

Erratum to “Lefschetz pencils and divisors in moduli space”

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We point out an error in Proposition 5.1 of the paper “Lefschetz pencils and divisors in moduli space” [2].

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Proposition 5.1 of the paper [2], which states a constraint on the self-intersection of a section of a genus two Lefschetz fibration in terms of the number of singular fibres, is incorrect. Indeed, the paper itself essentially contains a counterexample.

Namely, the genus two Lefschetz fibration $X \rightarrow S^2$ obtained as the fibre sum of those associated to the mapping class group relations

$$\begin{aligned}(a_1 b_1 a_2 b_2 a_3)^6 &= 1, \\ (a_1 b_1 a_2 b_2)^{10} &= 1,\end{aligned}$$

(as given in [2, Theorem 5.5], whose notation we follow) has a section of square -2 and 70 irreducible singular fibres, violating the dichotomy claimed in Proposition 5.1. The purported proof of Proposition 5.1 was based on the result (then conjectural, now established by Siebert and Tian in [1]) that a genus two Lefschetz fibration without reducible singular fibres and with transitive monodromy admits the structure of a holomorphic fibration. Though this is true, a given smooth section of the fibration need not be isotopic to a holomorphic section. This invalidates the argument given in the penultimate paragraph of the proof.

Proposition 5.1 was not used elsewhere in the paper.

Acknowledgements

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References

- [1] **B Siebert, G Tian**, *On the holomorphicity of genus two Lefschetz fibrations*, Ann. of Math. 161 (2005) 959–1020 [MR2153404](#)
- [2] **I Smith**, *Lefschetz pencils and divisors in moduli space*, Geom. Topol. 5 (2001) 579–608 [MR1833754](#)

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