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Problems around 3–manifolds

J H RUBINSTEIN

This is a personal view of some problems on minimal surfaces, Ricci flow, polyhedral geometric structures, Haken 4–manifolds, contact structures and Heegaard splittings, singular incompressible surfaces after the Hamilton–Perelman revolution.

57M50; 53C42, 53C44, 57N13, 57R17

We give sets of problems based on the following themes;

- Minimal surfaces and hyperbolic geometry of 3–manifolds. In particular, how do minimal surfaces give information about the geometry of hyperbolic 3–manifolds and conversely how does the geometry affect the types of minimal surfaces which can occur?
- Ricci flow. Here it would be good to be able to visualize Ricci flow and understand more about where and how singularities form. Also this leads into the next topic, via the possibility of a combinatorial version of Ricci flow.
- Combinatorial geometric structures. Various proposals have been made for combinatorial versions of differential geometry in dimension 3. However having a good model of sectional or Ricci curvature is a difficult challenge.
- Haken 4–manifolds. These are a class of 4–manifolds which admit hierarchies, in the same sense as Haken 3–manifolds. More information is needed to determine how abundant they are.
- Contact structures and Heegaard splittings. Contact structures are closely related to open book decompositions, which can be viewed as special classes of Heegaard splittings.
- Singular incompressible surfaces. A well-known and important conjecture is that all (closed or complete finite volume) hyperbolic 3–manifolds admit immersed or embedded closed incompressible surfaces. We give some variants of this problem.

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1 Minimal surfaces and hyperbolic geometry

Throughout, hyperbolic 3–manifolds will be either closed or complete with finite volume, unless indicated otherwise.

Problem 1 A quasi-Fuchsian subgroup of the fundamental group of a hyperbolic 3–manifold can be represented by a minimal (least area) surface by Schoen and Yau [52]. There are a number of natural questions about such minimal surfaces.

- *When is there a unique such surface representing a given quasi-Fuchsian subgroup, or equivalently in its homotopy class?*

Remark 1 Note that Ben Andrews has an unpublished flow which shows that if there is any surface in this homotopy class with all principal curvatures having absolute value strictly less than 1, then the flow produces a minimal surface with the same bounds on principal curvatures. (In fact, the maximum absolute value of principal curvature for this minimal surface is the smallest possible amongst all surfaces in the homotopy class). It then follows easily by a maximum principle argument due to Uhlenbeck [58] that this surface is indeed unique in its homotopy class. See also Taubes [57] and Krasnov and Schlenker [38] for more recent results on properties of minimal surfaces in hyperbolic 3–manifolds, with the latter reference concentrating on the quasi-Fuchsian case. See also Rubinstein [48] for some other properties of minimal surfaces, including construction of examples of parallel minimal surfaces in hyperbolic 3–manifolds, by using very short closed geodesics as barriers.

- *How does the geometry of such minimal surfaces correspond to the geometry of the corresponding quasi-Fuchsian 3–manifold, in the case that there is a unique minimal surface in the quasi-Fuchsian homotopy class? For example, how does the second fundamental form and area of such a minimal surface correspond to the volume of the convex core of the quasi-Fuchsian 3–manifold? Could there be a sequence of quasi-Fuchsian surface groups of fixed genus, with unique minimal surface and volume of the convex core approaching infinity?*
- *The generalised Gauss map (cf Krasnov and Schlenker [38] and Epstein [14]) for a quasi-Fuchsian 3–manifold, with a minimal surface with all principal curvatures having absolute value strictly less than one, defines a special homeomorphism between the two Riemann surfaces defined by the domains of discontinuity. Is there a way of using this homeomorphism to relate the distance between the two Riemann surfaces, using any of the usual distances on Teichmüller space, with properties of the minimal surface?*

Problem 2 (S T Yau) *Does every closed Riemannian (or in particular hyperbolic) 3–manifold admit infinitely many closed embedded minimal surfaces?*

Remark 2 One could also ask an easier question allowing merely immersions for the minimal surfaces—cf Rubinstein [48] for some discussion of this weaker question. Also note the analogy with the closed geodesic problem in Riemannian geometry, which asks to find infinitely many simple closed embedded or immersed geodesic loops in a hyperbolic 3–manifold. Note that it is not known even that a hyperbolic 3–manifold always has infinitely many such embedded loops—cf Kuhlmann [39] for some recent work on this latter problem. The techniques of Kapouleas [37] may be relevant to this problem, since if one could find two embedded intersecting minimal surfaces and perform a 'desingularisation' by adding arbitrarily many handles along the curves of intersection, then this would give a solution. Note also that it seems possible that any Heegaard splitting of sufficiently large genus (not necessarily irreducible or strongly irreducible) might be realisable by a minimal surface, but this seems to be a very difficult question.

Problem 3 *Can a hyperbolic 3–manifold, which is a closed surface bundle over a circle, have the property that every leaf of the bundle structure is a minimal surface?*

Remark 3 Sullivan [55] constructed metrics which have this property, but they are unlikely to be hyperbolic. Note that if such a hyperbolic 3–manifold M has some very short geodesic loops, as can happen by doing large longitudinal Dehn surgery along a loop which lies in a leaf, then it is straightforward to prove that not all the leaves of M can be minimal. For the barrier argument in [48] together with a simple maximum principle argument gives a contradiction, as minimal leaves cannot touch such a barrier on the convex side.

Problem 4 *Can minimal surfaces be constructed as suitable limits of normal surfaces?*

Remark 4 Note if a 3–manifold is embedded isometrically into a high dimensional Euclidean space, by Nash's theorem, then one can take a suitable net of closely spaced points so that the convex hull of subsets of 4 points gives a Euclidean triangulation, which is a good approximation to the Riemannian 3–manifold. For a fine enough net, any particular smooth surface will be well approximated by a normal surface, relative to such a triangulation. Does this give a useful way of producing sequences of triangulations and normal surfaces, which converge to all minimal surfaces?

Problem 5 (H Rosenberg) *R Bryant [7] showed that surfaces of constant mean curvature* $+1$ *have beautiful closed form representations in hyperbolic space* $\mathbf{H}^3$*. (This* *is analogous to the Weierstrass representation of minimal surfaces in* $\mathbf{R}^3$ *). There is a rich theory of complete Bryant surfaces, which is the usual term for surfaces of constant mean curvature* $+1$*, in hyperbolic geometry. Show that every closed or complete finite volume hyperbolic 3–manifold has an embedded closed Bryant surface.*

Remark 5 One approach would be to study a surface of smallest area, which divides the 3–manifold up into two regions of fixed volume. Such a surface will have constant mean curvature, but there are difficulties in establishing the existence of smooth solutions. For the 2–sphere, there are several ways of producing a simple closed geodesic loop. Two important ones are the minimax method constructing a curve of least length dividing the surface into two pieces of equal area. See Hass and Scott [23] and Croke [11]. One could start by studying the analogous problem of finding loops of constant curvature on a Riemannian 2–sphere.

Problem 6 *Find a 'topological' description of embedded closed minimal surfaces in hyperbolic 3–manifolds.*

Remark 6 In the seven Thurston's geometries which are non hyperbolic, one has a complete understanding of how minimal surfaces can occur topologically—essentially as either incompressible surfaces or as Heegaard surfaces or suitable generalisations. See [48] for a discussion of this. It is known that strongly irreducible splittings can be isotoped to be minimal, or to double cover a one-sided surface after a single handle pinch. Also incompressible surfaces can be isotoped to be minimal. On the other hand, by the barrier argument in [48], it is clear that there are many other topological types of minimal surfaces. Namazi [45] and Namazi and Souto [46] construct long cylindrical regions in hyperbolic 3–manifolds. Do these regions contain many disjoint parallel minimal surfaces?

2 Ricci flow

Problem 7 *Is there a 'weak' solution to Ricci flow which continues through singularities?*

Remark 7 Brakke [6] achieved such a setup for mean curvature flow and this has proved to be very useful. For a more recent account of generalized mean curvature flow, see Ecker [12]. In a brief numerical study (Rubinstein and Sinclair [50]), it is observed for rotationally symmetric neck pinching that one might be able to continue by allowing indefinite metrics in the part of the manifold where the pinching has occurred. If weak solutions could be found, then the advantage would be to continue flowing on manifolds, without the need to perform surgery. For an analysis of singularity formation in the rotationally symmetric case, see Simon [54] and Angenent and Knopf [5].

Problem 8 *For an atoroidal Haken 3–manifold, singularities will form away from a hierarchy consisting of least area incompressible surfaces. Is there a way of continuing the flow in a neighbourhood of the hierarchy after such singularities have formed, without surgery?*

Remark 8 Note that Hamilton [22] has analysed neck pinching and it is clear that least area incompressible surfaces will stay away from such necks. Moreover Gabai [19] has shown that even though minimal surfaces do not move continuously as metrics are deformed, one can interpolate between their positions in a useful way. So one could study the motion of the hierarchy right up to the moment of a singularity and possibly use this as a guide to continue the flow past the singularity, as all the topological structure of the manifold is contained in the hierarchy.

Problem 9 *Even if the initial curvature is strictly negative, it is not known if the Ricci flow develops singularities. The cross curvature flow of Chow–Hamilton [8] has been proposed to show that any negatively curved metric flows directly to a hyperbolic metric. The cross-curvature flow is a special flow which is only defined for 3–manifolds, since by the construction of Gromov–Thurston [21], it is known that there are arbitrarily pinched negatively curved metrics on higher dimensional manifolds, which cannot be deformed to be hyperbolic. If the cross curvature flow could be shown to work as above, this would show that the path components of the space of negatively curved metrics on a 3–manifold are contractible, following Gabai's solution of the Smale conjecture for hyperbolic 3–manifolds [19].*

Problem 10 *Find a combinatorial Ricci flow. To do this, we first need a robust combinatorial definition of sectional curvature or Ricci curvature in dimension 3. This is very challenging. Similarly, is there a combinatorial analogue of Perelman's entropy? Chow and Luo [9] have studied combinatorial Ricci flow for surfaces. Various proposals have been made for deforming shapes of tetrahedra by flows, but to obtain success, one would need to choose the 'right' triangulation to start with. Otherwise a feature of the flow should involve changing the cell decomposition.*

Problem 11 *If Ricci flow did not develop singularities, one could deduce a proof of the Smale conjecture for all geometric 3–manifolds simultaneously! Is there any way of using Ricci flow with surgery to deduce information about the homotopy type of the space of diffeomorphisms?*

Remark 9 Note that the space of Riemannian metrics on a manifold is homotopy equivalent to the manifold. Hence if a 3–manifold admits a geometric structure, then the space $\mathcal{D}$ of diffeomorphisms of the manifold to itself induce the collection of all geometric structures $\mathcal{D}/\mathcal{I}$, where $\mathcal{I}$ is the group of isometries of the geometric structure. If one could retract the metrics onto the geometric structures by Ricci flow, this would prove that the diffeomorphism group is homotopy equivalent to the isometry group. Various cases of the Smale conjecture have been established for classes of geometric 3–manifolds by Hatcher [24; 25; 26], Ivanov [30; 31], Gabai [19], McCullough and Rubinstein [42], and Hong, McCullough and Rubinstein [29].

Problem 12 *Can we visualise Ricci flow, eg, is there a natural way of embedding Ricci flow as a submanifold flow in $\mathbf{R}^N$ for N large? If so then level set methods (cf Sethian [53]) could be used for numerical study of Ricci flow.*

3 Combinatorial geometric structures

Problem 13 *Find hyperbolic structures on 3–manifolds by solving the gluing equations for a suitable triangulation. In the case of an irreducible atoroidal manifold with tori boundary, the natural choice is an ideal triangulation.*

Remark 10 Casson has suggested a flow along the gradient of volume in the ideal case, to change the shapes of tetrahedra in a suitable way. Feng Luo [41] has shown that a similar flow can be defined in the closed case. Lackenby [40] used taut foliations and sutured hierarchies (Gabai [18], Scharlemann [51]) to construct taut ideal triangulations in the ideal case. Kang and Rubinstein [36] showed that these deform to angle structures if and only if a certain normal surface theory obstruction vanishes for the triangulation. The challenge is to find ways of modifying a taut triangulation to eliminate this obstruction, to at least always obtain an angle structure. In the closed case, a related question is whether any hyperbolic 3–manifold admits a one vertex straight triangulation, i.e. all edges would then be geodesic loops with a possible corner at the single vertex? Properties of these triangulations are also interesting—for example are these 1–efficient (cf Jaco and Rubinstein [33; 34])?

Problem 14 *If an atoroidal 3–manifold has a cubing of non-positive curvature, does Ricci flow immediately produce a metric of strictly negative sectional curvature? Does the metric converge to a hyperbolic one without forming singularities?*

Remark 11 See Aitchison and Rubinstein [2] for background information on cubings of non-positive curvature. Mosher [44] proved that atoroidal 3–manifolds with such cubings have word hyperbolic fundamental groups. M Simon [54] has studied the behaviour of singular metrics under Ricci flow. One might hope to find a similar polyhedral metric of non-positive curvature on atoroidal Haken 3–manifolds, as in problem 16 below. If this could be done, one might hope to reprove Thurston's geometrisation theorem, using Ricci flow.

Problem 15 *Do all hyperbolic 3–manifolds M have cubical resolutions? These are compact cubical complexes $\mathcal{X}$ which have CAT(0) structures on their universal covers and are homotopy equivalent to M. See Rubinstein [47] and Rubinstein and Sageev [49] for discussion on how cubical resolutions can be constructed.*

Problem 16 *Haken 3–manifolds have very short hierarchies (Aitchison and Rubinstein [4]). Can these be used in the atoroidal case to get a combinatorial deformation theory to obtain CAT(0) or combinatorial hyperbolic metrics?*

Remark 12 Note that the gluing up of very short hierarchies is extremely simple. One can take a handlebody H and divide ∂H into two collections of subsurfaces, coloured black/white. This colouring has to satisfy the condition that every meridian disk for H has at least two black and two white arcs in its boundary. Then one glues together the black regions in pairs and the white regions become closed incompressible surfaces. Finally the 3–manifolds formed from such handlebodies are glued together along the closed incompressible surfaces to form M. It turns out that the characteristic variety has a simple description and so it is easy to detect whether the manifold is atoroidal (cf Rubinstein [47]). There is also a fairly straightforward argument that the fundamental group of M is word hyperbolic (cf Rubinstein [47] and Swarup [56]).

4 Haken 4–manifolds

Problem 17 (B Foozwell) *Show that every pair of Haken 3–manifolds M, M' are Haken cobordant, ie, there is a Haken 4–manifold W so that $\partial W = M \cup M'$ and the inclusions $M, M' \subset W$ induce injections on fundamental groups.*

Remark 13 As a special case, given a Haken 3-manifold M, construct a Haken 4–manifold W so that $\partial W = M$ and the inclusion $M \subset W$ is an injection on fundamental groups. Haken 4–manifolds are studied by Foozwell in [16] and are built from hierarchies of π_1–injective 3–manifolds. Note the recent examples of Ivansic, Ratcliffe and Tschantz [32] which are Haken 4–manifolds obtained by removing a knotted 2–torus from the 4–sphere!

Problem 18 *Show that if W_1, W_2 are Haken 4–manifolds and there is a homotopy equivalence between W_1 and W_2, then W_1 and W_2 are homeomorphic.*

Remark 14 Note that this can be viewed as a weak version of Waldhausen's classical result in dimension 3 (cf [59]). However here we need to assume that *both* manifolds are Haken, whereas for Waldhausen, only one is Haken and the other is assumed merely irreducible. Note also that Haken 4–manifolds have 'large' fundamental groups, so Freedman's surgery theory cannot be applied.

Problem 19 *Study π_1–injective 3–manifolds in hyperbolic 4–manifolds. Many examples of hyperbolic 4–manifolds come from Coxeter group constructions, so naturally contain interesting immersed totally geodesic 3–manifolds which are certainly π_1–injective. However the construction in Aitchison, Matsumoto and Rubinstein [1] appears possible to apply to some hyperbolic 4–manifolds with cusps, to construct many immersed 3–manifolds with principal curvatures at most $+1$, which are then π_1–injective. Also one could try to study Freedman–Freedman twisting [17] in dimension 4, assuming that suitable π_1–injective spanning 3–manifolds can be found for 4–manifolds with π_1–injective boundary.*

5 Contact structures and Heegaard splittings

Problem 20 *Find a topological characterisation of the class of 3–manifolds which admit the existence of tight contact structures. Many results have been obtained on this problem—see Etnyre and Honda [15] and Honda [27; 28]. There is a connection to the existence of taut foliations, following the argument of Eliashberg and Thurston [13], where a taut foliation is deformed to a tight contact structure.*

Problem 21 *Giroux [20] has shown that contact structures correspond to certain types of open book decompositions of 3–manifolds. There is an interesting connection between open book decompositions and Heegaard splitting theory. We say that a Heegaard splitting (M, H_+, H_-) can be* flipped *if there is an isotopy of M interchanging the two handlebodies H_+, H_-. Is it true that the Heegaard splitting can be flipped if and only if the Heegaard surface $S = \partial H_+ = \partial H_-$ consists of two pages of an open book decomposition, so that the binding lies on S? Another version of this question is to ask if the Heegaard surface S has an infinite automorphism group and M is atoroidal, then is it true that S comes from an open book decomposition?*

Remark 15 Note that in general, a Heegaard splitting might be two pages of several open book decompositions. Equivalently, there could be several fibred knots lying on the Heegaard surface dividing it into two pages. (Clearly reducible splittings can be constructed with this property.) In this case, there will be many flipping diffeomorphisms obtained by composition, which do not correspond in a simple way to open book structures. So this makes the problem quite challenging. Note also that this problem is closely related to the question of whether the natural map between the group G of isotopy classes of diffeomorphisms of the Heegaard surface, which extend over the two handlebodies and the group G' of isotopy classes of diffeomorphisms of M has a finite or infinite kernel. For more information on diffeomorphisms of M which preserve a Heegaard surface S, called automorphisms of the Heegaard splitting, see Johnson and Rubinstein [35].

Problem 22 *Find classes of closed orientable hyperbolic 3–manifolds, which only have a single irreducible or strongly irreducible Heegaard splitting, up to isotopy. Are such 3–manifolds common or rare?*

Remark 16 One likely class is obtained by large Dehn filling on the figure 8 knot space. It is easy to see that the figure 8 knot space has only one irreducible splitting, since the only almost normal surfaces are obtained by adding an unknotted tube parallel to an edge to the peripheral torus. By the technique in Moriah and Rubinstein [43], if one does suitably large Dehn filling, then any low genus irreducible/strongly irreducible splitting will actually be a splitting of the figure 8 knot space. Hence it is the unique splitting. However it is not clear that this is true for all splittings.

6 Singular incompressible surfaces

Problem 23 (W Thurston) *Does every closed or finite volume complete hyperbolic 3–manifold admit an immersed or embedded closed surface with all principal curvatures at most one? It is easy to show that such surfaces are π_1–injective. (Compare with problem 1.) Note in Aitchison, Matsumoto and Rubinstein [1], it is shown that the figure 8 knot space has a huge number of such surfaces.*

Remark 17 Here is a wonderful sketch of Thurston giving a way of studying this problem. Suppose we take a random point and a random totally geodesic plane Π through that point x in a closed hyperbolic 3–manifold M. As one takes an expanding domain in Π starting at x, the boundary curve Γ will grow exponentially, but will be in the bounded size manifold M. Consequently if one could arrange that points in Γ can be paired up to be reasonably close together, then by a small enough bending of Π, a new plane P with small enough principal curvature could be found and a new domain and boundary curve C, which could be glued to itself to form a smooth surface satisfying the required conditions.

Problem 24 (W Thurston) *Does every closed hyperbolic 3–manifold admit a finite sheeted cover by a surface bundle over a circle? Note in Aitchison and Rubinstein [3], a number of sets of examples of this type are constructed. The basic idea is to glue together a collection of fiberings of a fundamental domain. Can this method be made more systematic, to produce a suitable 'normal surface theory' of immersed foliations?*

Remark 18 In [3], the vertex link structure must be a *regular branched cover*, to ensure that the singular foliation glues up to an immersion at each vertex. So for example, for a cubing of non-positive curvature, one needs to suppose that every edge has degree a multiple of 4 and every vertex link is a regular branched cover of an octahedron, to deduce that the manifold has an immersed fibration which lifts to a surface bundle structure in a finite sheeted cover. See Rubinstein [47] for further discussion of this necessary condition.

Problem 25 *Suppose that a 3–manifold M has a triple handlebody decomposition satisfying a disk condition. Does M have an immersed incompressible surface without triple points? Are such surfaces separable, ie, do they lift to embeddings in finite sheeted coverings?*

Remark 19 Triple handlebody decompositions are obtained by gluing three handlebodies along subsurfaces of their boundaries. Triple curves are then the boundaries between the different boundary subsurfaces. A disk condition is the requirement that all meridian disks in the three handlebodies intersect the triple curves at least (m, n, p) times, where $\frac{1}{m} + \frac{1}{n} + \frac{1}{p} \leq \frac{1}{2}$. In [10], numerous properties and constructions of 3–manifolds of this type are given. The class can be viewed as generalising Seifert fibred spaces with infinite fundamental groups and Haken 3–manifolds. One can view this question as the analogue of the well-known fact that Seifert fibred spaces with infinite fundamental groups all have immersed incompressible tori without triple points.

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Department of Mathematics and Statistics, University of Melbourne
Parkville, 3010, Australia

rubin@ms.unimelb.edu.au

www.ms.unimelb.edu.au/~rubin

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