Algebra & Number Theory

Volume 12
2018
No. 2

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ALGEBRA AND NUMBER THEORY 12:2 (2018)
dx.doi.org/10.2140/ant.2018.12.491

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Volume **8**:7 (2014), 1647–1657

For the class of examples in Section 5 of the article in question, the proof of finite generation of cohomology is incomplete. We give here a proof of existence of a polynomial subalgebra needed there. The rest of the proof of finite generation given by the authors then applies.

Let k be a field of characteristic $p > 2$. Let A be the augmented k-algebra generated by a and b, with relations

$$a^p = 0, \quad b^p = 0, \quad ba = ab + \tfrac{1}{2}a^2,$$

and augmentation $\varepsilon : A \to k$ given by $\varepsilon(a) = \varepsilon(b) = 0$. Let G be a cyclic group of order p with generator g, acting on A by

$$g(a) = a, \quad g(b) = a + b.$$

The corresponding skew group algebra $A\#kG$ is a pointed Hopf algebra described in [Cibils et al. 2009, Corollary 3.14]. We remark that in Section 4 of the article we are correcting, referred to as [NW 2014], we used the left G-module structure with $g(a) = a$ and $g(b) = b - a$, whereas the authors in [Cibils et al. 2009; Nguyen et al. 2017] used the right G-module structure given as above. We will apply the results in [Nguyen et al. 2017] to prove that the cohomology $\mathrm{H}^*(A\#kG, k) := \mathrm{Ext}^*_{A\#kG}(k, k)$ is finitely generated, and this will fill a gap in the proof in [NW 2014, Section 5]. Thus we will now also adopt the choices of group actions in [Cibils et al. 2009; Nguyen et al. 2017] instead of that in [NW 2014]. This change does not affect the results discussed in [NW 2014, Section 4].

Let k be an $A\#kG$-module via the augmentation map ε. To prove finite generation of $\mathrm{H}^*(A\#kG, k)$, we wish to apply [NW 2014, Theorem 3.1]. We use results in [Nguyen et al. 2017], where the notation is slightly different, with x in place of a and y in place of b. There it is shown that there are 2-cocycles ξ_a, ξ_b in $\mathrm{H}^*(A, k)$ generating a polynomial subring $k[\xi_a, \xi_b]$. These 2-cocycles are not both G-invariant, as was claimed in [NW 2014]; specifically, in [Nguyen et al. 2017] it is shown that ξ_a is G-invariant while ξ_b is not. The claimed G-invariance was used in [NW 2014, Section 5] to show that ξ_a and ξ_b are

MSC2010: primary 16E40; secondary 16T05.
Keywords: cohomology, Hopf algebras, skew group algebras.

in the image $\mathrm{Im}(\mathrm{res}_{A\#kG,A})$ of the restriction map from $\mathrm{H}^*(A\#kG, k)$ to $\mathrm{H}^*(A, k)$. However, results in [Nguyen et al. 2017, Section 5.1] imply directly that ξ_a, ξ_b are in $\mathrm{Im}(\mathrm{res}_{A\#kG,A})$; the needed elements in $\mathrm{H}^*(A\#kG, k)$ are constructed explicitly using a twisted tensor product resolution in [Nguyen et al. 2017, Section 3.3]. Now the rest of the finite generation proof in [NW 2014, Section 5] can proceed as before, since it is shown there that the rest of the hypotheses of [NW 2014, Theorem 3.1] are satisfied. An alternative proof is given in [Nguyen et al. 2017, Section 5.1].

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Communicated by Susan Montgomery
Received 2017-10-27 Accepted 2018-02-15

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Algebra & Number Theory

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Algebra & Number Theory (ISSN 1944-7833 electronic, 1937-0652 printed) at Mathematical Sciences Publishers, 798 Evans Hall #3840, c/o University of California, Berkeley, CA 94720-3840 is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

ANT peer review and production are managed by EditFLOW® from MSP.

PUBLISHED BY

mathematical sciences publishers

nonprofit scientific publishing

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Algebra & Number Theory

Volume 12 No. 2 2018

1937-0652(2018)12:2;1-F