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Asymptotic H–Plateau problem in $\mathbb{H}^3$

BARIS COSKUNUZER

We show that for any Jordan curve Γ in $S^2_\infty(\mathbb{H}^3)$ with at least one smooth point, there exists an embedded H–plane $\mathcal{P}_H$ in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P}_H = \Gamma$ for any $H \in [0,1)$.

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1 Introduction

There are two versions of the asymptotic Plateau problem. The first version asks the existence of a least area plane $\mathcal{P}$ in $\mathbb{H}^3$ asymptotic to a given simple closed Γ in $S^2_\infty(\mathbb{H}^3)$, ie $\partial_\infty \mathcal{P} = \Gamma$. In this version, the surface $\mathcal{P}$ must be topologically a disk. The other version asks the existence of an area-minimizing surface Σ in $\mathbb{H}^3$ asymptotic to a given collection of Jordan curves $\widehat{\Gamma}$ in $S^2_\infty(\mathbb{H}^3)$, ie $\partial_\infty \Sigma = \widehat{\Gamma}$. In the latter version, there is no a priori topological restriction on the surface Σ, hence Σ can have positive genus depending on the given $\widehat{\Gamma}$. Anderson gave positive answers to both versions of the problem three decades ago [1; 2].

Constant mean curvature (CMC) surfaces are natural generalizations of minimal surfaces, and in many cases, results related to minimal surfaces are studied to see whether they can be generalized to CMC setting. We will call this natural generalization the *asymptotic H–Plateau problem.* A decade after Anderson's result, the second version of the asymptotic Plateau problem was generalized to the CMC case by Tonegawa [21]. He showed that for any given collection of Jordan curves $\widehat{\Gamma}$ in $S^2_\infty(\mathbb{H}^3)$, there exists a minimizing H–surface Σ_H in $\mathbb{H}^3$ with $\partial_\infty \Sigma_H = \widehat{\Gamma}$, where $H \in [0,1)$. Indeed, both Anderson and Tonegawa used geometric measure theory methods, and the solutions are automatically smoothly embedded surfaces by the regularity results of GMT. Our survey [7] gives a fairly complete account of the old and new results on the problem.

On the other hand the only result on the generalization of the first (plane) version to the CMC case was proved a few years ago by Cuschieri [10]. He showed the existence of *immersed* H–planes asymptotic to a given smooth Jordan curve in $S^2_\infty(\mathbb{H}^3)$ by using PDE techniques.

In this paper, we give a positive answer to the asymptotic H–Plateau problem for a larger family of curves. Furthermore, we show that these solutions are indeed *embedded*.

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Theorem 1.1 *Let Γ be a simple closed curve Γ in $S^2_\infty(\mathbb{H}^3)$ which is differentiable at least at one point. Then, for any $H \in [0,1)$, there exists a properly embedded H–plane $\mathcal{P}_H$ in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P}_H = \Gamma$.*

Our techniques are also valid for $H = 0$ case, and we are able to reprove the existence of least area planes in $\mathbb{H}^3$. Hence, with this result, we also fill a gap in Anderson's proof for existence of least area planes in [2] (see Remark 3.1 and Remark 3.4). Note that by using different techniques, Gabai [12] has also proved the existence of least area planes in $\mathbb{H}^3$ spanning a given Jordan curve in $S^2_\infty(\mathbb{H}^3)$.

On the other hand, our proof indeed works for $H \in (-1,1)$ by considering the orientation; see Section 2.2. If Γ is an oriented curve in $S^2_\infty(\mathbb{H}^3)$, it induces an orientation on the H–plane. Thus, with that induced orientation, there exist two H–planes, $\mathcal{P}^+_H$ and $\mathcal{P}^-_H$, with the same absolute value of the mean curvature, $|H| \in (0,1)$, but different sign. Hence, by forgetting the sign of the mean curvature, the above theorem shows that for a given Jordan curve Γ in $S^2_\infty(\mathbb{H}^3)$ and $H \in (0,1)$, there exists *a pair* of complete, embedded H–planes $\mathcal{P}^+_H$ and $\mathcal{P}^-_H$ with $\partial_\infty \mathcal{P}^\pm_H = \Gamma$ (see Remark 4.3).

Recently, Meeks, Tinaglia and the author [9] constructed nonproperly embedded H–planes in $\mathbb{H}^3$ for any $H \in [0,1)$, where the asymptotic boundary is a pair of infinite lines in $S^2_\infty(\mathbb{H}^3)$. Here we also show that if the Jordan curve Γ in $S^2_\infty(\mathbb{H}^3)$ is smooth enough, then the minimizing H–planes $\mathcal{P}_H$ with $\partial_\infty \mathcal{P}_H = \Gamma$ are *properly embedded* in $\mathbb{H}^3$ (Corollary 4.5).

In the final section, we discuss questions of the *generic uniqueness* of the H–planes in $\mathbb{H}^3$, and *foliations of* $\mathbb{H}^3$ with H–planes, and give an outline to answer them.

The organization of the paper is as follows. In Section 2, we go over the basic notions, and the related results. In Section 3, we prove the main theorem for least area planes ($H = 0$ case). In Section 4, we show the existence of embedded H–planes in $\mathbb{H}^3$. Finally in Section 5, we give some concluding remarks.

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2 Preliminaries

In this section, we will go over the related results which will be used in the following sections. For further details on the notions and results we use, see the survey [7].

Definition 2.1 Let M be a 3–manifold.

- A surface S in M is *minimal* if the mean curvature vanishes everywhere on S.

- A compact disk D in M with $\partial D = \Gamma$ is the *least area disk* in M if it has the smallest area among the disks in M with the boundary Γ.

Note that minimal surfaces are the critical points of the area functional. Least area disks and area-minimizing surfaces are the minima of the area functional in the corresponding spaces.

A natural generalization of minimal surfaces are CMC surfaces (H–surfaces). They can be defined as the critical points of the area functional with a volume constraint as follows. For an immersion $u : D^2 \to M$, the critical points of the variational problem

$$\mathcal{F}_H(u) = \int_{D^2} (|u_x|^2 + |u_y|^2) + \tfrac{4}{3} H[u \cdot (u_x \times u_y)]\, dx dy$$

are immersed disks with constant mean curvature H [14]. Here, the second summand in the integral represents the volume constraint.

We can reformulate this variational problem so that it will be independent of the parametrization of the surface [8]. Let Σ be a surface in M with boundary α. We fix a surface T in M with $\partial T = \alpha$, and define Ω to be the domain bounded by T and Σ. Again, let

$$\mathcal{I}_H(\Sigma) = \text{Area}(\Sigma) + 2H\,\text{Vol}(\Omega).$$

If Σ is a critical point of the functional $\mathcal{I}_H$ for any variation f, then this will imply that Σ has constant mean curvature H. Note also that a critical point of the functional $\mathcal{I}_H$ is independent of the choice of the surface T, since if $\widehat{I}_H$ is the functional which is defined by a different surface $\widehat{T}$, then $\mathcal{I}_H - \widehat{\mathcal{I}}_H = C$ for some constant C. Note that to keep the solution surface away from T, one needs a convexity condition on T (eg to be H_0–convex for $H_0 > H$) to employ the maximum principle [8].

Definition 2.2 Let M be a 3–manifold.

- A surface S in M is an *H–surface* if the mean curvature is equal to H everywhere on S.
- A compact disk D in M with $\partial D = \Gamma$ is a *minimizing H–disk* in M if $\mathcal{I}_H(D)$ (or equivalently $\mathcal{F}_H(D)$) has the smallest value among the disks in M with boundary Γ.

2.1 Embedded solutions to the H–Plateau problem

Here we quote a generalization of Meeks and Yau's embeddedness result [15] to H–disks.

Definition 2.3 (H_0–convex domains) Let Ω be a compact 3–manifold with piecewise smooth boundary. We call Ω an *H_0–convex domain* if:

- The mean curvature vector $\boldsymbol{H}$ always points into Ω along the smooth parts of $\partial\Omega$.
- The mean curvature satisfies $|\boldsymbol{H}(p)| \geq H_0$ for any smooth point $p \in \partial\Omega$.
- Along the nonsmooth parts of $\partial\Omega$, the inner dihedral angle is less than π.

With the definition above, 0–convex domains correspond to *mean convex* domains in [16]. A Jordan curve γ is called *extreme* if it is in the boundary of a convex domain. Following Meeks and Yau [16], we generalize notion of extremeness as follows: A Jordan curve γ is called *H_0–extreme* if γ is in the boundary of a H_0–convex domain.

Meeks and Yau showed that the solution to Plateau problem for 0–extreme curves must be embedded [16]. The following lemma is a generalization of their result, which shows that solutions of the H–Plateau problem for H_0–extreme curves are embedded.

Lemma 2.4 [8] *Let M be a compact H_0–convex ball. Let Γ be a Jordan curve in ∂M, ie Γ is H_0–extreme. Then, for any $H \in [0, H_0)$, there exists a minimizing H–disk Σ_H in M with $\partial\Sigma_H = \Gamma$, and any such Σ_H is embedded.*

Lemma 2.5 (Maximum principle [14]) *Let Σ_1 and Σ_2 be two surfaces in a Riemannian 3–manifold which intersect tangentially at a common point. Let H_i be the (signed) mean curvature of Σ_i at the common point with respect to the same normal vector $\boldsymbol{N}$, ie $\boldsymbol{H_i} = H_i\boldsymbol{N}$. If Σ_2 lie in positive side (the normal vector $\boldsymbol{N}$ direction) of Σ_1 nearby the common point, then H_1 is strictly less than H_2, ie $H_1 < H_2$.*

2.2 H–planes in $\mathbb{H}^3$

Now we restrict ourselves to $\mathbb{H}^3$. We will use the notion *mean curvature with sign*, and $H \in (-1, 1)$ throughout the paper. In particular, let Γ be a simple closed curve in $S^2_\infty(\mathbb{H}^3)$. Fix an orientation on Γ, and let $\mathcal{P}$ be a plane in $\mathbb{H}^3$ with $\partial_\infty\mathcal{P} = \Gamma$. Then the orientation on Γ naturally induces an orientation on $\mathcal{P}$, and we denote the induced normal vector of $\mathcal{P}$ by $\boldsymbol{N}$. If $\mathcal{P}_H$ is an H–plane in $\mathbb{H}^3$ with $\partial_\infty\mathcal{P}_H = \Gamma$, then the mean curvature vector is $\boldsymbol{H} = H\boldsymbol{N}$, where $H \in (-1, 1)$ [4; 21].

In other words, Γ separates $S^2_\infty(\mathbb{H}^3)$ into two open disks, D^+ and D^-. Consider the mean curvature H with sign depending on the direction of the mean curvature vector, ie $H \in (-1, 1)$ such that if H is close to $+1$ then $\mathcal{P}_H$ is "close" to D^-, and if H is close to -1 then $\mathcal{P}_H$ is "close" to D^+. For example, if γ is a round circle in

$S^2_\infty(\mathbb{H}^3)$, then for $H \in (0,1)$ there are two spherical caps $\mathcal{P}^+_H$ and $\mathcal{P}^-_H$ in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P}^\pm_H = \gamma$, where $\mathcal{P}^\pm_H$ is an H–plane. The mean curvature vectors on $\mathcal{P}^+_H$ and $\mathcal{P}^-_H$ point to each other. With the notation above, we will denote $\mathcal{P}^+_H$ by $\mathcal{P}_H$, and we will denote $\mathcal{P}^-_H$ by $\mathcal{P}_{-H}$.

By using the definitions above, we now define least area planes and minimizing H–planes in $\mathbb{H}^3$.

Definition 2.6 (Least area plane) Let $\mathcal{P}$ be a complete surface in $\mathbb{H}^3$ which is topologically a disk. We call $\mathcal{P}$ a *least area plane* in $\mathbb{H}^3$ if any compact subdisk D in $\mathcal{P}$ is a least area disk.

Definition 2.7 (Minimizing H–plane) Fix $H \in (-1,1)$. Let $\mathcal{P}_H$ be a complete surface in $\mathbb{H}^3$ which is topologically a disk. We call $\mathcal{P}_H$ a *minimizing H–plane* in $\mathbb{H}^3$ if any compact subdisk D in $\mathcal{P}_H$ is a minimizing H–disk.

For a given surface S in $\mathbb{H}^3$, we define the *asymptotic boundary* of S as follows. If $\overline{\mathbb{H}^3} = \mathbb{H}^3 \cup S^2_\infty(\mathbb{H}^3)$ is the natural (geodesic) compactification of $\mathbb{H}^3$, and $\overline{S}$ is the closure of S in $\overline{\mathbb{H}^3}$, then the asymptotic boundary $\partial_\infty S$ of S defined as $\partial_\infty S = \overline{S} \cap S^2_\infty(\mathbb{H}^3)$.

Now we define the shifted convex hulls $CH_H(\Gamma)$ as generalizations of the convex hulls in $\mathbb{H}^3$ [7; 4]. Fix $H \in (-1,1)$. Let Γ be a Jordan curve in $S^2_\infty(\mathbb{H}^3)$. Let α be a round circle in $S^2_\infty(\mathbb{H}^3)$ with $\alpha \cap \Gamma = \varnothing$. Let $\mathcal{P}^\alpha_H$ be the unique H–plane in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P}^\alpha_H = \alpha$. α separates $S^2_\infty(\mathbb{H}^3)$ into two open disks Δ^+_α and Δ^-_α. Similarly, $\mathcal{P}^\alpha_H$ divides $\mathbb{H}^3$ into two domains $\Omega^{\alpha+}_H$ and $\Omega^{\alpha-}_H$, where $\partial_\infty \Omega^{\alpha\pm}_H = \Delta^\pm_\alpha$. We will call these regions *H–shifted half-spaces*. If $\Gamma \subset \Delta^+_\alpha$, then we will call $\Omega^{\alpha+}_H$ a *supporting H–shifted half-space*. Similarly, if $\Gamma \subset \Delta^-_\alpha$, then we will call $\Omega^{\alpha-}_H$ a *supporting H–shifted half-space*.

Definition 2.8 (Shifted convex hull) Let Γ be a simple closed curve in $S^2_\infty(\mathbb{H}^3)$. Fix $H \in (-1,1)$. Then the *H–shifted convex hull* of Γ, $CH_H(\Gamma)$ is defined as the intersection of all supporting closed H–shifted half-spaces $\Omega^{\alpha\pm}_H$ of $\mathbb{H}^3$. For $H = 0$, this is the usual convex hull definition in $\mathbb{H}^3$, ie $CH(\Gamma) = CH_0(\Gamma)$.

The generalization of the convex hull property of minimal surfaces in $\mathbb{H}^3$ to H–surfaces in $\mathbb{H}^3$ is as follows [4; 21].

Lemma 2.9 [4; 21] *Let Σ be an H–surface in $\mathbb{H}^3$ with $\partial_\infty \Sigma = \Gamma$ and $H \in (-1,1)$. Then Σ is in the H–shifted convex hull of Γ, ie $\Sigma \subset CH_H(\Gamma)$.*

Remark 2.10 Note that the result is true for any H–surface. This is a straightforward generalization of the convex hull property for minimal surfaces. In particular, if Σ is an H–surface in $\mathbb{H}^3$ with $\partial_\infty \Sigma_H = \Gamma$, then Σ cannot go into a nonsupporting H–shifted half-space of Γ, as we can foliate such a half-space by H–planes, and the first point of touch gives a contradiction with the maximum principle.

Note also that if Γ is not a round circle in $S^2_\infty(\mathbb{H}^3)$, then $\partial CH_H(\Gamma)$ has two components, namely $\partial CH_H(\Gamma) = \partial^+ CH_H(\Gamma) \cup \partial^- CH_H(\Gamma)$. Let $S^2_\infty(\mathbb{H}^3) - \Gamma = D^+ \cup D^-$. Then for any $q \in D^+$, let τ_q be the largest round circle in D^+ with center p. Let $\mathcal{P}^q_H$ be the H–plane (spherical cap) in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P}^q_H = \tau_q$. Then, by construction of $CH_H(\Gamma)$, $\partial^+ CH_H(\Gamma)$ is a piecewise smooth (pleated) plane with $\mathcal{P}^q_H \cap \partial^+ CH_H(\Gamma) \neq \varnothing$ and each smooth part in $\partial^+ CH_H(\Gamma)$ belongs to $\mathcal{P}^q_H$ for some q [4; 21]. Similarly, $\partial^- CH_H(\Gamma)$ is a piecewise smooth plane with $\mathcal{P}^q_H \cap \partial^- CH_H(\Gamma) \neq \varnothing$, where $q \in D^-$. For further details, see [11].

3 Existence of least area planes

In this section, we will focus on the $H = 0$ case. In other words, we will consider the original asymptotic Plateau problem, and show the existence of smoothly embedded least area planes $\mathcal{P}$ in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P} = \Gamma$ for a given Jordan curve Γ in $S^2_\infty(\mathbb{H}^3)$.

Gabai showed the existence of least area planes in $\mathbb{H}^3$ in [12] by using Hass and Scott's techniques. Recently, Ripoll and Tomi also showed the existence of complete embedded minimal planes in Hadamard manifolds [19].

We will adapt Anderson's techniques from [2] to construct minimizing H–planes. To generalize his techniques, we need to fill a gap in the proof for least area plane case. The following remark explains the problem.

Remark 3.1 (Gap in [2, Theorem 4.1]) Anderson showed the existence of least area planes in [2, Theorem 4.1]. He basically generalized the techniques he used for absolutely area-minimizing surfaces to the plane case. In particular, let Γ be a Jordan curve in $S^2_\infty(\mathbb{H}^3)$, and $\{D_n\}$ be a sequence of least area disks in $B_n(0)$ with $\partial D_n = \gamma_n \subset \partial B_n(0)$, where $\gamma_n \to \Gamma$. Then the idea is to show the existence of a subsequence of $\{D_n\}$ converging to a least area plane $\mathcal{P}$ in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P} = \Gamma$. In particular, for every fixed compact domain K in $\mathbb{H}^3$, he showed the sequence $D_n \cap K = D^K_n$ has a subsequence converging to a smooth disk in K by using compactness and regularity results of GMT. Then, by using a diagonal sequence argument, he obtained a limit least area plane $\mathcal{P}$ with $\partial \mathcal{P} = \Gamma$.

However, to use the compactness result in this approach, one needs a uniform area bound on the disks $\{D_n^K\}$. Let Γ_n^K be the collection of simple closed curves $D_n \cap \partial K$. If $\{\Sigma_n\}$ were a sequence of area-minimizing surfaces, then the area of ∂K would be a uniform area bound for $\{\Sigma_n^K\}$ as Γ_n^K bounds a surface in ∂K and Σ_n^K is an absolutely area-minimizing surface with boundary Γ_n^K. Hence, for any n, $\operatorname{Area}(\Sigma_n^K) \leq \operatorname{Area}(\partial K)$. However, in the disk case, $D_n \cap K$ may contain many disks or planar surfaces in K, and the area of ∂K cannot give an upper bound for $\operatorname{Area}(D_n^K)$. In particular, the estimate (4.2) in [2, Theorem 4.1] (that is, $\boldsymbol{M}(D_i \llcorner \boldsymbol{B}_r) \leq \frac{1}{2}\operatorname{Area}(S(r))$) is not valid in general.

For example, if D_n^K is $2k$ disjoint disks close to the equator disk in $K = \boldsymbol{B}_R$, then the area of D_n^K would be close to the area of k equator disks, which is much larger than the area of $\partial \boldsymbol{B}_R$. The main difference with the area-minimizing case is that if we have k annuli $A_1, \dots, A_k$ in ∂K bounding Γ_n^K, we cannot compare the sum of the areas of the disks with the sum of the areas of the annuli, because if we replace two disks with an annulus in D_n we get a genus-1 surface, which is no longer a disk. So the area of an annulus cannot be compared with the area of the two disks, because of the restriction of the topology on $\{D_n\}$. Since there is no restriction on the topology of the surface for area-minimizing surfaces, $\operatorname{Area}(\partial K)$ gives a uniform bound, but in the least area disk case, $\operatorname{Area}(\partial K)$ does not give a uniform bound for $\{D_n^K\}$. In the following theorem, we will fix the proof by constructing a special (tight) sequence of least area disks $\{D_n\}$, where the intersection of sufficiently large balls contains only one component. Hence the uniform area bound holds, and the proof goes through. In the following section, we will generalize this idea to the CMC setting, and show the existence of H–planes in $\mathbb{H}^3$ spanning given Jordan curve in $S^2_\infty(\mathbb{H}^3)$.

We will show the existence of least area plane in $\mathbb{H}^3$ by using Anderson's techniques. To get a uniform area bound on $\{D_n^K\}$ for fixed compact set K, we will use ideas from [5].

Theorem 3.2 *Let Γ be a simple closed curve differentiable at least at one point in $S^2_\infty(\mathbb{H}^3)$. Then there exists a properly embedded least area plane $\mathcal{P}$ in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P} = \Gamma$.*

Proof We will define a special sequence of least area disks $\{D_n\}$ whose restriction to a compact subset K has a uniform area bound. Then, by following [2], we get a least area plane.

Notation and setting Let Γ be a Jordan curve differentiable at least at one point in $S^2_\infty(\mathbb{H}^3)$. Let $CH(\Gamma)$ be the convex hull of Γ in $\mathbb{H}^3$. Near the smooth point $p \in \Gamma$, we can have two sufficiently small and close round circles τ^+ and τ^- in the opposite sides of Γ such that $\tau^+ \cup \tau^-$ bounds a least area annulus $\mathcal{A}$ in $\mathbb{H}^3$ [22]. Hence, $\mathcal{A}$ cuts through $CH(\Gamma)$.

Fix a point O in $CH(\Gamma)$. Let $\boldsymbol{B}_n$ be the closed ball in $\mathbb{H}^3$ of radius n with center O. Then, for sufficiently large N_0, $\mathcal{A} \cap CH(\Gamma)$ is in $\boldsymbol{B}_{N_0}$. Let $\partial CH(\Gamma) = \partial^+ CH(\Gamma) \cup \partial^- CH(\Gamma)$ (see Remark 2.10). For $n > N_0$, let $\alpha_n^+ = \partial \boldsymbol{B}_n \cap \partial^+ CH(\Gamma)$ and $\alpha_n^- = \partial \boldsymbol{B}_n \cap \partial^- CH(\Gamma)$. Then, since $\mathcal{A}$ is a least area annulus, by [15, Theorem 1], the pair $\alpha_n^+ \cup \alpha_n^-$ bounds a least area annulus $\mathcal{A}_n$ in $CH(\Gamma)$. Then by [5, Lemma 4.1], the least area annuli $\mathcal{A}_n$ escapes to infinity. To see this, let $\{\Gamma_s^\pm\}$ be the foliation of a small neighborhood of Γ in $S^2_\infty(\mathbb{H}^3)$, where Γ_s^+ and Γ_s^- are in opposite sides of Γ and in between τ^+ and τ^- (Step 2 of [5, Lemma 4.1]). Let $\widehat{\mathcal{A}}_s$ be the least area annulus with $\partial_\infty \widehat{\mathcal{A}}_s = \Gamma_s^+ \cup \Gamma_s^-$. As $\alpha_n^\pm \to \infty$, for each n, there is an s_n such that $\alpha_n^\pm$ is in *outer side* of $\widehat{\mathcal{A}}_{s_n}$. As $\mathcal{A}_n$ is also least area annulus, $\mathcal{A}_n \cap \widehat{\mathcal{A}}_{s_n} = \varnothing$, which shows that the sequence $\{\mathcal{A}_n\}$ escapes to infinity too.

Let Ω_n be the compact region which $\mathcal{A}_n$ separates from $CH(\Gamma)$. Let γ_n be an essential, smooth, simple closed curve in $\mathcal{A}_n$. As $\mathcal{A}_n$ escapes to infinity and $\partial_\infty CH(\Gamma) = \Gamma$, then $\gamma_n \to \Gamma$. Let D_n be the least area disk in Ω_n with $\partial D_n = \gamma_n$ [15]. By construction, D_n is also a least area disk in $\mathbb{H}^3$.

We claim that there exists a subsequence of $\{D_n\}$ which converges to a least area plane $\mathcal{P}$ in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P} = \Gamma$. We will follow the proof of [2, Theorem 4.1]. Hence, if we show the estimate $\boldsymbol{M}(D_n \llcorner \boldsymbol{B}_r) \leq \frac{1}{2}\,\mathrm{Area}(\partial \boldsymbol{B}_r)$ (see [2, Theorem 4.1, (4.2)]) is valid for our sequence, we are done.

Let β be a transversal arc in $CH(\Gamma)$ connecting $\partial^+ CH(\Gamma)$ and $\partial^- CH(\Gamma)$ through the point O (center of the balls $\boldsymbol{B}_n$). Let l be the length of β. Consider the following lemma from [5], of which a sketch of a proof is provided. Refer to [5] for more details.

Let D_r be a least area disk in $\boldsymbol{B}_r$ with $\partial D_r \subset \partial \boldsymbol{B}_r \cap CH(\Gamma)$. Note that for sufficiently large $r > 0$, $\partial \boldsymbol{B}_r \cap CH(\Gamma)$ is an annulus near Γ as $\partial_\infty CH(\Gamma) = \Gamma$. Then we call D_r *nonseparating with respect to* Γ *in* $\boldsymbol{B}_r$ (or wrt-Γ) if ∂D_r is not an essential curve in the annulus $\partial \boldsymbol{B}_r \cap CH(\Gamma)$.

Lemma 3.3 [5, Lemma 4.1] *Let Γ be a Jordan curve in $S^2_\infty(\mathbb{H}^3)$ differentiable at least at one point. Let $N_0(\Gamma) > 0$ be as described above. Let D_r be a least area disk, nonseparating wrt-Γ in $\boldsymbol{B}_r$ for $r > N_0$. Then there is a monotone increasing function $F\colon [N_0, \infty) \to \mathbb{R}^+$ such that $F(r) \to \infty$ as $r \to \infty$, and $d(O, D_r) > F(r)$, where d is the distance.*

Let $R_0 > 0$ be sufficiently large so that $\beta \subset \boldsymbol{B}_{R_0}$ and $F(R_0) > l$. Now we will prove the uniform bound $|D_n \cap \boldsymbol{B}_r| < C_r$ for $r > R_0$, where $|\cdot|$ represents the area.

By [2, Lemma 4.2], for a given least area disk D_n, $D_n \cap \boldsymbol{B}_r$ is a collection of disjoint embedded disks for any generic $r > 0$. This is simply because $D_n \cap \boldsymbol{B}_r$ is a surface for generic r, and by the convexity of $\boldsymbol{B}_r$, any component in $D_n \cap \boldsymbol{B}_r$ must be a disk.

We claim that for $n > \max\{N_0, R_0\}$, $D_n \cap \boldsymbol{B}_r$ is just a disk (only one component) for any generic $r > R_0$. Assume not, and let E_1, E_2 be two such disks in $D_n \cap \boldsymbol{B}_{r_0}$. Since D_n is an embedded disk in $\boldsymbol{B}_n$, there is a path α connecting E_1 and E_2 in D_n. Let $r_0 < r' < n$ be the smallest radius such that E_1 and E_2 are in the same component $\widehat{E}$ of $D_n \cap B_{r'}$. Hence $\widehat{E}$ is a least area disk, nonseparating wrt-Γ in $\boldsymbol{B}_{r'+\epsilon}$ for sufficiently small $\epsilon > 0$. Hence by Lemma 3.3, $d(O, \widehat{E}) > F(r')$. However, $d(O, E_i) \leq l$ as β intersects E_i by construction. Hence we get $d(O, \widehat{E}) \leq d(O, E_i) \leq l < F(R_0) < F(r')$, which gives a contradiction.

This proves that for $n > \max\{N_0, R_0\}$ and for any $r > R_0$, $D_n \cap \boldsymbol{B}_r$ is a disk (only one component). Hence, for any fixed $r > R_0$, $|D_n \cap \boldsymbol{B}_r| \leq |\partial \boldsymbol{B}_r| = C_r$, which gives the desired uniform bound. This proves the estimate (4.2) of [2, Theorem 4.1] is valid for our sequence $\{D_n\}$. The proof follows. □

Remark 3.4 As explained in Remark 3.1, Anderson's proof has a gap because the sequence of disks he constructed does not satisfy a uniform area estimate in a ball, as there might be more than one disk component. However, each component does satisfy an area estimate and thus curvature estimates by a result of Schoen and Simon [20]. Therefore, one still can obtain convergence to a lamination σ with $\partial_\infty \sigma = \Gamma$. One then needs to show that each leaf in σ is in fact topologically a disk. For this, see for instance [3]. There, they deal with the harder case of possible singularities forming. In fact, Schoen and Simon curvature estimates can also be used in the proof of Theorem 4.2 to avoid working with integral currents.

4 Minimizing *H*–planes

In this section, we will show the existence of solutions of the asymptotic H–Plateau problem in $\mathbb{H}^3$ for $H \in (-1, 1)$. In particular, we will generalize the techniques in the previous section to the CMC case, and show that for any simple closed curve Γ in $S^2_\infty(\mathbb{H}^3)$ with one smooth point, there exists an embedded H–plane $\mathcal{P}_H$ with $\partial_\infty \mathcal{P}_H = \Gamma$. First, we need to generalize Lemma 3.3 proven in [5] to the minimizing H–planes in $\mathbb{H}^3$.

We will use the same notation, ie let $O, \boldsymbol{B}_r, \mathcal{A}_r, N_0$ be as in the previous section. Fix $H \in (-1, 1)$. Let $\widehat{A}^H_\Gamma = \partial \boldsymbol{B}_r \cap CH_H(\Gamma)$ be an annulus in $\partial \boldsymbol{B}_r$. Again, let D_r be a minimizing H–disk in $\boldsymbol{B}_r$ with $\partial D_r \subset \widehat{A}^H_\Gamma$. Then we call D_r *nonseparating wrt-*Γ *in* $\boldsymbol{B}_r$ if ∂D_r is *not* an essential curve in the annulus $\widehat{A}^H_\Gamma$.

Lemma 4.1 *Fix $H \in (-1,1)$. Let Γ be a Jordan curve differentiable at least at one point in $S^2_\infty(\mathbb{H}^3)$. For $r > N_0$, let D_r be a minimizing H–disk, and nonseparating wrt-Γ in $\boldsymbol{B}_r$. Then there is a monotone increasing function $F\colon [N_0,\infty) \to \mathbb{R}^+$ such that $F(r) \to \infty$ as $r \to \infty$, and $d(O, D_r) > F(r)$, where d is the distance.*

Proof We will adapt the proof of Lemma 3.3 to this case. Lemma 3.3 finishes the $H=0$ case. Hence, we can take $H>0$. For $H<0$, the same proof works by changing the orientation. Fix $H>0$.

Recall that $\mathcal{A}_r$ is the least area annulus in $\boldsymbol{B}_r$ with $\partial \mathcal{A}_r = \alpha_r^+ \cup \alpha_r^-$ for generic $r > N$. Let $\widehat{A}_r$ be the annulus in $\partial \boldsymbol{B}_r$ with $\partial \widehat{A}_r = \alpha_r^+ \cup \alpha^-$. Let $F(r) = d(O, \mathcal{A}_r)$.

We claim that for any minimizing H–disk D_r which is nonseparating wrt-Γ in $\boldsymbol{B}_r$, $D_r \cap \mathcal{A}_r = \varnothing$, ie D_r stays in the solid torus $\mathcal{U}_r$ in $\boldsymbol{B}_r$ with $\partial \mathcal{U}_r = \mathcal{A}_r \cup \widehat{A}_r$; see Figure 1. In particular, this shows that $d(O, D_r) > F(r)$, and the proof will follow.

Now, let D_r be a minimizing H–disk and nonseparating wrt-Γ in $\boldsymbol{B}_r$ with $\partial D_r = \gamma_r \subset \widehat{A}_r$. Since D_r is nonseparating wrt-Γ, γ_r is a not an essential curve in $\widehat{A}_r$. In other words, γ_r bounds a disk E_r in $\widehat{A}_r$; see Figure 1.

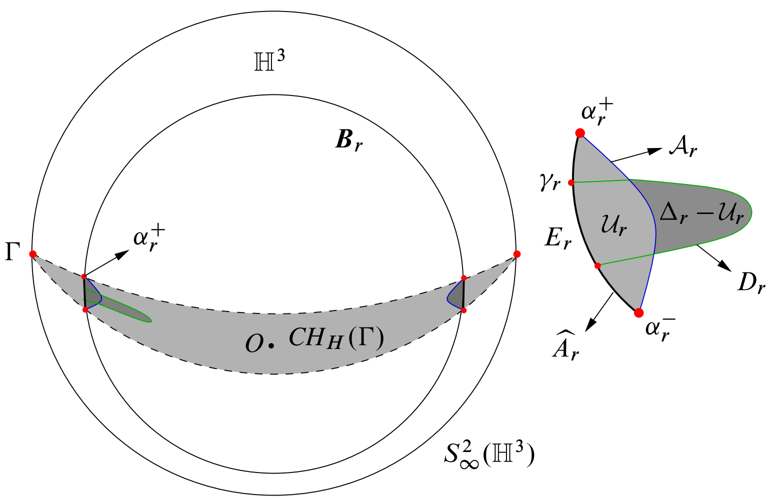

Figure 1: If γ_r is a nonessential curve in $\widehat{A}_r$, the nonseparating minimizing H–disk D_r in $\boldsymbol{B}_r$ with $\partial D_r = \gamma_r$ must belong to $\mathcal{U}_r$. This shows that for any such D_r in $\boldsymbol{B}_r$, $d(O, D_r) > d(O, \mathcal{A}_r) = F(r)$.

Let Δ_r be the region in $\boldsymbol{B}_r$ with $\partial \Delta_r = D_r \cup E_r$. Since D_r is a minimizing H–disk in $\boldsymbol{B}_r$ with $H>0$, $I_H(D_r) = |D_r| + 2H|\Delta_r|$ is the smallest among all the disks in $\boldsymbol{B}_r$ with boundary γ_r. Here $|\cdot|$ represents the area or the volume of the corresponding region.

Assume that $d(O, D_r) < d(O, \mathcal{A}_r)$. Recall that $\mathcal{U}_r$ is the region in $\boldsymbol{B}_r$ with $\partial\mathcal{U}_r = \mathcal{A}_r \cup \widehat{A}_r$. Let $\Delta_r' = \Delta_r \cap \mathcal{U}_r$. Then as $d(O, D_r) < d(O, \mathcal{A}_r)$, $|\Delta_r'| < |\Delta_r|$. Furthermore, let $T' = \mathcal{A}_r \cap \Delta_r'$ be the planar region in $\mathcal{A}_r$ with $\partial T' = \beta$. Let T be the planar region in D_r with $\partial T = \beta$. Since D_r is a nonseparating disk, and $\mathcal{A}_r$ is an annulus, $D' = (D_r - T) \cup T'$ is also a disk in $\boldsymbol{B}_r$ with $\partial D' = \gamma_r$. Furthermore, as $\mathcal{A}_r$ is a least area annulus, $|T'| < |T|$ and hence $|D'| < |D_r|$. This implies $\mathcal{I}_H(D') = |D'| + 2H|\Delta_r'| < |D_r| + 2H|\Delta_r'| = \mathcal{I}_H(D_r)$. However, D_r is a minimizing H–disk in $\boldsymbol{B}_r$. This is a contradiction. This proves $D_r \cap \mathcal{A}_r = \varnothing$, and $D_r \subset \mathcal{U}_r$. Hence, this shows that $d(O, D_r) > d(O, \mathcal{A}_r) = F(r)$. The lemma follows. □

Now we prove the main result of the paper.

Theorem 4.2 (Existence of H–planes) *Let Γ be a Jordan curve in $S^2_\infty(\mathbb{H}^3)$ differentiable at least at one point. Let $H \in (-1, 1)$. Then there exists a properly embedded minimizing H–plane $\mathcal{P}_H$ in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P}_H = \Gamma$.*

Proof Fix $H \in (-1, 1)$. Given Γ in $S^2_\infty(\mathbb{H}^3)$, we will use the same setup as before, ie let $\mathcal{A}, \mathcal{A}_r, \widehat{A}_r, O, \boldsymbol{B}_r, N_0$ be as in the previous lemma.

Let $\Omega_n = \boldsymbol{B}_n \cap CH_H(\Gamma)$ for $n > N_0$. Let γ_n be an essential smooth curve in $\widehat{A}_n \subset \partial\Omega_n$. Then by construction, $\gamma_n \to \Gamma$ as $\partial_\infty CH_H(\Gamma) = \Gamma$. Recall that the mean curvature of the geodesic sphere of radius R is $\coth R$. Hence, $\boldsymbol{B}_n$ is 1–convex for any n. Then, by Lemma 2.4, there exists an embedded minimizing H–disk D_n in $\boldsymbol{B}_n$ with $\partial D_n = \gamma_n$. Here, the sign of H determines the minimizing H–disk as there are two minimizing H–disks in $\boldsymbol{B}_n$ facing each other for $H \neq 0$ [8]. Furthermore, since $\gamma_n \subset CH_H(\Gamma)$, by Lemma 2.9 (see also Remark 2.10), $D_n \subset CH_H(\Gamma)$. Hence, this implies $D_n \subset \Omega_n = \boldsymbol{B}_n \cap CH_H(\Gamma)$.

We claim that the sequence of embedded minimizing H–disks $\{D_n\}$ converges to a minimizing H–plane $\mathcal{P}_H$. First, we claim that the sequence of minimizing H–disks $\{D_n\}$ has a convergent subsequence in any compact set K in $\mathbb{H}^3$. Then, by using the diagonal sequence argument as before, we will get a limit minimizing H–plane in $\mathbb{H}^3$.

Consider the closed balls $\boldsymbol{B}_K$ with center O. By using convexity of $\boldsymbol{B}_K$, for generic $K > N_0$, we can assume $D_n \cap \boldsymbol{B}_K$ is a collection of disks for any n by [2, Lemma 4.2].

Now, let β be a transversal arc in $CH_H(\Gamma)$ connecting $\partial^+ CH_H(\Gamma)$ and $\partial^- CH_H(\Gamma)$ through the point O. Let l be the length of β. Fix a generic $K_0 > N_0$ such that $F(K_0) > l$. Then by Lemma 4.1, and the proof of Theorem 3.2, $D_n^0 = D_n \cap \boldsymbol{B}_{K_0}$ is a closed disk (only one component) for any n. Hence, the sequence of minimizing H–disks $\{D_n^0\}$ has a uniform area bound, say $|D_n^0| < |\partial \boldsymbol{B}_{K_0}|$. Then, by following the proof of [2, Theorem 4.1], the existence of smoothly embedded minimizing H–plane Σ_H can be shown as follows:

With the uniform area bound, the sequence of minimizing H–disks $\{D_n^0\}$ has a convergent subsequence in B_{K_0} by the compactness theorem for integral currents. Hence by considering them as integral currents, a subsequence of $\{D_n^0\}$ converges to a properly embedded minimizing H–disk D^0 in $\boldsymbol{B}_{K_0}$. Let K_i be a monotone increasing sequence with $K_i \nearrow \infty$. For $K_1 > K_0$, by starting with this subsequence, get another subsequence converging on $\boldsymbol{B}_{K_1}$. By iterating this process and the diagonal sequence argument, we get a sequence of integral currents $\{D_n\}$ that converges on compacts to the integral current Σ_H in $\mathbb{H}^3$. By Allard's regularity, the convergence is smooth on compact sets. Also, the asymptotic boundary of the support of Σ_H is Γ by the convex hull property (Lemma 2.9), ie $\partial_\infty \Sigma_H = \Gamma$ as $\partial_\infty CH_H(\Gamma) = \Gamma$.

The limit of minimizing H–disks Σ_H is a minimizing H–surface. Hence, by [14], for any point p in the support of Σ_H, there exists $\epsilon > 0$ with $\boldsymbol{B}_\epsilon(p) \cap \Sigma_H$ is a smooth embedded disk. Hence, the support of Σ_H is smoothly embedded surface. Finally, since the convergence is smooth in compact sets, and $\{D_n\}$ is a sequence of embedded disks, Σ_H is a complete minimizing H–plane in $\mathbb{H}^3$ with $\partial_\infty \Sigma_H = \Gamma$. The proof follows. □

Remark 4.3 (Pairs of H–planes) Notice that if we forget the sign of the mean curvature H, the theorem above shows that for a given Jordan curve Γ in $S^2_\infty(\mathbb{H}^3)$, there exist two minimizing H–planes $\mathcal{P}_H^+$ and $\mathcal{P}_H^-$ with $\mathcal{P}_H^\pm = \Gamma$ for $H \in (0,1)$. Furthermore, these H–planes are disjoint, and the convex sides are facing each other.

Remark 4.4 The above result was shown for special families of curves in $S^2_\infty(\mathbb{H}^3)$ like star-shaped curves [13], "mean convex" curves [17], and graph over a line [18], where they showed that the area-minimizing surface Σ_H is indeed a graph over a geodesic subspace in $\mathbb{H}^3$.

Corollary 4.5 (Proper embeddings) *Let Γ be a $C^{3,\alpha}$ smooth Jordan curve in $S^2_\infty(\mathbb{H}^3)$. Let $\mathcal{P}_H$ be a minimizing H–plane in $\mathbb{H}^3$ with $\partial_\infty \mathcal{P}_H = \Gamma$. Then $\mathcal{P}_H$ is properly embedded in $\mathbb{H}^3$.*

Proof By [21], since Γ is $C^{3,\alpha}$ smooth, $\mathcal{P}_H$ is regular near infinity. In particular, there exists a $\rho > 0$ such that in the upper half-space model of $\mathbb{H}^3$, $\mathcal{P}_H \cap \{z < \rho\}$ is a graph over $\Gamma \times (0,\rho)$. Then, for sufficiently large N, $\partial \boldsymbol{B}_N \cap \mathcal{P}_H$ is a Jordan curve γ in $\partial \boldsymbol{B}_N$. Hence, $\boldsymbol{B}_N \cap \mathcal{P}_H = \mathcal{D}$ is a minimizing H–disk in $\boldsymbol{B}_N$ with $\partial \mathcal{D} = \gamma$ by the definition of minimizing H–plane. By Lemma 2.4, $\mathcal{D}$ is properly embedded. Since $\mathcal{P}_H - \mathcal{D}$ is a graph over $\Gamma \times (0,\rho)$ in the upper half-space model [21], the proof follows. □

Remark 4.6 Indeed, it can be showed that the above result is true for far more generality. By using the techniques in [5], one can naturally generalize the above result to Jordan curves in $S^2_\infty(\mathbb{H}^3)$ differentiable at least at one point. Even though there is no regularity near infinity in that case, by using Lemma 4.1, the arguments in [5] can easily be adapted.

Note also that Meeks, Tinaglia and the author [9] recently showed that there exists a *nonproperly* embedded complete H–plane Σ_H in $\mathbb{H}^3$ for any $H \in [0,1)$. In particular, Σ_H is an H–plane between two rotationally invariant H–catenoids $\mathcal{C}_1$ and $\mathcal{C}_2$ where Σ_H spirals into $\mathcal{C}_1$ in one end, and spirals into $\mathcal{C}_2$ in the other end. Hence, $\partial_\infty \Sigma_H$ is a pair of infinite lines l^+ and l^- in $S^2_\infty(\mathbb{H}^3)$. Here, if $\partial_\infty \mathcal{C}_i = \alpha_i^+ \cup \alpha_i^-$, and $A^\pm$ is the annuli in $S^2_\infty(\mathbb{H}^3)$ with $\partial A_i^\pm = \alpha_1^\pm \cup \alpha_2^\pm$, then $l^+ \subset A^+$ and $l^- \subset A^-$ where $l^\pm$ spirals into $\alpha_1^\pm$ in one end, and spirals into $\alpha_2^\pm$ in the other end.

5 Final remarks

5.1 Generic uniqueness of minimizing H–planes

The generic uniqueness results for minimizing H–surfaces in $\mathbb{H}^3$ [4] can naturally be generalized to our context, ie minimizing H–planes. In particular, for fixed $H \in (-1,1)$, let Σ_1 and Σ_2 be minimizing H–planes with $\partial_\infty \Sigma_i = \Gamma_i$, where Γ_1 and Γ_2 are disjoint simple closed curves in $S^2_\infty(\mathbb{H}^3)$. Then, by using the Meeks–Yau exchange roundoff trick, it can be showed Σ_1 and Σ_2 are disjoint, too. By using this, and similar ideas to [4], it can be showed that any simple closed curve Γ bounds either a unique minimizing H–plane Σ in $\mathbb{H}^3$, or there are two canonical disjoint minimizing H–planes Σ^+ and Σ^- with $\partial_\infty \Sigma^\pm = \Gamma$. Hence, foliating an annular neighborhood of Γ in $S^2_\infty(\mathbb{H}^3)$ with simple closed curves, and considering the canonical H–planes constructed, one can get generic uniqueness result as in [4]. In particular, this shows that *for fixed $H \in (-1,1)$, a generic Jordan curve Γ in $S^2_\infty(\mathbb{H}^3)$ bounds a unique minimizing H–plane Σ in $\mathbb{H}^3$ with $\partial_\infty \Sigma = \Gamma$.*

5.2 Foliations of $\mathbb{H}^3$ by H–planes

Similar to the previous part, it is also possible to show that two H–planes in $\mathbb{H}^3$ with different H values, which are asymptotic to the same asymptotic curve in $S^2_\infty(\mathbb{H}^3)$ are disjoint. In particular, for a given Jordan curve Γ in $S^2_\infty(\mathbb{H}^3)$, and given $-1 < H_1 < H_2 < 1$, it can be showed that the minimizing H–planes $\mathcal{P}_{H_1}$ and $\mathcal{P}_{H_2}$ in $\mathbb{H}^3$ with $\mathcal{P}_{H_i} = \Gamma$ are disjoint by using the ideas in [6]. Hence, if Γ bounds a unique minimizing H–plane for any $H \in (-1,1)$, it can be showed that the family of planes

$\mathcal{F}_\Gamma = \{\mathcal{P}_H \mid \partial_\infty \mathcal{P}_H = \Gamma \text{ and } -1 < H < 1\}$ foliates $\mathbb{H}^3$. By the disjointness of the planes $\{\mathcal{P}_H\}$ for different H, in order to get the foliation, all one needs to show is that there is no gap between the planes $\{\mathcal{P}_H\}$ by using the uniqueness. By the arguments in [6], a gap between the planes, say between $\{\mathcal{P}_H \mid -1 < H \leq H_0\}$ and $\{\mathcal{P}_H \mid H_0 < H < 1\}$, implies the nonuniqueness for H_0–planes with $\mathcal{P}_{H_0} = \Gamma$. This gives a contradiction, and shows that $\mathcal{F}_\Gamma$ foliates $\mathbb{H}^3$. For example, if Γ is a star-shaped curve in $S^2_\infty(\mathbb{H}^3)$, then the family $\mathcal{F}_\Gamma$ foliates $\mathbb{H}^3$ [6; 13; 18].

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Massachusetts Institute of Technology, Mathematics Department
Cambridge, MA 02139, USA

Department of Mathematics, Koç University
Istanbul 34450, Turkey

`bcoskunuzer@ku.edu.tr`

Proposed: Tobias H Colding — Received: 20 March 2015
Seconded: Dmitri Burago, Gang Tian — Revised: 7 May 2015