y)/(x - c_i y)^m$ where $m \geq 0$ and p_k is a homogenous polynomial of degree $k+m$ that is prime to $(x - c_i y)^m$. Writing $g_k(z_j)(x - c_j y)^k = q_k(x, y)/(x - c_j y)^l$ for $l \geq 0$ and q_k a homogenous polynomial of degree $k+l$ that is prime to $(x - c_i y)^l$, the equality

$$\frac{p_k(x, y)}{(x - c_i y)^m} = \frac{q_k(x, y)}{(x - c_j y)^l}$$

implies that $m = l = 0$ and hence that $f_k(z_i)(x - c_i y)^k \in K[x, y]$ for all $k \geq 0$. It follows that $f = \sum_{k=0}^{\infty} f_k(z_i)(x - c_i y)^k \in K[\![x, y]\!]$, as required. □

Let us complete the proof of Proposition 1.10:

Proposition. *Suppose $J, J' \subseteq I$. Then $D_J \cap D_{J'} = D_{J \cap J'}$.*

Proof. Clearly $D_{J \cap J'} \subseteq D_J \cap D_{J'}$. For the converse inclusion, we distinguish between two cases. First suppose that $J \cap J' \neq \varnothing$ and fix $j \in J \cap J'$. Then $D_J = K[z_k \mid k \in J][\![x - c_j y]\!]$, $D_{J'} = K[z_k \mid k \in J'][\![x - c_j y]\!]$ and hence

$$D_J \cap D_{J'} = (K[z_k \mid k \in J] \cap K[z_k \mid k \in J'])[\![x - c_j y]\!].$$

By Lemma 1.9, $K[z_k \mid k \in J] \cap K[z_k \mid k \in J'] = K[z_k \mid k \in J \cap J']$.

Now suppose that $J \cap J' = \varnothing$. If $|J| = |J'| = 1$, then the claim follows from the Lemma. Assume without loss of generality $|J| \geq 2$. For distinct $j_1, j_2 \in J$, we have by the previous case $D_J \cap D_{J' \cup \{j_i\}} = D_{\{j_i\}}$, for $i = 1, 2$. In particular, $D_J \cap D_{J'} \subseteq D_{\{j_1\}} \cap D_{\{j_2\}}$. By the Lemma, $D_{\{j_1\}} \cap D_{\{j_2\}} = D_{\varnothing}$ implying that $D_J \cap D_{J'} = D_{\varnothing}$ as required. □

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