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We present a minor correction, which leaves the main results unchanged.

In the introduction of [Sher 2013], a theorem of Cheng, Li, and Yau has been misstated. Here is the correct version:

Theorem 1 [Cheng et al. 1981]. *For any $T > 0$, there exist nonzero constants C_1 and C_2 such that the heat kernel on M, denoted by $H^M(t, z, z')$, satisfies, for any $z, z' \in M$ and any $t \in (0, T]$,*

$$H^M(t, z, z') \leq \frac{C_1}{t^{n/2}} e^{-|z-z'|^2/(C_2 t)}. \tag{1}$$

Note in particular that the constants C_1 and C_2 may depend on T. The incorrect version of Theorem 1 was used later in [Sher 2013], in Section 2.5, in the proof of Theorem 2. Specifically, Theorem 8 does not give the claimed order-n decay of the heat kernel at the face zf, and the corrected version of Theorem 1 is not sufficient to do so.

However, Theorem 2 is still true. An alternative approach to this proof is already outlined in [Sher 2013], but we also present another version suggested by Pierre Albin (personal communication, 2016). Specifically, since we know the heat kernel is polyhomogeneous, it has an expansion at zf. If that expansion is trivial, the leading order of the heat kernel at zf is ∞ and we are done. Otherwise, the expansion must be of the form

$$F(w, z, z') = w^{s_0} (\log w)^j a_0(z, z') + \text{ (lower order terms)}$$

for some $s_0 \in \mathbb{R}$, $j \in \mathbb{N}_0$, with $a_0(z, z')$ not identically zero. Applying the heat operator to this heat kernel gives zero by definition, and in these coordinates the heat operator is $-w^2 \partial_w + \Delta_z$. Since the kernel has a polyhomogeneous conormal expansion we may apply this operator term by term. The leading-order term of the result is

$$w^{s_0} (\log w)^j \Delta_z a_0(z, z'),$$

with all other terms lower order. This term must be zero, so $\Delta_z a_0(z, z')$ must be zero, and therefore $a_0(z, z')$ is harmonic for each z' and nonvanishing for at least some z'. By the maximum principle, $a_0(z, z')$ cannot decay at infinity. So the leading-order term of $F(w, z, z')$ at zf is $w^{s_0}(\log w)^j$ times a

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term which *does not vanish* at the left face lf_0. Since $w^{s_0}(\log w)^j$ has index set (s_0, j) at lf_0, the index set of $F(w, z, z')$ at lf_0 must contain a term no better than (s_0, j) — in particular $F(w, z, z')$ cannot decay to any order better than $w^{s_0}(\log w)^j$ at lf_0. However, we already know that the leading order of $F(w, z, z')$ at lf_0 is n. Thus $s_0 \geq n$, with $j = 0$ if $s_0 = n$. This shows that the leading order of the heat kernel at zf must actually be at least n, filling the gap in the proof of Theorem 2.

Acknowledgements

The author would like to thank Pierre Albin for finding this error and suggesting this alternative approach.

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