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Correction to the article  
Finite generation of the cohomology of  
some skew group algebras

Van C. Nguyen and Sarah Witherspoon





# Correction to the article Finite generation of the cohomology of some skew group algebras

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For the class of examples in Section 5 of the article in question, the proof of finite generation of cohomology is incomplete. We give here a proof of existence of a polynomial subalgebra needed there. The rest of the proof of finite generation given by the authors then applies.

Let  $k$  be a field of characteristic  $p > 2$ . Let  $A$  be the augmented  $k$ -algebra generated by  $a$  and  $b$ , with relations

$$a^p = 0, \quad b^p = 0, \quad ba = ab + \frac{1}{2}a^2,$$

and augmentation  $\varepsilon : A \rightarrow k$  given by  $\varepsilon(a) = \varepsilon(b) = 0$ . Let  $G$  be a cyclic group of order  $p$  with generator  $g$ , acting on  $A$  by

$$g(a) = a, \quad g(b) = a + b.$$

The corresponding skew group algebra  $A\#kG$  is a pointed Hopf algebra described in [Cibils et al. 2009, Corollary 3.14]. We remark that in Section 4 of the article we are correcting, referred to as [NW 2014], we used the left  $G$ -module structure with  $g(a) = a$  and  $g(b) = b - a$ , whereas the authors in [Cibils et al. 2009; Nguyen et al. 2017] used the right  $G$ -module structure given as above. We will apply the results in [Nguyen et al. 2017] to prove that the cohomology  $H^*(A\#kG, k) := \text{Ext}_{A\#kG}^*(k, k)$  is finitely generated, and this will fill a gap in the proof in [NW 2014, Section 5]. Thus we will now also adopt the choices of group actions in [Cibils et al. 2009; Nguyen et al. 2017] instead of that in [NW 2014]. This change does not affect the results discussed in [NW 2014, Section 4].

Let  $k$  be an  $A\#kG$ -module via the augmentation map  $\varepsilon$ . To prove finite generation of  $H^*(A\#kG, k)$ , we wish to apply [NW 2014, Theorem 3.1]. We use results in [Nguyen et al. 2017], where the notation is slightly different, with  $x$  in place of  $a$  and  $y$  in place of  $b$ . There it is shown that there are 2-cocycles  $\xi_a, \xi_b$  in  $H^*(A, k)$  generating a polynomial subring  $k[\xi_a, \xi_b]$ . These 2-cocycles are not both  $G$ -invariant, as was claimed in [NW 2014]; specifically, in [Nguyen et al. 2017] it is shown that  $\xi_a$  is  $G$ -invariant while  $\xi_b$  is not. The claimed  $G$ -invariance was used in [NW 2014, Section 5] to show that  $\xi_a$  and  $\xi_b$  are

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in the image  $\text{Im}(\text{res}_{A\#kG,A})$  of the restriction map from  $H^*(A\#kG, k)$  to  $H^*(A, k)$ . However, results in [Nguyen et al. 2017, Section 5.1] imply directly that  $\xi_a, \xi_b$  are in  $\text{Im}(\text{res}_{A\#kG,A})$ ; the needed elements in  $H^*(A\#kG, k)$  are constructed explicitly using a twisted tensor product resolution in [Nguyen et al. 2017, Section 3.3]. Now the rest of the finite generation proof in [NW 2014, Section 5] can proceed as before, since it is shown there that the rest of the hypotheses of [NW 2014, Theorem 3.1] are satisfied. An alternative proof is given in [Nguyen et al. 2017, Section 5.1].

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