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Irreducible characters with bounded root Artin conductor

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We prove that the best possible lower bound for the Artin conductor is exponential in the degree.

1. Introduction

Let K be an algebraic number field such that $K/\mathbb{Q}$ is Galois and let χ be the character of a linear representation of $\mathrm{Gal}(K/\mathbb{Q})$. We denote by f_χ the Artin conductor of χ. Odlyzko [1977] found lower bounds for f_χ by applying analytic methods to the Artin L-function. We have improved Odlyzko's lower bounds in [Pizarro-Madariaga 2011] by using explicit formulas for Artin L-functions. In particular, if χ is an irreducible character of $\mathrm{Gal}(K/\mathbb{Q})$ by assuming that $\chi\bar{\chi}$ satisfies the Artin conjecture, we obtained

$$f_\chi^{1/\chi(1)} \geq 4.73(1.648)^{(a_\chi - b_\chi)^2/\chi(1)^2} e^{-(13.34/\chi(1))^2},$$

where a_χ and b_χ are nonnegative integers giving the Γ-factors of the completed Artin L-function. Namely, $a_\chi + b_\chi = \chi(1)$ and $a_\chi - b_\chi = \chi(\sigma)$, with $\sigma \in \mathrm{Gal}(K/\mathbb{Q})$ the complex conjugation. This bound is even better when we assume that $L(s, \chi\bar{\chi})$ satisfies the generalized Riemann hypothesis. We have to point out that, throughout this article, no additional hypothesis are needed.

A natural question now is how far from being optimal these bounds are. This problem has been studied for the discriminant of a number field. If $n_0 = r_1 + 2r_2$, let d_n be the minimal discriminant of the field K with degree n such that n is a multiple of n_0 and $r_1(K)$ and $r_2(K)$ are in the same ratio as r_1, r_2. Let $\alpha(r_1, r_2) = \liminf_{n\to\infty} d_n^{1/n}$. Martinet [1978] considered number fields with infinite 2-class field towers and proved that

$$\alpha(0, 1) < 93 \quad \text{and} \quad \alpha(1, 0) < 1059.$$

In this work, we follow this idea and consider a number field K with infinite p-class field tower for some prime p. Under some technical conditions on K, we find an upper bound (depending only on K) for the root Artin conductor of the irreducible characters of $\mathrm{Gal}(K_n/\mathbb{Q})$ (given by $f_\chi^{1/\chi(1)}$), where K_n is the Hilbert p-class field of K_{n-1} with $K_0 = K$.

This work is organized as follow. In Section 2, we propose a technique obtained from Clifford's theory which is useful to classify the irreducible characters of $\mathrm{Gal}(K_n/\mathbb{Q})$ in terms of a certain normal subgroup.

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Keywords: Artin character, Artin conductor, Hilbert class field tower.

This characterization is convenient in order to obtain upper bounds for root Artin conductors. In Section 3, we conclude that there exists an infinite sequence $\{\chi_n\}$ of irreducible Artin characters with $\chi_n(1) \to \infty$ and such that $f_{\chi_n}^{1/\chi_n(1)} \le C$, where $C > 0$ is an effective computable constant. In Section 4, we apply the results obtained in Section 2 and 3 to the number field $K = \mathbb{Q}(\zeta_{11} + \zeta_{11}^{-1}, \sqrt{2}, \sqrt{-23})$. This field was found by Martinet [1978] and has infinite 2-class field tower and lowest known discriminant. In particular, we prove that for each $n \ge 1$ it is possible to find an irreducible character of $\mathrm{Gal}(K_n/\mathbb{Q})$ with large degree and

$$f_\chi^{1/\chi(1)} \le C, \quad \text{where } C \le 11^4 \cdot 2^{15} \cdot 23.$$

2. Irreducible characters of large degree

In this section, we develop a technique to classify the irreducible characters of groups with a normal subgroup of prime index. Also, by using a result from [Isaacs 1976], we obtain conditions that ensure the existence of irreducible characters of large degree. We believe these results are of independent interest.

Let us consider a finite group G and a normal subgroup H of G. We denote the set of irreducible characters of G by $\mathrm{Irr}(G)$. If χ and θ are characters of G and H respectively, we denote the restriction of χ to H by $\mathrm{Res}_H^G \chi$ and the induced character of θ to G by $\mathrm{Ind}_H^G \theta$. If $\theta \in \mathrm{Irr}(H)$, we define the conjugate character to θ in G by $\theta^g : H \to \mathbb{C}$, where $\theta^g(h) = \theta(ghg^{-1})$. The inertia group of θ in G is given by

$$I_G(\theta) = \{g \in G : \theta^g = \theta\}.$$

G acts on $\mathrm{Irr}(H)$ by conjugation and $I_G(\theta)$ is the stabilizer of θ under this action. The next result of Clifford will be the main argument allowing us to give a classification of the irreducible characters of G.

Theorem 1 (Clifford, [Huppert 1998, page 253]). *Let H be a normal subgroup of G and $\theta \in \mathrm{Irr}(H)$, $\chi \in \mathrm{Irr}(G)$ such that θ is an irreducible constituent of $\mathrm{Res}_H^G \chi$, with $\langle \mathrm{Res}_H^G \chi, \theta\rangle = e > 0$. Suppose that $\theta = \theta^{g_1}, \theta^{g_2}, \ldots, \theta^{g_t}$ are the distinct conjugates of θ in G. Assume also that*

$$G = \bigcup_{j=1}^{t} I_G(\theta) g_j, \quad \text{with } t = [G : I_G(\theta)].$$

Then:

(a) $\mathrm{Res}_H^G \mathrm{Ind}_H^G \theta = |I_G(\theta)/H| \sum_{j=1}^t \theta^{g_j}$.

(b) $\langle \mathrm{Ind}_H^G \theta, \mathrm{Ind}_H^G \theta\rangle = |I_G(\theta)/H|$. *In particular*, $\mathrm{Ind}_H^G \theta \in \mathrm{Irr}(G)$ *if and only if* $I_G(\theta) = H$.

(c) $\mathrm{Res}_H^G \chi = e \sum_{j=1}^t \theta^{g_j}$. *In particular,*

$$\chi(1) = et\theta(1) \quad \text{and} \quad \langle \mathrm{Res}_H^G \chi, \mathrm{Res}_H^G \chi\rangle = e^2 t.$$

Also, $e^2 \le |I_G(\theta)/H|$ *and* $e^2 t \le |G/H|$.

In order to ensure the existence of a sequence of irreducible characters of growing degrees, let us consider the following corollary which is given as an exercise in [Isaacs 1976, page 98]. The proof is a consequence of Clifford's theorem.

Corollary 2. *Let G be a group with a chain of normal subgroups*

$$1 = H_0 \trianglelefteq H_1 \trianglelefteq H_2 \cdots \trianglelefteq H_n = G$$

such that H_i/H_{i-1} is nonabelian for $i = 1, \ldots, n$. Then, there exists an irreducible character ϕ of G, such that $\phi(1) \geq 2^n$.

Now we state the following result which is crucial for the proof of Theorem 14.

Proposition 3. *Let H be a subgroup of a finite group G. Let $\theta \in \mathrm{Irr}(H)$. Then there exists $\rho \in \mathrm{Irr}(G)$ such that*:

(i) $\rho(1) \geq \theta(1)$.

(ii) $\langle \mathrm{Ind}_H^G \theta, \rho \rangle = a \geq 1$.

Proof. It is enough take ρ to be any irreducible constituent of $\mathrm{Ind}_H^G(\theta)$. □

We say that an irreducible character θ of H *is extendible to* G if there is an irreducible character χ of G such that $\mathrm{Res}_H^G \chi = \theta$. The following result gives us a criterion to decide when a character is extendible.

Theorem 4 [Gallagher 1962, page 225]. *Let G be a finite group with a normal subgroup H of prime index q in G. If $\theta \in \mathrm{Irr}(H)$ is invariant in G (i.e., $I_G(\theta) = G$), then θ is extendible to G.*

Lemma 5. *Suppose that G is a finite group with a normal subgroup H such that $[G : H] = q$, where q is a prime number. If $\theta \in \mathrm{Irr}(H)$, then the inertia group of θ is either*

(i) $I_G(\theta) = G$, *or*

(ii) $I_G(\theta) = H$.

Proof. See [Isaacs 1976, page 82]. □

Theorem 6. *Under the conditions of Lemma 5, let χ be an irreducible character of G. Then, either*

(i) $\mathrm{Res}_H^G \chi = \theta$, *for some* $\theta \in \mathrm{Irr}(H)$ *or*

(ii) $\chi = \mathrm{Ind}_H^G \theta$, *for some* $\theta \in \mathrm{Irr}(H)$.

Proof. Let $\chi \in \mathrm{Irr}(G)$ and take $\theta \in \mathrm{Irr}(H)$ an irreducible constituent of $\mathrm{Res}_H^G \chi$. The proof follows directly from Theorem 4, [Huppert 1998, Theorem 19.4] and Lemma 5. □

3. Estimation for the root Artin conductor of irreducible characters of G_n

Let L/M be a Galois extension and χ be the character of a linear representation of $\mathrm{Gal}(L/M)$. The Artin conductor attached to χ is given by the ideal

$$f_\chi = \prod_{\mathfrak{p} \nmid \infty} \mathfrak{p}^{f_\chi(\mathfrak{p})},$$

where

$$f_\chi(\mathfrak{p}) = \frac{1}{|G_0|} \sum_{j \geq 0} (|G_j| \chi(1) - \chi(G_j))$$

and G_i is the i-th ramification group of the local extension $L_{\mathfrak{b}}/M_{\mathfrak{p}}$ with $\mathfrak{b}$ a prime over $\mathfrak{p}$ and $\chi(G_j) = \sum_{g\in G_j} \chi(g)$.

It is well-known that if L is an unramified extension of M, then f_χ is the trivial ideal. Then, in order to find a family of irreducible representations with bounded root Artin conductor, let us consider a number field K with infinite p-class field tower for some prime p. Let K_n be the Hilbert p-class field of K_{n-1} with $K_0 = K$ and $G_n = \mathrm{Gal}(K_n/\mathbb{Q})$.

The main objective of this section is to prove that, under some conditions over K and applying the results of the previous section, there exists an upper bound for the root Artin conductor of the irreducible characters of G_n. This bound depends only on the base field K. In addition, we obtain that for each $n > 1$ it is possible to find an irreducible character of G_n with degree increasing with n.

Proposition 7. *Let K be a Galois extension of $\mathbb{Q}$ with infinite p-class field tower, for some prime p. Suppose that K has a subfield $\tilde{k}$ satisfying the following conditions*:

(a) *$\tilde{k}$ is Galois over $\mathbb{Q}$.*

(b) *$[\tilde{k} : \mathbb{Q}] = q$, with q a prime number.*

Let $\chi \in \mathrm{Irr}(G_n)$, where $G_n = \mathrm{Gal}(K_n/\mathbb{Q})$. If $\tilde{H}_n = \mathrm{Gal}(K_n/\tilde{k})$, then either

(i) $\mathrm{Res}^{G_n}_{\tilde{H}_n} \chi = \theta$, *for some $\theta \in \mathrm{Irr}(\tilde{H}_n)$, or*

(ii) $\chi = \mathrm{Ind}^{G_n}_{\tilde{H}_n} \theta$, *for some $\theta \in \mathrm{Irr}(\tilde{H}_n)$.*

Proof. The proof follows directly from Theorem 6 with $G = G_n$ and $H = \tilde{H}_n$. □

Proposition 8. *Let K be a number field with infinite p-class field tower for some prime p. If $T_n = \mathrm{Gal}(K_n/K)$, then for each $n \geq 1$ there exists $\phi \in \mathrm{Irr}(T_n)$ such that*

$$\phi(1) > 2^{(n-1)/2}.$$

Proof. Let us consider the following chain of subgroups. If n is even, we take for $1 \leq j \leq \frac{n}{2}$:

$$\begin{aligned}
&H_0 = \{1\}, \\
&H_1 = \mathrm{Gal}(K_n/K_{n-2}), && H_1/H_0 \cong H_1 \\
&H_2 = \mathrm{Gal}(K_n/K_{n-4}), && H_2/H_1 \cong \mathrm{Gal}(K_{n-2}/K_{n-4}) \\
&\quad\vdots \\
&H_j = \mathrm{Gal}(K_n/K_{n-2j}), && H_j/H_{j-1} \cong \mathrm{Gal}(K_{n-2(j-1)}/K_{n-2j}) \\
&\quad\vdots \\
&H_{n/2} = T_n = \mathrm{Gal}(K_n/K), && H_{n/2}/H_{n/2-1} \cong \mathrm{Gal}(K_2/K).
\end{aligned}$$

If $l < i - 1$ then K_i/K_l is a nonabelian group, so by Corollary 2, there exists $\phi \in \mathrm{Irr}(T_n)$ with $\phi(1) \geq 2^{n/2} > 2^{(n-1)/2}$.

If n is odd, for $j < (n-1)/2$ we take H_j and H_j/H_{j-1} as in the even case. For $j = (n-1)/2$ we take $H_{(n-1)/2}=T_n$ and $H_{(n-1)/2}/H_{(n-1)/2-1} \cong \mathrm{Gal}(K_3/K)$. Hence, there exists $\phi \in \mathrm{Irr}(G)$ such that $\phi(1) > 2^{(n-1)/2}$. □

Corollary 9. *Let G_n be as in Proposition 7. Then for each $n > 1$, there exists $\chi \in \mathrm{Irr}(G_n)$ such that*

$$\chi(1) > 2^{(n-1)/2}.$$

Proof. Note that if $T_n = \mathrm{Gal}(K_n/K)$ has an irreducible character θ with $\theta(1) > 2^{(n-1)/2}$, then there exists $\chi \in \mathrm{Irr}(G)$ with $\chi(1) > 2^{(n-1)/2}$. In fact, let $\theta \in \mathrm{Irr}(T_n)$ with $\theta(1) > 2^{(n-1)/2}$ and choose $\chi \in \mathrm{Irr}(G_n)$ such that θ is an irreducible constituent of $\mathrm{Res}^{G_n}_{T_n}\chi$. By Theorem 1, $\chi(1) = et\theta(1)$, where $e = \langle \mathrm{Res}^{G_n}_{T_n}\chi, \theta\rangle$ and $t = [G_n : I_g(\theta)]$. As $e, t \geq 1$, then $\chi(1) \geq \theta(1) > 2^{(n-1)/2}$. □

Now, we obtain upper bounds for the root Artin conductor of irreducible characters of G_n.

Theorem 10. *Assume G_n as in Proposition 7 and $\chi \in \mathrm{Irr}(G_n)$:*

(i) *If $\mathrm{Res}^{G_n}_{\tilde{H}_n}\chi = \theta$, for some $\theta \in \mathrm{Irr}(\tilde{H}_n)$ then*

$$f_\chi^{1/\chi(1)} \leq |D_{\tilde{k}/\mathbb{Q}}| N_{\tilde{k}/\mathbb{Q}}(f_\theta)^{1/\theta(1)}.$$

(ii) *If $\chi = \mathrm{Ind}^{G_n}_{\tilde{H}_n}\theta$, for some $\theta \in \mathrm{Irr}(\tilde{H}_n)$ then*

$$f_\chi^{1/\chi(1)} = |D_{\tilde{k}/\mathbb{Q}}|^{1/q} N_{\tilde{k}/\mathbb{Q}}(f_\theta)^{1/q\theta(1)}.$$

Proof. In the first case, we have $\chi(1) = \theta(1)$ and

$$\mathrm{Ind}^{G_n}_{\tilde{H}_n}\theta = \sum_{i=1}^{q} \psi_i(1)\cdot\chi\psi_i,$$

where $\mathrm{Irr}(G_n/\tilde{H}_n) = \{\psi_1, \psi_2, \ldots, \psi_q\}$ (see [Huppert 1998, Theorem 19.5]). Since $G_n/\tilde{H}_n$ is isomorphic to the abelian group $\mathbb{Z}/q\mathbb{Z}$, it follows that $\mathrm{Ind}^{G_n}_{\tilde{H}_n}\theta = \sum_{i=1}^{q}\chi\psi_i$. The Artin conductor of this induced character is, on the one hand,

$$f_{\mathrm{Ind}^{G_n}_{\tilde{H}_n}\theta} = |D_{\tilde{k}/\mathbb{Q}}|^{\theta(1)} N_{\tilde{k}/\mathbb{Q}}(f_\theta),$$

where the ideal f_θ is the Artin conductor of θ. On the other hand, assuming that ψ_1 is the trivial character,

$$f_{\mathrm{Ind}^{G_n}_{\tilde{H}_n}\theta} = f_{\sum_{i=1}^{q}\chi\psi_i} = f_\chi \cdot \prod_{i=2}^{q} f_{\chi\psi_i}.$$

Now, combining these expressions we get

$$f_\chi = |D_{\tilde{k}/\mathbb{Q}}|^{\theta(1)} N_{\tilde{k}/\mathbb{Q}}(f_\theta)\cdot\left(\prod_{i=2}^{q} f_{\chi\psi_i}\right)^{-1},$$

so

$$f_\chi^{1/\chi(1)} \leq |D_{\tilde{k}/\mathbb{Q}}| N_{\tilde{k}/\mathbb{Q}}(f_\theta)^{1/\theta(1)}.$$

In the second case,

$$\chi(1) = [G_n : \tilde{H}_n]\theta(1) = q\theta(1)$$

and we can see that the root Artin conductor of χ is given by the expression

$$f_\chi^{1/\chi(1)} = |D_{\tilde{k}/\mathbb{Q}}|^{1/q} N_{\tilde{k}/\mathbb{Q}}(f_\theta)^{1/q\theta(1)}. \qquad \square$$

In order to obtain a bound for the root Artin conductors, we need the following result.

Lemma 11. *Assume K_n and K as in the Proposition 7. Let $\mathfrak{p}$ be a prime in $\tilde{k}$, with $\mathfrak{b}$ and $\mathfrak{q}$ primes over $\mathfrak{p}$ in K_n and K respectively. Let $G_i(K_{n,\mathfrak{b}}/\tilde{k}_\mathfrak{p})$ and $G_i(K_\mathfrak{q}/\tilde{k}_\mathfrak{p})$ be the i-th ramification groups of the local extensions $K_{n,\mathfrak{b}}/\tilde{k}_\mathfrak{p}$ and $K_\mathfrak{q}/\tilde{k}_\mathfrak{p}$. Then, for $i \geq 0$:*

(a) $G_i(K_{n,\mathfrak{b}}/K_\mathfrak{q}) = G_i(K_{n,\mathfrak{b}}/\tilde{k}_\mathfrak{p}) \cap G(K_{n,\mathfrak{b}}/K_\mathfrak{q}) = \{1\}$.

(b) $|G_i(K_{n,\mathfrak{b}}/\tilde{k}_\mathfrak{p})| = |G_i(K_\mathfrak{q}/\tilde{k}_\mathfrak{p})|$.

The proof of this lemma follows directly from properties of higher ramification groups (see for example [Neukirch 1999, pages 177–180]) and by the fact that K_n/K is an unramified extension.

Corollary 12. *There is an infinite sequence $\{\chi_n\}_{n\in\mathbb{N}}$ of irreducible Artin characters with $\chi_n(1) \to \infty$ and with*

$$f_{\chi_n}^{1/\chi_n(1)} \leq C,$$

where $C > 0$ is an effective computable constant.

Proof. By the Corollary 9 and Theorem 10, we know that for each n there is an irreducible character χ_n of G_n with $\chi_n(1) \to \infty$ and

$$f_{\chi_n}^{1/\chi_n(1)} \leq |D_{\tilde{k}/\mathbb{Q}}| N_{\tilde{k}/\mathbb{Q}}(f_\theta)^{1/\theta(1)},$$

for some $\theta \in \mathrm{Irr}(\tilde{H}_n)$. By the properties of the higher ramification groups stated in Lemma 11 and considering that the primes ramifying in K are the only ones that appears in $N_{\tilde{k}/\mathbb{Q}}(f_\theta)$, it is possible to find a constant $T > 0$ depending only on the base field K, such that $N_{\tilde{k}/\mathbb{Q}}(f_\theta) \leq T^{\theta(1)}$. Hence,

$$f_{\chi_n}^{1/\chi_n(1)} \leq |D_{\tilde{k}/\mathbb{Q}}| T := C. \qquad \square$$

Remark 13. As the referee pointed out, it is possible to avoid the hypothesis about the degree of $\tilde{k}/\mathbb{Q}$ and obtain the same type of bounds for the asymptotic behavior of $f_\chi^{1/\chi(1)}$. This is accomplished in Theorem 14 below.

Theorem 14. *Let K be a Galois extension of $\mathbb{Q}$ with infinite p-class field tower. Let $m = [K : \mathbb{Q}]$. Then there exists an infinite sequence $\{\chi_n\}_{n\in\mathbb{N}}$ of irreducible Artin characters such that $\chi_n(1) \to \infty$ and*

$$f_{\chi_n}^{1/\chi_n(1)} \leq |D_{K/\mathbb{Q}}|.$$

Proof. Let $G_n = \mathrm{Gal}(K_n/\mathbb{Q})$ and $T_n = \mathrm{Gal}(K_n/K)$. We can choose $\theta_n \in \mathrm{Irr}(T_n)$ as in Proposition 8. Then, by the Proposition 3, there exists $\chi_n \in \mathrm{Irr}(G_n)$ such that $\langle \mathrm{Ind}_{H_n}^{G_n} \theta_n, \chi_n \rangle = a \geq 1$ and with $\chi_n(1) \geq \theta_n(1)$, so

$$\chi_n(1) > 2^{(n-1)/2}.$$

Hence, by the properties of the Artin conductor we get

$$f_{\chi_n}^a \leq f_{\mathrm{Ind}_{H_n}^{G_n} \theta_n} = |D_{K/\mathbb{Q}}|^{\theta_n(1)},$$

and therefore,

$$f_{\chi_n}^{1/\chi_n(1)} \leq f_{\chi_n}^{a/\chi_n(1)} \leq |D_{K/\mathbb{Q}}|.$$

□

4. Number fields with infinite 2-class field tower

Golod and Shafarevich [1964] proved that a number field K has an infinite p-class field tower if the p-rank of the class group of K is large enough. In this case,

$$\alpha(r_1, r_2) \leq |D_K|^{1/[K:\mathbb{Q}]},$$

where D_K is the discriminant of K.

In addition, Martinet has constructed a number field with infinite Hilbert class field towers and lowest known root discriminant and proved that

$$\alpha(0, 1) < 93 \quad \text{and} \quad \alpha(1, 0) < 1059.$$

In particular, he found that $K = \mathbb{Q}(\zeta_{11} + \zeta_{11}^{-1}, \sqrt{2}, \sqrt{-23})$ has infinite 2-class field tower. Since $\tilde{k} = \mathbb{Q}(\zeta_{11} + \zeta_{11}^{-1})$ is a subfield of K of degree 5 over $\mathbb{Q}$, K satisfies the conditions of the Theorem 10. The discriminant of $\tilde{k}$ is

$$|D_{\tilde{k}/\mathbb{Q}}| = 14641 = 11^4$$

and the only rational primes that ramify in K are 2, 11 and 23. Using PARI/GP [PARI 2014], we can estimates the sizes of the higher ramification groups. Thus, we get the upper bound

$$N_{\tilde{k}/\mathbb{Q}}(f_\theta) \leq (2^{15} 23)^{\theta(1)}.$$

With this estimation, we get the following explicit result:

Corollary 15. *For each $n \geq 1$, there exists a irreducible character χ_n such that $\chi_n(1) \to \infty$ and*

$$f_{\chi_n}^{1/\chi_n(1)} \leq C, \quad \textit{where } C \leq 11^4 \cdot 2^{15} \cdot 23.$$

An open problem now is to improve the constant C.

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