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DAVID A. SHER

**CORRECTION TO THE ARTICLE
THE HEAT KERNEL ON AN ASYMPTOTICALLY CONIC
MANIFOLD**

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We present a minor correction, which leaves the main results unchanged.

In the introduction of [Sher 2013], a theorem of Cheng, Li, and Yau has been misstated. Here is the correct version:

Theorem 1 [Cheng et al. 1981]. *For any $T > 0$, there exist nonzero constants C_1 and C_2 such that the heat kernel on M , denoted by $H^M(t, z, z')$, satisfies, for any $z, z' \in M$ and any $t \in (0, T]$,*

$$H^M(t, z, z') \leq \frac{C_1}{t^{n/2}} e^{-|z-z'|^2/(C_2 t)}. \quad (1)$$

Note in particular that the constants C_1 and C_2 may depend on T . The incorrect version of Theorem 1 was used later in [Sher 2013], in Section 2.5, in the proof of Theorem 2. Specifically, Theorem 8 does not give the claimed order- n decay of the heat kernel at the face zf , and the corrected version of Theorem 1 is not sufficient to do so.

However, Theorem 2 is still true. An alternative approach to this proof is already outlined in [Sher 2013], but we also present another version suggested by Pierre Albin (personal communication, 2016). Specifically, since we know the heat kernel is polyhomogeneous, it has an expansion at zf . If that expansion is trivial, the leading order of the heat kernel at zf is ∞ and we are done. Otherwise, the expansion must be of the form

$$F(w, z, z') = w^{s_0} (\log w)^j a_0(z, z') + (\text{lower order terms})$$

for some $s_0 \in \mathbb{R}$, $j \in \mathbb{N}_0$, with $a_0(z, z')$ not identically zero. Applying the heat operator to this heat kernel gives zero by definition, and in these coordinates the heat operator is $-w^2 \partial_w + \Delta_z$. Since the kernel has a polyhomogeneous conormal expansion we may apply this operator term by term. The leading-order term of the result is

$$w^{s_0} (\log w)^j \Delta_z a_0(z, z'),$$

with all other terms lower order. This term must be zero, so $\Delta_z a_0(z, z')$ must be zero, and therefore $a_0(z, z')$ is harmonic for each z' and nonvanishing for at least some z' . By the maximum principle, $a_0(z, z')$ cannot decay at infinity. So the leading-order term of $F(w, z, z')$ at zf is $w^{s_0} (\log w)^j$ times a

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term which *does not vanish* at the left face lf_0 . Since $w^{s_0}(\log w)^j$ has index set (s_0, j) at lf_0 , the index set of $F(w, z, z')$ at lf_0 must contain a term no better than (s_0, j) — in particular $F(w, z, z')$ cannot decay to any order better than $w^{s_0}(\log w)^j$ at lf_0 . However, we already know that the leading order of $F(w, z, z')$ at lf_0 is n . Thus $s_0 \geq n$, with $j = 0$ if $s_0 = n$. This shows that the leading order of the heat kernel at zf must actually be at least n , filling the gap in the proof of Theorem 2.

Acknowledgements

The author would like to thank Pierre Albin for finding this error and suggesting this alternative approach.

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