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Correction to the article
An infinite-rank summand of topologically slice knots

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We describe an error in the proof of a key proposition of our paper *An infinite-rank summand of topologically slice knots* (Geom. Topol. 19 (2015) 1063–1110), which was necessary for the proof of the main result. Alternative proofs of the main result are given by Ozsváth, Stipsicz and Szabó, and Dai, Hom, Stoffregen and Truong.

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In [4], we constructed a homomorphism

$$\mathcal{C} \to \mathcal{CFK},$$

where $\mathcal{C}$ denotes the smooth knot concordance group and $\mathcal{CFK}$ consists of knots Floer complexes modulo ε–equivalence. We then applied the Hahn embedding theorem to show that

$$\mathcal{CFK} \hookrightarrow \mathbb{R}^X.$$

These results still stand.

However, Proposition 1.3 of [4], which claimed that this embedding could be used to construct an infinite family of $\mathbb{Z}$–valued homomorphisms from $\mathcal{CFK}$, is false. The error is that we quoted a version of the Hahn embedding theorem for vector spaces — see Hausner and Wendel [3] — not groups. In particular, condition (2) above Definition 1.2 of [4] does not hold for groups.

To see that Proposition 1.3 is false for groups, consider the following example, due to B Gordon and found in Clifford [1]. Order $\mathbb{Q}^2$ lexicographically. Let G be the (ordered) subgroup of $\mathbb{Q}^2$ generated by (p_n^{-1}, np_n^{-1}), where p_n denotes the n^{th} prime. Then $(0,1) \in G$ satisfies the hypothesis of Proposition 1.3, but not the conclusion. In particular, $(0,1) \in G$ satisfies Property A but the image of projection from G onto its second coordinate is not isomorphic to $\mathbb{Z}$.

The main purpose of defining $\mathbb{Z}$–valued concordance homomorphisms was to prove the following result about $\mathcal{C}_{TS}$, the subgroup of the smooth concordance group generated by topologically slice knots.

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Theorem 1 *The group $\mathcal{C}_{TS}$ contains a direct summand isomorphic to $\mathbb{Z}^\infty$.*

An independent proof of Theorem 1 is given by Ozsváth, Stipsicz and Szabó [5], using their concordance homomorphism Υ. (Ironically, Ozsváth, Stipsicz and Szabó state in their introduction that Υ was inspired by [4].)

Recent work of Dai, Stoffregen, Truong and the author [2] provides a new infinite family of $\mathbb{Z}$–valued concordance homomorphisms φ_n that factor through $\mathcal{CFK}$. The definition of φ_n still relies on the total order of $\mathcal{CFK}$, although it does not rely on the Hahn embedding theorem. The homomorphisms φ_n give an alternative proof of Theorem 1, using the same knots as in [4].

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