Erratum to “Deriving Deligne–Mumford stacks with perfect obstruction theories”

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The main result in [2] is the erroneous claim that for every commutative algebra object \( A \) in a suitable \( \infty \)-category \( C \) equipped with an obstruction theory \( E \to L_A \), there exists a commutative algebra object \( B \) inducing the obstruction theory [2, Definition 2.7]. The mistake is caused by a missing assumption in [2, Lemma 2.15].

For the lemma to hold, it is necessary to additionally assume that the obstruction theory lifts along the square-zero extension \( A \xrightarrow{\eta} A \) defined by the derivation \( \eta: L_A \to K \). Here \( \eta \) is defined by completing the obstruction theory to a cofiber sequence.

The main result then has to be rephrased as a necessary and sufficient condition for an obstruction theory to be induced by a derived structure. If a compatible system of liftings of the obstruction theory to inductively defined square-zero extensions exists, then it is induced by a derived structure. Conversely, if the obstruction theory is induced by a derived structure, such an inductive system exists by using the Postnikov decomposition. The precise statement is the following:

**Theorem**  Let \( C \) be an \( \infty \)-category as in [2, Assumption 2.1], and let \( A \in \text{CAlg}(C) \) be a connective commutative algebra object. Assume that \((A, \phi: E \to L_A)\) is an \( n \)-connective obstruction theory with \( n \geq 1 \), and let \( \text{cofib}(\phi) = K \).

Then a pair

\[
(f: B \to A, \delta: K \to L_{A/B})
\]

inducing the obstruction theory exists if and only if an inductive system of lifts of the obstruction theory exists.

In the special case of an \( n \)-connective and \( n \)-perfect obstruction theory (the most important case being \( n = 1 \), which was studied by Behrend and Fantechi [1]) it is possible to define obstruction classes that precisely measure whether an obstruction theory lifts to the square-zero extension \( A^\eta \to A \).
The same applies to all geometric versions of the above theorem which were proved in [2, Section 3]. Thus an obstruction theory on a Deligne–Mumford stack is induced by a derived structure on the same underlying topos if and only if a compatible system of liftings of the obstruction theory to inductively defined square-zero extensions exists. In the case of an $n$–connective and $n$–perfect obstruction theory analogous obstruction classes can be defined.

All details and precise statements can be found in the arXiv version of the paper with the same title [3].

References