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For a group $G$ with generating set $S = \{s_1, s_2, \ldots, s_k\}$, the $\mathbb{G}$-graph of $G$, denoted by $\Gamma(G, S)$, is the graph whose vertices are distinct cosets of $\langle s_i \rangle$ in $G$. Two distinct vertices are joined by an edge when the set intersection of the cosets is nonempty. In this paper, we explore the planarity of $\Gamma(G, S)$.

## 1. Introduction

Let $G$ be a group with a generating set $S = \{s_1, \ldots, s_k\}$. We say that the subset $T_{\langle s_i \rangle} \subset G$ is a *left transversal* for the subgroup $\langle s_i \rangle$ of $G$ if $\{x \langle s_i \rangle \mid x \in T_{\langle s_i \rangle}\}$ is precisely the set of all left cosets of $\langle s_i \rangle$ in $G$. As in [Bauer et al. 2008], we associate with $(G, S)$ a simple graph $\Gamma(G, S)$ with vertex set $V(\Gamma(G, S)) = \{x_j \langle s_i \rangle \mid x_j \in T_{\langle s_i \rangle}\}$. Two distinct vertices $x_j \langle s_i \rangle$ and $x_l \langle s_k \rangle$ in $V(\Gamma(G, S))$ are joined by an edge if $x_j \langle s_i \rangle \cap x_l \langle s_k \rangle$ is nonempty. The edge set, $E(\Gamma(G, S))$, consists of pairs $(x_j \langle s_i \rangle, x_l \langle s_k \rangle)$. $\Gamma(G, S)$ defined this way has no multiedge or loop.

Let $V_i = \{x_j \langle s_i \rangle \mid x_j \in T_{s_i}\}$. Then $V = \bigcup_{i=1}^{k} V_i$. The number of vertices in $V_i$ is simply the order of $G$ divided by the order of $s_i$ which is the index of $\langle s_i \rangle$ in $G$, denoted $[G : \langle s_i \rangle]$. The minimum number of elements required to generate a finite group $G$ is called the *rank of* $G$. A *minimal generating set for* $G$ is a subset $S = \{s_1, \ldots, s_k\}$ such that $G = \langle S \rangle$, where $k$ is the rank of $G$. This concept is not to be confused with nonredundancy. A *nonredundant* set of generators is a set $S$ such that $S$ generates all of $G$, that is, $\langle S \rangle = G$, but no proper subset of $S$ generates all of $G$.

The main object of this paper is to explore the planarity of $\Gamma(G, S)$.

**Definition 1.1.** A group $G$ is $\mathbb{G}$-planar if there exists a generating set $S$ such that the graph, $\Gamma(G, S)$, is a planar graph.

We recall a fundamental criterion for the $\mathbb{G}$-planarity of a group:

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*Keywords:* groups, graphs, generators.

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**Theorem 1.2** (Wagner). *A finite graph is planar if and only if it does not have $K_5$ or $K_{3,3}$ as a minor.*

## 2. Examples of $\mathbb{G}$-planar groups

The next two theorems give us two classes of $\mathbb{G}$-planar groups.

**Theorem 2.1.** *All cyclic groups are $\mathbb{G}$-planar.*

*Proof.* Let $G$ be a cyclic group. Since $G$ is cyclic, there exists an element $b \in G$ such that $\langle b \rangle = G$. Let $S = \{b\}$ be the generating set of $G$. Then $\Gamma(G, S)$ contains only one vertex and $\Gamma(G, S)$ is a planar graph. Therefore $G$ is a $\mathbb{G}$-planar group. □

For the dihedral group, $D_n$, let $r$ be a rotation of $360°/n$ and let $f$ be any reflection.

**Proposition 2.2.** *For $S = \{f, rf\}$, the graph $\Gamma(G, S)$ of the dihedral group $D_n$ is the cycle of length $2n$, $C_{2n}$.*

*Proof.* Write

$$V_1 = \{\langle f\rangle, r\langle f\rangle, r^2\langle f\rangle, \ldots, r^{n-1}\langle f\rangle\},$$
$$V_2 = \{\langle rf\rangle, r\langle rf\rangle, r^2\langle rf\rangle, \ldots, r^{n-1}\langle rf\rangle\}.$$

Since $f$ and $rf$ are both reflections, their composition is a rotation. Denote this rotation by $r^m$.

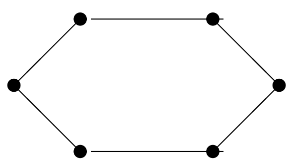
Choose a vertex from $V_1$, $r^s\langle f\rangle$. Since

$$r^s \in r^s\langle f\rangle \cap r^s\langle rf\rangle,$$

the edge $(r^s\langle f\rangle, r^s\langle rf\rangle)$ is in $E$. Now we need to show that there is another edge between $r^s\langle f\rangle$ and $V_2$. By simple calculation, we have $r^s f = r^{(s+m) \bmod n} rf$; moreover $(r^s\langle f\rangle, r^{(s+m) \bmod n}\langle rf\rangle)$ is in $E$.

Therefore the degree of each vertex in $V_1$ is 2. By similar arguments, the degree of each vertex in $V_2$ is 2 and $\Gamma(G, S)$ is a cycle. □

**Example 2.3.** Let $G = D_3$ and $S = \{f, rf\}$. Then the $\mathbb{G}$-graph is the cycle $C_6$:



**Theorem 2.4.** *All dihedral groups are $\mathbb{G}$-planar.*

*Proof.* Let $G = D_n$ and $S = \{f, rf\}$. Since $\Gamma(G, S)$ is a cycle, $\Gamma(G, S)$ is a planar graph and $G$ is a $\mathbb{G}$-planar group. □

From [DeWitt et al. ≥ 2010], we have a few other examples of $\mathbb{G}$-planar groups.

**Example 2.5.** The modular group $M$ has presentation

$$\langle s, t \mid s^8 = t^2 = e, st = ts^5\rangle.$$

Let $S = \{s, ts\}$. From [DeWitt et al. ≥ 2010], $\Gamma(M, S)$ is $K_{2,2}$. Therefore $\Gamma(M, S)$ is a planar graph and $M$ is a $\mathbb{G}$-planar group.

**Example 2.6.** The quasihedral group $QS$ has presentation

$$\langle s, t \mid s^8 = t^2 = e, st = ts^3\rangle.$$

Let $S = \{s, ts\}$. From [DeWitt et al. ≥ 2010], $\Gamma(QS, S)$ is $K_{2,4}$. Therefore $\Gamma(QS, S)$ is a planar graph and $QS$ is a $\mathbb{G}$-planar group.

Recall that the generalized quaternion group $Q_{2^n}$ has presentation

$$\langle s, t \mid s^{2^{n-1}} = e, s^{2^{n-2}} = t^2, tst^{-1} = s^{-1}\rangle.$$

**Theorem 2.7.** *The generalized quaternion group $Q_{2^n}$ is $\mathbb{G}$-planar.*

*Proof.* Let $G = Q_{2^n}$ and $S = \{ts^k, ts^m\}$, where $k$ is odd and $m$ is even. $\Gamma(G, S)$ is a bipartite connected graph with every vertex of degree 2 [DeWitt et al. ≥ 2010]. Therefore, $\Gamma(G, S)$ is a cycle and $Q_{2^n}$ is $\mathbb{G}$-planar. □

## 3. Finite abelian groups

The fundamental theorem of finite abelian groups tells us that every finite abelian group of rank $k$ is isomorphic to a direct product of cyclic groups of prime-power order, that is, $G \cong \mathbb{Z}_{m_1} \times \mathbb{Z}_{m_2} \times \cdots \times \mathbb{Z}_{m_k}$. A *standard generating set for $G$* is a subset $S = \{s_1, \ldots, s_k\}$ such that $G = \langle s_1\rangle \times \cdots \times \langle s_k\rangle$. Let $G$ be an abelian group with standard generating set $S = \{s_1, \ldots, s_k\}$, then $G$ is isomorphic to

$$\mathbb{Z}_{|s_1|} \times \mathbb{Z}_{|s_2|} \times \cdots \times \mathbb{Z}_{|s_k|}.$$

From Theorem 2.1, we know that all finite abelian groups with 1 generator are $\mathbb{G}$-planar. We now consider three cases: finite abelian groups with 4 or more generators, 3 generators or 2 generators.

Let $G$ be a group with generating set $S$. There exists a subset of $S$, $S'$, that is nonredundant and generates $G$. From [Bretto and Gillibert 2004], $\Gamma(G, S')$ is necessarily a subgraph of $\Gamma(G, S)$. If $\Gamma(G, S')$ is not a planar graph, then $\Gamma(G, S)$ is not planar. Therefore, it is only necessary to consider generating sets that are nonredundant.

**Example 3.1.** Let $G = \mathbb{Z}_2 \times \mathbb{Z}_6$ and $S = \{(1, 0), (0, 0), (0, 2), (0, 3), (0, 4)\}$. The subset $S' = \{(1, 0), (0, 2), (0, 3)\}$ of $S$ is a nonredundant generating set of $G$. The set $S'' = \{(1, 0), (0, 1)\}$ is a minimal generating set of $G$ that is also nonredundant.

**Lemma 3.2.** *Let $G$ be a finite abelian group and let $S = \{s_1, s_2, s_3, \ldots, s_k\}$ be a nonredundant generating set, then $|s_i| \geq 2$ for all $i$.*

*Proof.* Assume $|s_i| < 2$. Then $|s_i| = 1$ and $\langle s_i \rangle = \{e\}$. Therefore $s_i$ is not needed to generate $G$ and $S \setminus \{s_i\}$ generates $G$. This is a contradiction. Therefore, $|s_i| \geq 2$. □

***Finite abelian groups G with 4 or more generators.***

**Lemma 3.3.** *Let $G$ be a finite abelian group and let $S = \{s_1, s_2, s_3, s_4, \ldots, s_k\}$ be a nonredundant generating set of $G$ with $k \geq 4$. Consider the subgroup $H$ of $G$ that is generated by $S' = \{s_1, s_2, s_3, s_4\}$. The vertices $\langle s_1 \rangle$, $\langle s_2 \rangle$, $\langle s_3 \rangle$, $\langle s_4 \rangle$, $s_1\langle s_2 \rangle$, $s_2\langle s_1 \rangle$, $s_2\langle s_3 \rangle$, $s_3\langle s_2 \rangle$, $s_3\langle s_4 \rangle$, $s_4\langle s_3 \rangle$ of $\Gamma(H, S')$ are all unique.*

*Proof.* To see that each of these vertices is unique, assume $\langle s_1 \rangle, s_2\langle s_1 \rangle \in V_1$ are not distinct, that is, $\langle s_1 \rangle = s_2\langle s_1 \rangle$. So there exists $k \in \mathbb{Z}^+$ such that $s_2 = s_1^k$ which contradicts the fact that $S$ is a nonredundant generating set of $G$. The proofs of the other cases are similar. □

**Theorem 3.4.** *Let $G$ be a finite abelian group and let $S = \{s_1, s_2, s_3, s_4, \ldots, s_k\}$ be a nonredundant generating set of $G$ with $k \geq 4$. Then $\Gamma(G, S)$ is not a planar graph.*

*Proof.* Consider the subgroup $H$ of $G$ generated by $S' = \{s_1, s_2, s_3, s_4\}$. Define a contraction $\Gamma$ of $\Gamma(H, S')$ in this way: Let $\overline{V}_1, \overline{V}_2, \overline{V}_3, \overline{V}_4, \overline{V}_5 \in V(\Gamma)$ with

$$\{\langle s_1 \rangle\} = \overline{V}_1, \quad \{\langle s_2 \rangle\} = \overline{V}_2, \quad \{\langle s_3 \rangle\} = \overline{V}_3, \quad \{\langle s_4 \rangle\} = \overline{V}_4,$$
$$\{s_1\langle s_2 \rangle, s_2\langle s_1 \rangle, s_2\langle s_3 \rangle, s_3\langle s_2 \rangle, s_3\langle s_4 \rangle, s_4\langle s_3 \rangle\} = \overline{V}_5.$$

Then, $e \in (\overline{V}_1 \cap \overline{V}_2)$, $e \in (\overline{V}_1 \cap \overline{V}_3)$, $e \in (\overline{V}_1 \cap \overline{V}_4)$, $s_1 \in (\overline{V}_1 \cap \overline{V}_5)$, $e \in (\overline{V}_2 \cap \overline{V}_3)$, $e \in (\overline{V}_2 \cap \overline{V}_4)$, $s_2 \in (\overline{V}_2 \cap \overline{V}_5)$, $e \in (\overline{V}_3 \cap \overline{V}_4)$, $s_3 \in (\overline{V}_3 \cap \overline{V}_5)$, and $s_4 \in (\overline{V}_4 \cap \overline{V}_5)$. Then $(\overline{V}_i, \overline{V}_j) \in E(\Gamma)$ for all $i \neq j$ and $\Gamma = K_5$. So, $\Gamma(H, S')$ has $K_5$ as a minor and $\Gamma(H, S')$ is not planar. From [Bretto et al. 2005], $\Gamma(H, S')$ is a subgraph of $\Gamma(G, S)$. Therefore, $\Gamma(G, S)$ is not a planar graph. □

**Corollary 3.5.** *Let $G$ be a finite abelian group of rank 4 or more. Then $G$ is not $\mathbb{G}$-planar.*

***Finite abelian groups G with 3 generators.***

**Example 3.6.** Let $G = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ with standard generating set

$$S = \{s_1, s_2, s_3\} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}.$$

The graph $\Gamma(G, S)$, illustrated in Figure 1, is a planar graph; hence $G$ is a $\mathbb{G}$-planar group.

Next we show that this example is the only abelian group of rank three that is $\mathbb{G}$-planar.

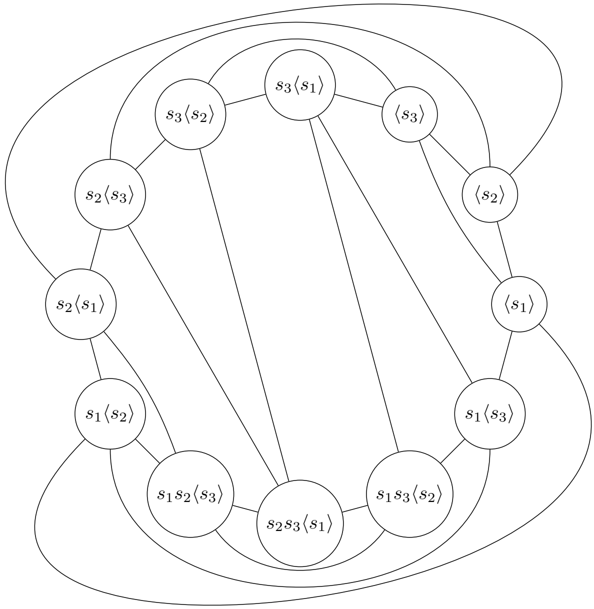



**Figure 1.** The graph $\Gamma(G, S)$, with $G = {\mathbb{Z}_2}^3$ and $S = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$.

**Lemma 3.7.** *Let $G$ be a finite abelian group with nonredundant generating set $S = \{s_1, s_2, s_3\}$ such that $|s_i| \geq 3$ for at least one $i$. Then the graph $\Gamma(G, S)$ contains at least 16 vertices.*

*Proof.* Without loss of generality, assume that $|s_3| \geq 3$. There are at least 6 vertices in $V_1$. They are $\langle s_1\rangle$, $s_2\langle s_1\rangle$, $s_3\langle s_1\rangle$, $s_2s_3\langle s_1\rangle$, $s_3^2\langle s_1\rangle$, $s_2s_3^2\langle s_1\rangle$. To see that each of these vertices is unique, assume $\langle s_1\rangle, s_2s_3\langle s_1\rangle \in V_1$ are not distinct, that is, $\langle s_1\rangle = s_2s_3\langle s_1\rangle$. So there exists $k \in \mathbb{Z}^+$ such that $s_2s_3 = s_1^k$ which contradicts the fact that $S$ is a nonredundant generating set of $G$. The proofs of the other cases are similar.

Likewise, there are at least 6 unique vertices in $V_2$ and 4 unique vertices on $V_3$. They are $\langle s_2\rangle$, $s_1\langle s_2\rangle$, $s_3\langle s_2\rangle$, $s_1s_3\langle s_2\rangle$, $s_3^2\langle s_2\rangle$, $s_1s_3^2\langle s_2\rangle$ and $\langle s_3\rangle$, $s_1\langle s_3\rangle$, $s_2\langle s_3\rangle$, $s_1s_2\langle s_3\rangle$. □

**Theorem 3.8.** *Let $G$ be a finite abelian group with nonredundant generating set $S = \{s_1, s_2, s_3\}$ such that $|s_i| \geq 3$ for at least one $i$. Then $\Gamma(G, S)$ is not a planar graph.*

*Proof.* Define a contraction $\Gamma$ of $\Gamma(G, S)$ by setting

$$\overline{V}_1 = \{\langle s_1\rangle, \langle s_2\rangle\}, \qquad \overline{V}_2 = \{s_1\langle s_2\rangle, s_1s_2\langle s_3\rangle, s_1s_3\langle s_2\rangle\},$$
$$\overline{V}_3 = \{s_1\langle s_3\rangle, s_3^2\langle s_1\rangle, s_3^2\langle s_2\rangle\}, \qquad \overline{V}_4 = \{\langle s_3\rangle, s_3\langle s_2\rangle, s_3\langle s_1\rangle, s_2s_3\langle s_1\rangle\},$$
$$\overline{V}_5 = \{s_2\langle s_1\rangle, s_2\langle s_3\rangle, s_2s_3^2\langle s_1\rangle, s_1s_3^2\langle s_2\rangle\}.$$

Then

$$\begin{aligned} s_1 &\in (\overline{V}_1 \cap \overline{V}_2), & s_1 &\in (\overline{V}_1 \cap \overline{V}_3), \\ e &\in (\overline{V}_1 \cap \overline{V}_4), & s_2 &\in (\overline{V}_1 \cap \overline{V}_5), \\ s_1 &\in (\overline{V}_2 \cap \overline{V}_3), & s_1 s_2 s_3 &\in (\overline{V}_2 \cap \overline{V}_4), \\ s_1 s_2 &\in (\overline{V}_2 \cap \overline{V}_5), & s_3^2 &\in (\overline{V}_3 \cap \overline{V}_4), \\ s_3^2 s_2 &\in (\overline{V}_3 \cap \overline{V}_5), & s_2 s_3 &\in (\overline{V}_4 \cap \overline{V}_5). \end{aligned}$$

It follows that $(\overline{V}_i, \overline{V}_j) \in E(\Gamma)$ for all $i \neq j$ and $\Gamma = K_5$. So, $\Gamma(G, S)$ has $K_5$ as a minor and is not a planar graph. □

**Corollary 3.9.** *Let $G$ be a finite abelian group of rank 3 such that $G \ncong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. Then $G$ is not a $\mathbb{G}$-planar group.*

***Finite abelian groups $G$ with 2 generators.*** Since we have results for groups of rank 1 and for groups of rank 3 or more, the only case left to consider is that of groups of rank 2. Notice that any finite abelian group of rank 2 is isomorphic to the direct product $\mathbb{Z}_m \times \mathbb{Z}_n$ with $gcd(m, n) \neq 1$.

**Lemma 3.10.** *Let $G$ be a finite abelian group of rank 2 and let $S$ be a nonredundant generating set of $G$. If $|S| \geq 3$, then $\Gamma(G, S)$ is not a planar graph.*

*Proof.* If $|S| > 3$, then $\Gamma(G, S)$ is not planar by Theorem 3.4. Assume that $|S| = 3$, that is, $S = \{s_1, s_2, s_3\}$ and that $|s_i| < 3$ for $i = 1, 2, 3$. Since $S$ is nonredundant $|s_i| > 1$ and therefore $|s_i| = 2$ for $i = 1, 2, 3$. Consider the subset

$$H = \langle s_1 \rangle \langle s_2 \rangle = \{hk \mid h \in \langle s_1 \rangle, k \in \langle s_2 \rangle\} = \{e, s_1, s_2, s_1 s_2\}$$

of $G$. Since $G$ is abelian, this subset is a subgroup. Now consider the subset

$$K = H\langle s_3 \rangle = \{hk \mid h \in H, k \in \langle s_3 \rangle\} = \{e, s_1, s_2, s_1 s_2, s_3, s_1 s_3, s_2 s_3, s_1 s_2 s_3\}$$

of $G$. Again $K$ is necessarily a subgroup of $G$.

Now assume that $g \in G$. Since $S$ generates $G$, there exists $n, m, l$ such that $g = s_1^n s_2^m s_3^l$. Since the order of each generator is 2, $n, m, l$ are congruent to 0 or 1 modulo 2 and $g \in K$. Therefore $G = K$. Since the order of each element in $G$ is two, $G \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. This is a contradiction since $G$ is a group of rank 2. Therefore, $|s_i| \geq 3$ for at least one $i$ and by Theorem 3.8 the graph, $\Gamma(G, S)$, is not planar. □

**Theorem 3.11.** *Let $G$ be a finite abelian group of rank 2. $G$ is $\mathbb{G}$-planar if and only if $G \cong \mathbb{Z}_2 \times \mathbb{Z}_k$, for some $k \in \mathbb{N}$.*

*Proof.* ($\Leftarrow$) Let $G \cong \mathbb{Z}_2 \times \mathbb{Z}_k$ and let $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_k, S)$ be the associated $\mathbb{G}$-graph of $\mathbb{Z}_2 \times \mathbb{Z}_k$ with $S = \{(1, 0), (0, 1)\}$. There exist an isomorphism $\phi : \mathbb{Z}_2 \times \mathbb{Z}_k \to G$. Let $(x, y) \in \mathbb{Z}_2 \times \mathbb{Z}_k$. There exists $a, b$ such that $(x, y) = a(1, 0) + b(0, 1)$. Then $\phi(x, y) = \phi(a(1, 0) + b(0, 1)) = a\phi(1, 0) \oplus b\phi(0, 1)$. So $\phi(S) = \{\phi(1, 0), \phi(0, 1)\}$

| Rank | Group | Planarity |
|---|---|---|
| 1 | all $G$ | planar |
| 2 | $G \cong \mathbb{Z}_2 \times \mathbb{Z}_k$<br>$G \ncong \mathbb{Z}_2 \times \mathbb{Z}_k$ | planar<br>not planar |
| 3 | $G \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$<br>$G \ncong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ | planar<br>not planar |
| 4 or more | all $G$ | not planar |

**Table 1.** $\mathbb{G}$-planarity of finite abelian groups.

generates $G$. $\Gamma(\mathbb{Z}_2 \times \mathbb{Z}_k, \text{S})$ is $K_{k,2}$, so $K_{k,2} \cong \Gamma(G, \phi(S))$. Since $K_{k,2}$ is planar, $\Gamma(G, \phi(S))$ is planar. Therefore $G$ is $\mathbb{G}$-planar.

($\Rightarrow$) Let $G$ be a finite abelian $\mathbb{G}$-planar group of rank 2 and let $S$ be a generating set such that $\Gamma(G, S)$ is a planar graph. From Lemma 3.10, $|S| = 2$, that is, $S = \{s_1, s_2\}$.

**Case 1**. Assume that $|s_1| = 2$. Let $|G| = n$, $|V_1| = [G : \langle s_1\rangle] = n/2$. So

$$V_1 = \{\langle s_1\rangle, s_2\langle s_1\rangle, s_2^2\langle s_1\rangle, \cdots, s_2^{n/2-1}\langle s_1\rangle\},$$

and the elements of $G$ are of the form

$$s_2, s_2^2, \ldots, s_2^{n/2-1}, e \quad \text{and} \quad s_1s_2, s_1s_2^2, \ldots, s_1s_2^{n/2-1}, s_1.$$

Therefore $|s_2| = n/2$ and $G$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{n/2}$.

**Case 2**. Assume that $|s_1|, |s_2| > 2$. Consider the vertex induced subgraph generated by the six vertices $\langle s_1\rangle, s_2\langle s_1\rangle, s_2^2\langle s_1\rangle, \langle s_2\rangle, s_1\langle s_2\rangle, s_1^2\langle s_2\rangle$. This graph is $K_{3,3}$. Since this subgraph is not planar, $\Gamma(G, S)$ is not planar. This contradicts the supposition that $S$ is a generating set such that $\Gamma(G, S)$ is a planar graph. Therefore, if $G$ is $\mathbb{G}$-planar, then $G \cong \mathbb{Z}_2 \times \mathbb{Z}_k$. □

Table 1 summarizes the results for all finite abelian groups.

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