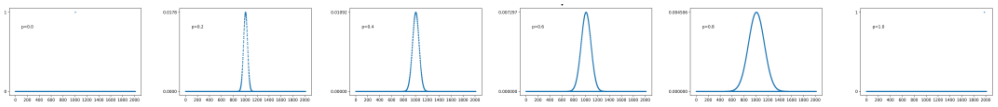
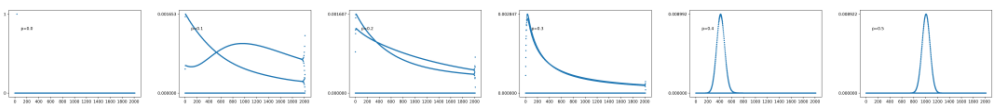
Appendix : Numerical Simulations

The three sets of graphs below represent numerical simulations in each of the three variants of the probabilistic Josephus problem considered so far, namely the first variant (referred to as R1) following the rule stated in the introduction, the second one (R2) with the alternative rule introduced in the final section and the third one (R3) with the general rule depending on two parameters p and q. The graphs have all been produced with simulations comprising $N = 2000$ participants. They show the probability each of the 2000 players has of being the survivor. In particular, when the elimination process is deterministic (that is, when the parameters p and q take the extremal values 0 or 1), the survivor is uniquely determined by the number of participants. The graphs display in such cases a fixed value achieved with probability 1.

- **Numerical simulations for the probabilistic rule of elimination R1 with $N = 2000$ persons.** The parameter p takes successively the values 0, 0.2, 0.4, 0.6, 0.8 and 1. All the non-deterministic cases (i.e. when $p \neq 0, 1$) show the convergence in probability to the constant $1/2$.



- **Numerical simulations for the alternative probabilistic rule of elimination R2 with $N = 2000$ persons.** The parameter p takes successively the values 0, 0.1, 0.2, 0.3, 0.4 and 0.5 (restricting the parameter p to the interval $[0, 1/2]$ is without loss of generality : the elimination process is indeed left unchanged upon swapping p with $1 - p$ and, correspondingly, the right and left moves). In the non-deterministic cases (i.e. when $p \neq 0, 1$), the graphs indicate a convergence of the process in probability to the constant $3p - 1$ in the middle range $p \in (1/3, 2/3)$ and a divergence outside this range.



- **Numerical simulations for the general probabilistic rule of elimination R3 depending on two parameters (p, q) with $N = 2000$ players.** The distributional limit of the process displays here a more subtle dependency on the parameters p and q which is left to conjecture.