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Behrend's function is not constant on $\mathrm{Hilb}^n(\mathbb{A}^3)$

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We prove that Behrend's function is not constant on $\mathrm{Hilb}^n(\mathbb{A}^3)$ for $n \geq 24$.

14C05, 14N35; 14L30

1 Introduction

The Behrend function ν_X of a scheme X of finite type over $\mathbb{C}$ is a constructible function introduced in the seminal paper [2]. It plays a central role in Donaldson–Thomas theory: if X is proper and admits a symmetric perfect obstruction theory, then the degree of the virtual cycle $[X]^{\mathrm{vir}}$ equals the ν_X-weighted Euler characteristic of X. It has been conjectured that $\nu_{\mathrm{Hilb}^n(\mathbb{A}^3)}$ is constant, equal to $(-1)^n$ (see Ricolfi [15, Conjecture A] and Jelisiejew [10, Problem XXVI]), although the conjecture is older than these references. A reason for this expectation is the fact that the value of the Behrend function is indeed $(-1)^n$ on all smooth points of the smoothable component and on all monomial ideals of $\mathrm{Hilb}^n(\mathbb{A}^3)$; see Maulik, Nekrasov, Okounkov and Pandharipande [11]. Constancy would imply that $\mathrm{Hilb}^n(\mathbb{A}^3)$ is generically reduced, as explained in [15], which would solve a major open problem; see Coskun [4] and Fogarty [6]. The Behrend function is notoriously difficult to compute. For X 0-dimensional, an algebraic method was developed by Graffeo and Ricolfi [9]. In general, a powerful localization method was introduced by Behrend and Fantechi:

Theorem 1 [3, Theorem 3.4] *Let X be an affine $\mathbb{C}^*$-scheme with an isolated fixed point P. Assume that X admits an equivariant symmetric obstruction theory. Then $\nu_X(P) = (-1)^{\dim_{\mathbb{C}} T_X|_P}$.*

The assumption that X is affine can be removed by using (a very special case of) a theorem of Alper, Hall and Rydh [1, Theorem 4.4]. In our applications X embeds into a smooth variety, so one can use work of Sumihiro [18] instead.

For more on symmetric equivariant obstruction theories, we refer to Behrend [2], Behrend and Fantechi [3] and Ricolfi [14]. The Hilbert scheme $\mathrm{Hilb}^n(\mathbb{A}^3)$ admits such an obstruction theory provided that $\mathbb{C}^*$ lies inside the Calabi–Yau torus $\{(t_1, t_2, t_3) \in (\mathbb{C}^*)^3 \mid t_1 t_2 t_3 = 1\}$. For any $\mathbb{C}^*$ (in the usual 3-dimensional torus) acting on $\mathrm{Hilb}^n(\mathbb{A}^3)$ such that the fixed points are isolated, the fixed points correspond to monomial ideals. For these, the value of the Behrend function is known to be $(-1)^n$, by combining Theorem 1 with the virtual localization calculation of Maulik, Nekrasov, Okounkov and Pandharipande [11]. For a smooth point P of the smoothable component of $\mathrm{Hilb}^n(\mathbb{A}^3)$, we have $\nu_{\mathrm{Hilb}^n(\mathbb{A}^3)}(P) = (-1)^n$. The tools above are enough to prove that $\nu_{\mathrm{Hilb}^n(\mathbb{A}^3)}$ is constant for $n \le 4$, where $n = 4$ is the first interesting case.[1] Other than that, little is known. Here we provide a first example where $\nu_{\mathrm{Hilb}^n(\mathbb{A}^3)}$ is *not* constant.

Theorem 2 (Theorem 6) *For $n \ge 24$, there exist points of $\mathrm{Hilb}^n(\mathbb{A}^3)$ on which the Behrend function is equal to $-(-1)^n$. In particular, the Behrend function is not constant. For $n = 24$ one such point is $[I]$, where $I \subset \mathbb{C}[x, y, z]$ is the ideal*

$$I = ((x^2) + (y, z)^2)^2 + (y^3 - x^3 z).$$

Although the point P of the theorem is a fixed point of a suitable $\mathbb{C}^*$-action, it is *nonisolated*, so we cannot apply Theorem 1 directly. We circumvent this by taking a quotient by another transverse free $\mathbb{C}^*$-action such that P becomes an isolated fixed point on the quotient. Since this transverse $\mathbb{C}^*$ is not in the Calabi–Yau torus, the usual symmetric obstruction theory is not equivariant and it is not clear whether it descends to the quotient. We construct another superpotential on the quotient, using an "average" of the usual superpotential, and apply Theorem 1 to the symmetric obstruction theory obtained from this other superpotential.

Our argument is quite general and does not use much about the Hilbert scheme, only some equivariance properties of the superpotential; see Section 3.

1.1 Parity of the tangent space

The constancy of the Behrend function is related to another conjecture disproved earlier this year. The conjecture predicted that for every point $P \in \mathrm{Hilb}^n(\mathbb{A}^3)$,

$$\dim_{\mathbb{C}} T_{\mathrm{Hilb}^n(\mathbb{A}^3)}|_P \equiv n \pmod 2. \tag{1-1}$$

The relation between the two conjectures is Theorem 1. Equation (1-1) was established for monomial ideals in [11] and recently for homogeneous ideals (even with respect to some nonstandard gradings) by Ramkumar and Sammartano [13]. However, a counterexample to (1-1) was given by F Giovenzana, L Giovenzana, Graffeo and Lella [8].

Although their counterexample does not directly fit the hypotheses in Section 3, our example in Theorem 2 is very closely related and the present paper would not exist without their example.

1.2 Open questions

It is unknown whether $\mathrm{Hilb}^n(\mathbb{A}^3)$ is irreducible[2] for $n = 12$; see also Jelisiejew [10, Problem V]. Proving irreducibility for $n = 24$ seems hopeless, however in [7], F Giovenzana, L Giovenzana, Graffeo and Lella

[1] As explained by Andrea Ricolfi in a private conversation.

[2] There is an ongoing project by Sema Gunturkun to prove this.

show that the point described in Theorem 2 lies in the smoothable component. Due to the connection with reducedness (see Ricolfi [15]) it is natural to ask: *is* $\mathrm{Hilb}^{24}(\mathbb{A}^3)$ *reduced at* $[I]$? We do not know the answer. In general, it is not known whether $\mathrm{Hilb}^n(\mathbb{A}^3)$ is reduced; see [10, Problem I]. See also [7] for results and questions for the Quot scheme.

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2 Behrend function in the equivariant setup

All schemes we consider are of finite type over $\mathbb{C}$. Let P be a fixed point for an action of an algebraic group G on a scheme X. We say that P is *isolated* if P is equal *as a scheme* to a connected component of the fixed locus X^G. If P is smooth and G is linearly reductive, then P is isolated if and only if $\{P\}$ is a connected component of the topological space X^G. However, even on a normal variety X, it may happen that a singular point P is isolated topologically, but X^G is nonreduced at P.

For a scheme X over $\mathbb{C}$, the Behrend function [2] is a constructible function $\nu_X\colon X(\mathbb{C}) \to \mathbb{Z}$ which gives rise to an "Euler characteristic description" of Donaldson–Thomas invariants when X is a moduli space of stable sheaves on a smooth projective Calabi–Yau 3-fold. The value $\nu_X(x)$ depends only on (any) étale neighbourhood of $x \in X$ [2, page 1309], and for a morphism $f\colon X \to Y$ smooth at $x \in X$ we have $\nu_X(x) = (-1)^{\dim f^{-1}(f(x))}\nu_Y(f(x))$; see [2, page 1309; 17, Tag 039P]. It is generally very hard to determine the value of the Behrend function at a given point; see [9] for results in the case when X is 0-dimensional.

2.1 Critical loci

For generalities on critical loci and symmetric obstruction theories, we refer to [3; 14]. We recall some basics for completeness. For a smooth variety X and a global 1-form $\omega \in H^0(\Omega_X)$, we have a vanishing scheme $Z(\omega)$. Over an open subset U on which we have a trivialization $\Omega_X|_U \simeq \bigoplus_{i=1}^{\dim X} \mathcal{O}_U\, dz_i$ with $\omega = \sum f_i\, dz_i$, the vanishing scheme $Z(\omega) \cap U = Z(\omega|_U)$ is given by $f_1 = \cdots = f_{\dim X} = 0$. For a *smooth* morphism $\varphi\colon Y \to X$ and the form df, for some global regular function $f\colon X \to \mathbb{C}$, we have

$$\varphi^{-1}(Z(df)) = Z(d(f \circ \varphi)). \tag{2-1}$$

The following notation will be useful to us.

Definition 3 Let G be an algebraic group, X a G-variety and $f\colon X \to \mathbb{C}$ a morphism. Let $\chi\colon G \to \mathbb{C}^*$ be a character. We say that f is a *G-semi-invariant with weight* χ if $f(g \cdot P) = \chi(g) f(P)$ for all $g \in G$ and $P \in X$.

Proposition 4 (critical locus of a semi-invariant) *Let G be an algebraic group and $\chi\colon G \to \mathbb{C}^*$ be a character that is nontrivial on the connected component*[3] *of $1_G \in G$. Let B be a smooth variety and $f\colon G \times B \to \mathbb{C}$ a G-semi-invariant with weight χ, where G acts trivially on B. Take $\bar{f} := f(1_G, -)\colon B \to \mathbb{C}$. Then*

$$Z(df) = \mathrm{pr}_B^{-1}(Z(d\bar{f}) \cap Z(\bar{f})),$$

where $\mathrm{pr}_B\colon G \times B \to B$ is the projection.

We observe that if χ above is trivial, then $Z(df) = \mathrm{pr}_B^{-1}(Z(d\bar{f}))$, so the claim fails.

Proof By equivariance of f, we have $f(g, b) = \chi(g) f(1, b) = \chi(g)\bar{f}(b)$. Therefore

$$df = \chi d\bar{f} + \bar{f} d\chi. \tag{2-2}$$

The character $\chi\colon G \to \mathbb{C}^*$ is a surjection of algebraic groups, and hence it is a smooth morphism onto a curve, so its differential is nowhere vanishing and $Z(\bar{f} d\chi) = Z(\bar{f})$. The function χ is also nowhere vanishing, so $Z(\chi d\bar{f}) = Z(d\bar{f})$. The two summands in (2-2) are elements of distinct summands of the direct sum $\mathrm{pr}_G^* \Omega_G \oplus \mathrm{pr}_B^* \Omega_B \simeq \Omega_{G \times B}$, and thus

$$Z(df) = Z(\chi d\bar{f}) \cap Z(\bar{f} d\chi) = \mathrm{pr}_B^{-1}(Z(d\bar{f}) \cap Z(\bar{f})),$$

as claimed. □

For examples of functions f such that $Z(df)$ does not contain $Z(f)$ and for related pathologies, see for example [12, Appendix A; 16].

2.2 General setup

Let $f\colon A \to \mathbb{C}$ be a regular function on a smooth variety A. Let $M := Z(df)$ be the critical locus of f and $P \in M$ a closed point. We are interested in the value $\nu_M(P)$ of the Behrend function.

For any $d \geq 0$, let G be a d-dimensional affine algebraic group acting regularly on A. Let $T_0 \cong \mathbb{C}^*$ be an algebraic torus acting regularly on A, and commuting with G, such that:

[3] In particular, the connected component of $1_G \in G$ is not 0-dimensional.

- f is T_0-invariant and f is a G-semi-invariant (Definition 3).
- $\mathrm{Stab}_G(P) = \{1\}$ and P lies in the T_0-fixed locus.
- The T_0-fixed part of the tangent space at P, $T_M|_P^{T_0} \subset T_M|_P$, is d-dimensional.

Suppose further that there is another linearly reductive group H acting regularly on A, and commuting with G and T_0, such that P lies in the H-fixed locus and f is an H-semi-invariant with character $\chi\colon H \to \mathbb{C}^*$ which is *nontrivial* on the connected component of $1_H \in H$.

Theorem 5 *In the setup above, we have* $\nu_M(P) = (-1)^{\dim_{\mathbb{C}} T_M|_P}$.

Proof First we reduce to the case of A being a product $G \times B$. By assumption, the stabilizer $\mathrm{Stab}_{G\times H\times T_0}(P)$ is equal to $H \times T_0$, and hence is linearly reductive. A strong version of Luna's étale slice theorem [1, Theorem 4.5] provides an affine scheme B with a $(H\times T_0)$-action, a $(H\times T_0)$-fixed point $Q \in B$ and a $(G\times H\times T_0)$-equivariant étale morphism $\mu\colon G \times B \to A$ mapping $P' := (1_G, Q)$ to P. Since A is smooth, it follows that B is a smooth variety. Let $f' := f \circ \mu\colon G \times B \to \mathbb{C}$. Since $\mu^{-1}(Z(df)) = Z(df')$, we obtain an étale morphism $\mu\colon M' := Z(df') \to M$ and $\nu_{M'}(P') = \nu_M(P)$. We note that f' is T_0-invariant, and is a G- and H-semi-invariant with the same characters as before. Moreover $\mathrm{Stab}_G(P') = \{1\}$, P' lies in the T_0- and H-fixed loci, and $T_{M'}|_{P'}^{T_0}$ is d-dimensional.

Let $\psi = f'(1_G, -)\colon B \to \mathbb{C}$. We claim that

$$\mathrm{pr}_B^{-1}(Z(d\psi)) = Z(df'). \tag{2-3}$$

Then $N := Z(d\psi) \subset B$ has a T_0-equivariant symmetric perfect obstruction theory by [3, Example 1.19]. Moreover, since $T_{M'}|_{P'}^{T_0}$ is d-dimensional, it follows that $T_N|_Q^{T_0}$ is 0-dimensional and $Q \in N$ is an isolated T_0-fixed point, so we may apply Theorem 1. Therefore, assuming (2-3), we get that $Z(df') \simeq G \times N$ and so we obtain

$$\nu_M(P) = \nu_{M'}(P') = (-1)^d \nu_N(Q) = (-1)^{d+\dim_{\mathbb{C}} T_N|_Q} = (-1)^{\dim_{\mathbb{C}} T_{M'}|_{P'}} = (-1)^{\dim_{\mathbb{C}} T_M|_P},$$

which concludes the proof.

It remains to prove (2-3). Let $\sigma_H\colon H \times B \to B$ be the action map, so $\sigma_H(h,b) = h \cdot b$. The action map is smooth, since it is a composition of the isomorphism $(h,b) \mapsto (h, h\cdot b)$ and a projection $\mathrm{pr}_B\colon H \times B \to B$ where H is smooth. Let $\sigma\colon G\times H\times B \to G\times B$ be the induced smooth map, defined by $\sigma(g,h,b) = (g, h\cdot b)$. We have the following situation:

$$\begin{tikzcd}
G \times H \times B \arrow[d, "\sigma"] & & \\
G \times B \arrow[r, "\text{étale}"] \arrow[rr, bend left, "f'"] \arrow[d, "\mathrm{pr}_B"] & A \arrow[r] & \mathbb{C} \\
B \arrow[rr, "\psi"] & & \mathbb{C}
\end{tikzcd}$$

Consider the functions $f'' = f' \circ \sigma$ and $\psi'' = \psi \circ \mathrm{pr}_B \circ \sigma$. They are both $(G \times H)$-semi-invariants with characters that are nontrivial on the connected component of $(1_G, 1_H) \in G \times H$. Moreover, the functions $f''(1_G, 1_H, -) = \psi''(1_G, 1_H, -)$ are both equal to ψ. Applying Proposition 4 to each of them, we obtain

$$Z(df'') = p^{-1}(Z(d\psi) \cap Z(\psi)) = Z(d\psi''),$$

where $p\colon G \times H \times B \to B$ is the projection to B. The maps σ and pr_B are smooth, so

$$\sigma^{-1}(Z(df')) = Z(df'') = Z(d\psi'') = \sigma^{-1}(Z(d(\psi \circ \mathrm{pr}_B)) = \sigma^{-1} \mathrm{pr}_B^{-1} Z(d\psi).$$

So the preimages of both sides of (2-3) under a smooth surjective map σ are equal. Hence both sides of (2-3) are equal. The map σ has a natural section, so the equality also follows by restricting to this section. $\square$

3 Applications to Hilbert schemes

Let $M := \mathrm{Hilb}^n(\mathbb{A}^3)$ be the Hilbert scheme of n points on $\mathbb{A}^3$. We recall the well-known fact [5; 14; 19] that there exists a smooth variety A with regular function $f\colon A \to \mathbb{C}$ such that $M = Z(df)$. Let V be an n-dimensional complex vector space and consider the quiver in Figure 1.

We consider representations of this quiver with dimension vector $(1, n)$, ie we put the vector space $\mathbb{C}$ at the node ∞ and V at the node 0. Representations with this dimension vector correspond to elements of $W = \mathbb{C}^3 \otimes \mathrm{End}(V) \oplus V$. The group $\mathrm{GL}(V)$ acts on W by

$$g \cdot (X, Y, Z, v) := (gXg^{-1}, gYg^{-1}, gZg^{-1}, gv).$$

Let $\widetilde{A} \subset W$ be the open subset consisting of representations satisfying

$$\mathbb{C}\langle X, Y, Z\rangle \cdot \langle v\rangle_{\mathbb{C}} = V.$$

Then $\mathrm{GL}(V)$ acts freely on $\widetilde{A}$, and the noncommutative Hilbert scheme is defined as the smooth variety

$$A := \mathrm{ncHilb}^n(\mathbb{A}^3) = \widetilde{A}/\mathrm{GL}(V).$$

The regular function $\tilde{f}\colon \widetilde{A} \to \mathbb{C}$ given by $\tilde{f}(X, Y, Z, v) = \mathrm{tr}(X[Y, Z])$ is invariant under the action of $\mathrm{GL}(V)$ and descends to a regular function $f\colon A \to \mathbb{C}$, and $M = Z(df)$. Furthermore, we have a regular action of $(\mathbb{C}^*)^3$ on $\widetilde{A}$ defined by

$$(t_1, t_2, t_3) \cdot (X, Y, Z, v) = (t_1 X, t_2 Y, t_3 Z, v),$$

which descends to an action on A. Clearly

$$f((t_1, t_2, t_3) \cdot P) = t_1 t_2 t_3 f(P) \quad \text{for all } (t_1, t_2, t_3) \in (\mathbb{C}^*)^3 \text{ and } P \in A.$$

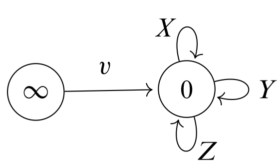

Figure 1: Quiver for the noncommutative Hilbert scheme.

Theorem 6 *For $n \geq 24$, there exist points of $\mathrm{Hilb}^n(\mathbb{A}^3)$ on which the Behrend function is equal to $-(-1)^n$. In particular, the Behrend function is not constant. For $n = 24$ one such point is $[I]$, where I is the following ideal in $\mathbb{C}[x, y, z]$:*

$$I = ((x^2) + (y, z)^2)^2 + (y^3 - x^3 z).$$

Proof We write $M = \mathrm{Hilb}^{24}(\mathbb{A}^3)$ and $P = [I]$. By hand or using Macaulay2, one shows that $\mathbb{C}[x, y, z]/I$ is 24-dimensional and $T_M|_P = \mathrm{Hom}_{\mathbb{C}[x,y,z]}(I, \mathbb{C}[x, y, z]/I)$ is 99-dimensional; see the appendix. In particular, $(-1)^{\dim_{\mathbb{C}} T_M|_P} \neq (-1)^{24}$.

We want to apply Theorem 5 with $P = [I]$, $M = \mathrm{Hilb}^{24}(\mathbb{A}^3)$ and A, f as above. For this, we define the tori

$$T_0 := \{(t^2, t, t^{-3}) \in (\mathbb{C}^*)^3\}, \quad G := \{(1, 1, t) \in (\mathbb{C}^*)^3\}, \quad H := \{(t, t, 1) \in (\mathbb{C}^*)^3\}.$$

Clearly f is T_0-invariant and is a G-semi-invariant with weight 1. Moreover, f is an H-semi-invariant with weight 2. We also note that $\mathrm{Stab}_G(P) = \{1\}$ and P is T_0-fixed and H-fixed. Again by hand or Macaulay2, we check that $T_M|_P^{T_0}$ is 1-dimensional in the appendix. The monomial $x^3 z$ maps to a socle element in the quotient $\mathbb{C}[x, y, z]/I$ and $y^3 - x^3 z$ is a minimal generator of I, so the transformation that sends $y^3 - x^3 z$ to $x^3 z$ and other minimal generators to zero extends to a nonzero tangent vector in $T_M|_P$. This vector spans the 1-dimensional space $T_M|_P^{T_0}$.

Therefore, by Theorem 5, $\nu_M(P) = -1$. Clearly any $Q \in M$ corresponding to a reduced scheme is a smooth point with $(3{\cdot}24)$-dimensional tangent space, so it satisfies $\nu_M(Q) = 1$. Thus the Behrend function is not constant on M. To obtain the claim for $n \geq 24$, add a tuple of disjoint points in $\mathbb{A}^3$ to $[I]$: étale locally the neighbourhood of the scheme obtained in this way is a product of the neighbourhood of a disjoint tuple of points and the neighbourhood of $[I]$. By the properties of the Behrend function on étale neighbourhoods and smooth maps, we conclude the result. □

Appendix Macaulay2 code

```
kk = QQ;
S = kk[x, y, z, Degrees=>{{1, 2}, {2, 1}, {3, -3}}];
I = ideal mingens((ideal(x^2) + ideal(y^2, y*z, z^2))^2 + ideal(y^3 - x^3*z));
tgI = Hom(I, S^1/I);
assert(rank source basis(S^1/I) == 24); -- Hilbert scheme of 24 points
assert(rank source basis(tgI) == 99); -- 99-dimensional tangent space
for i in -1000..1000 do (
ress := rank source basis({i, 0}, tgI);
if ress != 0 then
print((i,0), ress);
);
```

Macaulay2 technical note One must work with an auxiliary grading, since $(2, 1, -3)$ raises a `no heft vector` error. Alternatively, one can work with the purely nonnegative degrees $\{\{1, 0\}, \{1, 1\}, \{0, 3\}\}$.

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