ERRATA FOR "NEF CONES OF HILBERT SCHEMES OF POINTS ON SURFACES"

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In Section 5 it was erroneously claimed that the Weyl group acts transitively on the extremal rays of the nef cone of a del Pezzo surface X of degree 1. In fact, there are two orbits.

Proposition. *For $k \geq 2$, let X_k be the del Pezzo surface $X_k \cong \mathrm{Bl}_{p_1,\ldots,p_k} \mathbb{P}^2$, where $p_1,\ldots,p_k$ are distinct points such that $-K_{X_k}$ is ample. Then the Weyl group acts on the extremal rays of $\mathrm{Nef}(X_k)$, and there are two orbits. The classes H and $H-E_1$ are orbit representatives.*

Proof. Since the Weyl group preserves the intersection pairing and H and $H-E_1$ are extremal nef divisors with different self-intersections, there are at least two orbits.

For $k=2$, the nef cone is spanned by $H, H-E_1, H-E_2$. The Weyl group $\mathbb{Z}/2\mathbb{Z}$ fixes H and exchanges $H-E_1$ and $H-E_2$, so there are two orbits.

Now suppose $k>2$ and N is an extremal nef divisor on X_k. Then N is orthogonal to a face of $\mathrm{NE}(X)$, so there is a (-1)-curve orthogonal to N. Since $k>2$, we may use the Weyl group to assume this (-1)-curve is E_k. Then N is a pullback $N=\pi^*N'$ along the blowdown map $\pi : X_k \to X_{k-1}$ contracting E_k. Since N is an extremal nef divisor on X_k, N' is an extremal nef divisor on X_{k-1}: a nontrivial decomposition $N' = A+B$ with A, B nef would pullback to a nontrivial decomposition of N. Continuing in this fashion we see that up to the action of the Weyl group N is the pullback of H or $H-E_1$ from X_2. □

In Section 5B, we may then assume the nef class N is either $H-E_1$ or H; the analysis of this second possibility must also be carried out. The corresponding $F_{[n]}$-orthognal ray is

$$(n-1)(-K_X)^{[n]} + \frac{1}{3}H^{[n]} - \frac{1}{2}B,$$

and we have to show this ray is nef. If we choose

$$Q = \left(n - \frac{3}{2}\right)(-K_X) + \frac{1}{3}H,$$

this amounts to showing that the Gieseker wall for the (Q, K_X)-slice has center $(s_W, 0) = (-1, 0)$. This is easily proved by arguments identical to those used for the (P, K_X)-slice in section 5C.

After these modifications, all the stated results in Section 5 remain true without change.

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