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Erratum to “Deriving Deligne–Mumford stacks with perfect obstruction theories”

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This erratum corrects a mistake in “Deriving Deligne–Mumford stacks with perfect obstruction theories” published in Geom. Topol. 17 (2013) 73–92.

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The main result in [2] is the erroneous claim that for every commutative algebra object A in a suitable ∞–category $\mathcal{C}$ equipped with an obstruction theory $E \to L_A$, there exists a commutative algebra object B inducing the obstruction theory [2, Definition 2.7]. The mistake is caused by a missing assumption in [2, Lemma 2.15].

For the lemma to hold, it is necessary to additionally assume that the obstruction theory lifts along the square-zero extension $A^\eta \to A$ defined by the derivation $\eta\colon L_A \to K$. Here η is defined by completing the obstruction theory to a cofiber sequence.

The main result then has to be rephrased as a necessary and sufficient condition for an obstruction theory to be induced by a derived structure. If a compatible system of liftings of the obstruction theory to inductively defined square-zero extensions exists, then it is induced by a derived structure. Conversely, if the obstruction theory is induced by a derived structure, such an inductive system exists by using the Postnikov decomposition. The precise statement is the following:

Theorem *Let $\mathcal{C}$ be an ∞–category as in [2, Assumption 2.1], and let $A \in \mathrm{CAlg}(\mathcal{C})$ be a connective commutative algebra object. Assume that $(A, \phi\colon E \to L_A)$ is an n–connective obstruction theory with $n \geq 1$, and let $\mathrm{cofib}(\phi) = K$.*

Then a pair

$$(f\colon B \to A, \tilde{\delta}\colon K \to L_{A/B})$$

inducing the obstruction theory exists if and only if an inductive system of lifts of the obstruction theory exists.

In the special case of an n–connective and n–perfect obstruction theory (the most important case being $n = 1$, which was studied by Behrend and Fantechi [1]) it is possible to define obstruction classes that precisely measure whether an obstruction theory lifts to the square-zero extension $A^\eta \to A$.

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The same applies to all geometric versions of the above theorem which were proved in [2, Section 3]. Thus an obstruction theory on a Deligne–Mumford stack is induced by a derived structure on the same underlying topos if and only if a compatible system of liftings of the obstruction theory to inductively defined square-zero extensions exists. In the case of an n–connective and n–perfect obstruction theory analogous obstruction classes can be defined.

All details and precise statements can be found in the arXiv version of the paper with the same title [3].

References

[1] **K Behrend**, **B Fantechi**, *The intrinsic normal cone*, Invent. Math. 128 (1997) 45–88

[2] **T Schürg**, *Deriving Deligne–Mumford stacks with perfect obstruction theories*, Geom. Topol. 17 (2013) 73–92

[3] **T Schürg**, *Deriving Deligne–Mumford stacks with perfect obstruction theories* (2014) arXiv:1005.3945v4

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