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Sarah Mason and Mali Zhang

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A graph-theoretical approach to solving Scramble Squares puzzles

Sarah Mason and Mali Zhang

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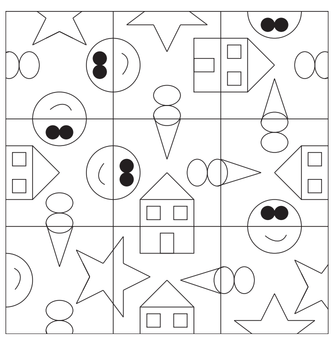
A Scramble Squares puzzle is made up of nine square pieces such that each edge of each piece contains half of an image. A solution to the puzzle is obtained when the pieces are arranged in a 3×3 grid so that the adjacent edges of different pieces together make up a complete image. We describe a graph-theoretical approach to solving Scramble Squares puzzles and a method for decreasing randomness in the backtracking solution algorithm.

1. Introduction

A Scramble Squares® puzzle (created and marketed by B. Dazzle, Inc.) consists of nine square pieces, each of which contains half of an image on each side. A solution to a Scramble Squares puzzle is an arrangement of the nine pieces into a 3×3 grid so that the adjacent half images on adjacent pieces together create a complete image. Here is an example of a solution to a Scramble Squares puzzle:



There are many different ways to arrange the pieces in an attempt to solve a Scramble Squares puzzle. There are nine different positions in the 3×3 grid and therefore 9! different ways to place the pieces into the grid, assuming that the pieces are pairwise distinct. Once the pieces have been placed, there are 4

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different orientations for each piece. This means that there are a total of $4^9 \times 9!$ different arrangements of the pieces. Taking into account rotational symmetry, if there is a solution there must be at least four, but still the probability of finding one of them by laying the pieces down at random can be as low as $4/(4^9 \times 9!)$, or about 4.2×10^{-11}. It would therefore be desirable to have an efficient algorithm for solving Scramble Squares puzzles, but this turns out to be quite a steep request since Scramble Squares are *constraint satisfaction problems* (CSPs) and many CSPs are known to belong to the NP-complete complexity class. The most efficient known algorithm for solving Scramble Squares puzzles is a depth first backtracking search developed by Brandt, Burger, Downing, and Kilzer [Brandt et al. 2002].

A visual representation of a problem can often provide key insights into the nature of the solution(s). The graph-theoretical solution to the Instant Insanity puzzle is a wonderful example of this phenomenon [Busacker and Saaty 1965; Carteblanche 1947; Grecos and Gibberd 1971; Van Deventer 1969]. The Instant Insanity puzzle consists of four unit cubes whose faces are colored arbitrarily with four colors. A solution is obtained by stacking the cubes into a vertical rectangular prism with dimensions $4 \times 1 \times 1$ so that each color appears exactly once on each side of the prism. Van Carteblanche [1947] introduces a method (elaborated upon by many [Busacker and Saaty 1965; Grecos and Gibberd 1971; Van Deventer 1969]) for representing the cubes as edges in a graph whose vertices correspond to the four colors. A solution is determined by choosing an appropriate subgraph. This graph-theoretical solution to Instant Insanity is the inspiration for this paper. We provide a graph-theoretical solution to a simplified Scramble Squares puzzle, following a similar approach. We also provide a method for ordering the pieces used in the backtracking algorithm in [Brandt et al. 2002] as a way to potentially improve upon its efficiency.

2. Restricted Scramble Squares puzzles

We begin by introducing the terminology and notations which will appear throughout this paper. A *pattern* is a complete image in the puzzle. Each pattern is comprised of two *pictures*, which are halves of the image. The *complement* of a picture is the other half of the pattern. A *piece* is one of the nine squares that make up a puzzle. See Figure 1 for an example.

In this section, we will restrict to puzzles containing four or fewer patterns. We do this for the sake of simplicity, but it would not be difficult to extend these results to puzzles with more patterns; in essence, it involves considering graphs with more vertices but whose solution graphs satisfy the same set of restrictions.

The recording graph. We provide a method to represent any Scramble Squares puzzle mathematically as a graph. Begin by assigning a number to each pattern.

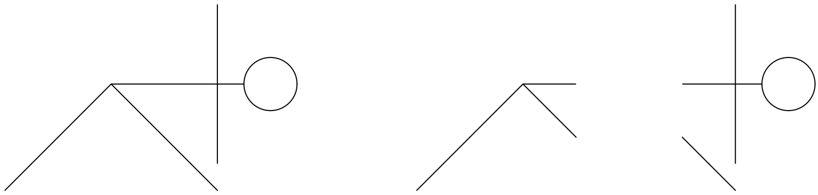


Figure 1. The two pictures on the right are *complements*, which together make up the *pattern* on the left.

Each pattern consists of two pictures, so associate a plus sign to one of the pictures and a minus sign to the other. This assigns a number and a sign to each picture appearing in the puzzle. Notice that two pictures with the same number but opposite signs together form a complete pattern. Also note that if $X+$ is the signed number corresponding to a given picture (of pattern $|X|$), then its complement X^c, is given by $X-$. For example, in Figure 2 the number 1 represents the star, the number 2 represents the ice cream cone, the number 3 represents the house, and the number 4 represents the smiling face. For this reason, we use the absolute value notation to denote the underlying pattern, so that $|X+| = |X-|$, and we frequently refer to a pair of complementary pictures as X and X^c.

A *repetition* in a puzzle piece is a picture which appears more than one time on the piece. Note that a picture $X+$ and its complement $X-$ appearing on the same piece do not constitute a repetition. We say that a puzzle is *repetition-free* if no piece of the puzzle contains a repetition. This means that a particular picture may appear multiple times in the puzzle, provided that each appearance is on a different piece. We restrict to 2×2 repetition-free puzzles but it would be interesting to extend these results to larger puzzles or puzzles containing repetitions. See Section 4 for details on this and other related open problems.

We construct a graph, called the *recording graph* $G(P)$, corresponding to a given Scramble Squares puzzle P as follows. The vertices of $G(P)$ are the symbols associated to the pictures appearing in the puzzle pieces. They are arranged into

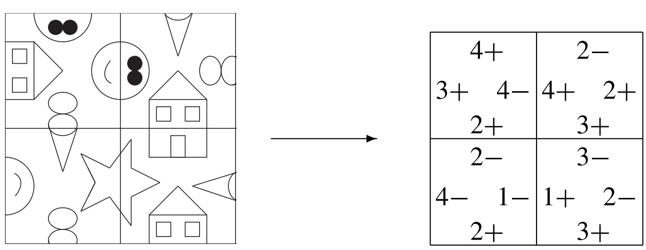


Figure 2. Converting pictures to symbols.

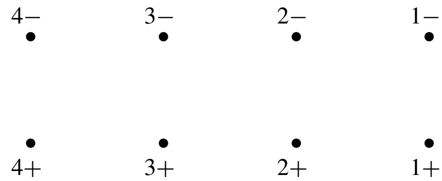


Figure 3. An edgeless graph.

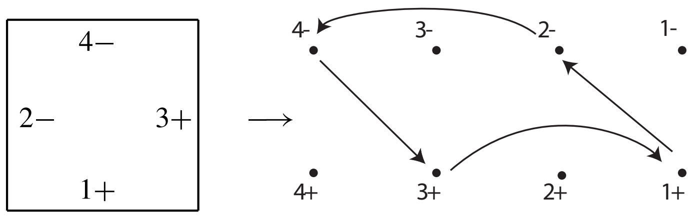


Figure 4. One piece of the puzzle becomes a length-4 directed cycle.

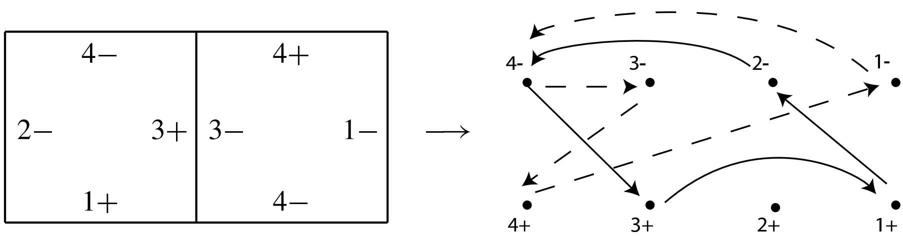


Figure 5. The left-hand piece is represented by solid lines, while the right-hand piece is represented by dashed lines.

two rows so that the top row contains the pictures with negative sign and the bottom row contains the pictures with positive sign. The vertices are written in decreasing order in both rows, as shown in Figure 3.

The edges of the recording graph are colored directed edges obtained from the pieces in the puzzle. Each piece is assigned a color. (Note that the numbers represent patterns while the colors represent pieces.) Construct four directed edges for each piece by drawing an arrow from each picture appearing in the piece to the picture which is ninety degrees away clockwise. Therefore each piece contributes four edges to the recording graph. The vertex from which this arrow originates is called the *tail* of the edge, while the vertex to which it points is called the *head* of the edge. Figure 4 demonstrates the construction of the four edges corresponding to one puzzle piece, and Figure 5 demonstrates the recording graph for a Scramble Squares puzzle with two pieces.

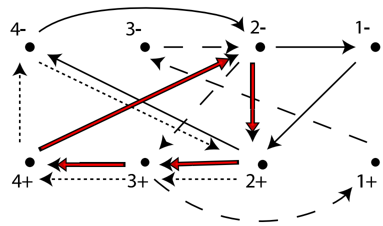


Figure 6. Recording graph for the puzzle shown in Figure 2.

The pieces of the puzzle are distinguished from one another by the color (or shading) of their edges. Once all the pieces have been represented in the graph, the resulting figure is called the *recording graph*. Figure 6 shows an example. We may now discard the original pieces since the recording graph encodes all of the information necessary to solve the puzzle. We determine a solution by finding a subgraph of the recording graph which satisfies certain properties.

Solution graphs for $\mathbf{2\times 2}$ ***repetition-free puzzles.*** Every solution to a 2×2 Scramble Squares puzzle without repetitions is an arrangement of the pieces such that each picture not on the boundary is adjacent to its complement. Every subgraph of the recording graph which contains four edges of distinct colors represents an arrangement of the pieces. (Note that we need exactly one edge of each color to represent an arrangement of the pieces since each color represents a piece.) Recall that an arrow $A \to B$ in the recording graph represents the corner between sides A and B, where A is 90 degrees counterclockwise from B. When that edge is present in a subgraph, it means that this corner will be the corner of that piece which is in the middle of the arrangement, adjacent to the other pieces. Since not every arrangement of the pieces constitutes a solution, not every four-colored subgraph of the recording graph constitutes a solution. See Figure 6 for an example of a recording graph, and Figure 7 for examples of a solution subgraph and a subgraph which does not correspond to a solution. We provide necessary and sufficient conditions on a subgraph to guarantee that it constitutes a solution.

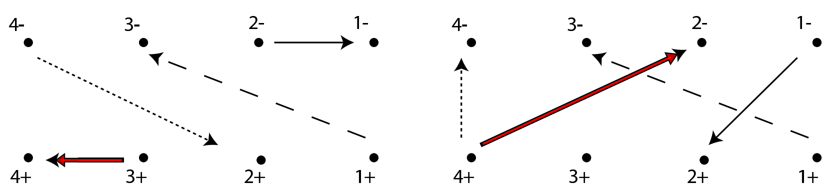


Figure 7. Left: a subgraph of Figure 6 representing the solution shown in Figure 2. Right: graph of an arrangement of the pieces that does not constitute a solution.

In order to state these conditions, we need the notion of *pseudoconnectedness*. Two distinct connected components of a recording graph are said to be *pseudoconnected* if the intersection of the set of absolute values of their vertices is nonempty. Write $C_1 \simeq C_2$ if C_1 and C_2 are pseudoconnected. A *pseudo-path* between two connected components C and D is a collection of connected components $\{C_0 = C, C_1, \dots, C_k = D\}$ such that $C_0 \simeq C_1 \simeq \dots \simeq C_k$. A subgraph of a recording graph is said to be *pseudoconnected* if there is a pseudo-path between every pair of connected components in the graph.

For example, let C_1, C_2, C_3 be the three connected components of a recording graph G, with respective vertex sets $\{1+, 3-, 4-\}$, $\{2+, 3-, 3+\}$, and $\{1-, 4+\}$. The graph G is pseudoconnected even though C_2 is not pseudoconnected to C_3, since there exists a pseudo-path $C_2 \simeq C_1 \simeq C_3$.

Theorem 2.1. *A subgraph of the recording graph $G(P)$ consisting of four edges is a solution graph $G_s(P)$ for a repetition-free 2×2 puzzle if and only if it is a pseudoconnected subgraph satisfying the following properties*:

(1) *Each edge is a different color.*

(2) *The in-degree of each vertex is equal to the out-degree of its complement.*

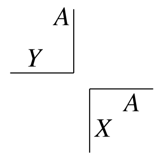
(3) *If $X \to A \to Y$ is a directed path in $G_s(P)$, then Y must be the complement of X.*

Proof. We begin by proving that every subgraph which corresponds to a solution must be of the form described in Theorem 2.1. The subgraph must contain exactly four distinctly colored edges since a solution must use each of the four pieces.

Next consider the pseudoconnectedness property. In a solution to the puzzle, every pair of pieces is either adjacent or diagonally opposite one another. If two pieces are adjacent, then their corresponding vertices are pseudoconnected in the solution graph since the adjacent edges of the pieces must contain the same pattern. If two pieces are diagonally opposite one another, there is a piece between them whose edges share a pattern with each. The edges of this piece will form a pseudo-path between the two corresponding patterns contained in the diagonally opposite pieces. Hence every solution graph must be pseudoconnected.

Every vertex appearing in the solution graph corresponds to a picture which is matched to its complement. Since at every such matching, one picture is represented by the head of a directed edge and the other is represented by the tail of a directed edge, the matching contributes 1 to the in-degree of one picture and 1 to the out-degree of its complement. Therefore the in-degree of a vertex must equal the out-degree of its complement.

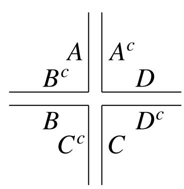
Finally consider a graph that does not satisfy the third property. This implies that the graph contains a length 2 directed path $X \to A \to Y$ such that Y is not the complement of X. Without loss of generality, let the corner $A \to Y$ be the upper-left corner of the solution. Then the corner represented by $X \to A$ cannot be placed in a position adjacent to the corner represented by $A \to Y$ since the complement of A is not A and the complement of Y is not X. Therefore the corner represented by $X \to A$ must be positioned in the lower right-hand corner (diagonally opposite the corner represented by $A \to Y$):



In this case, the picture A^c must appear twice in the piece located in the upper right-hand corner. This contradicts the assumption that the puzzle is repetition-free, and therefore in this case no solution exists. Thus if there is a length two directed path $X \to A \to Y$, then Y must be the complement of X. This implies that the conditions listed in Theorem 2.1 are necessary.

Next we must prove that all subgraphs satisfying the given conditions are indeed solution graphs. Let G be a subgraph satisfying the hypotheses of Theorem 2.1. We prove that the pieces represented by G constitute a solution to the puzzle.

First assume that four distinct patterns appear in the pieces represented by G. Without loss of generality let A, B, C, and D denote the pictures with out-degree one. We can't have $X \to X^c$ for any X by the pseudoconnectedness condition. (Since four distinct patterns appear, if we had $X \to X^c$ then the pattern represented by X would not be pseudoconnected to any of the other patterns, violating pseudoconnectedness.) Therefore without loss of generality assume $A \to B^c$ is one of the pieces. If $B \to A^c$ is a piece, then pseudoconnectedness fails since the patterns $|A|$ and $|B|$ would not be pseudoconnected to the patterns $|C|$ and $|D|$. So, again without loss of generality, assume $B \to C^c$. Then $C \to D^c$ since pattern D must be pseudoconnected to one of the other patterns and if $C \to A^c$ then pattern D would be isolated. Then $D \to A^c$ by process of elimination. Therefore a solution is given by the following arrangement.



Next assume that three distinct patterns appear in the pieces represented by the subgraph G. Let $|A|$ be the repeated pattern. Then the tails are either given by A, A, B, C or A, A^c, B, C for some pictures A, B, C. Assume that the tails are A, A, B, C. By pseudoconnectedness one of B, C must appear on the same piece as A^c. Assume without loss of generality that this piece is $B \to A^c$. Then C appears (also by pseudoconnectedness) on the same piece as either A^c or B^c. If $C \to A^c$, then the other pieces must be $A \to B^c$ and $A \to C^c$ and a solution is given below.

$$\begin{array}{r||l} B & B^c \\ A^c & A \\ \hline\hline A & A^c \\ C^c & C \end{array}$$

If $C \to B^c$, then the other two directed edges appearing in G must be $A \to C^c$ and $A \to A^c$, which together with $C \to B^c$ and $B \to A^c$ represent a solution.

If the tails are A, A^c, B, C, then one of B or C must be on the same piece as either A or A^c. Without loss of generality assume this piece is $B \to A$. Then the third condition implies that $A \to B^c$ is another piece. Pseudoconnectedness implies that the remaining two pieces are represented by $C \to A^c$ and $A^c \to C^c$ since $C \to C^c$ would isolate pattern C, keeping it from being pseudoconnected to A or B. Therefore a solution is obtained by placing the pieces as shown below.

$$\begin{array}{r||l} B & B^c \\ A & A \\ \hline\hline A^c & A^c \\ C^c & C \end{array}$$

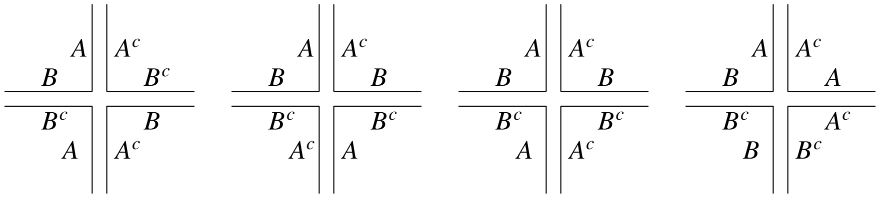
Next assume that two distinct patterns appear in the pieces represented by G. This can happen with each pattern appearing twice or one pattern repeated three times. If each of two patterns $|A|$ and $|B|$ is repeated twice, we may assume without loss of generality that $A \to B$ appears in G. Condition 3 implies that none of $A^c \to A$, $B \to B^c$, or $B \to A$ appears in G, but there must be at least one more piece involving both $|A|$ and $|B|$ since each pattern occurs an even number of times. This piece could be any of $B^c \to A$, $B^c \to A^c$, $B \to A^c$, $A \to B$, $A \to B^c$, $A^c \to B$, or $A^c \to B^c$.

Case 1: If this piece is $B^c \to A$ then A^c must appear at least two more times, once as a head and once as a tail by condition 2. Similarly, B must appear as a head and B^c as a tail by condition 2. This means that the other pieces appearing are either

$$\text{(a)}\ \ A^c \to B \text{ and } B^c \to A^c, \qquad \text{or} \qquad \text{(b)}\ \ A^c \to B^c \text{ and } B \to A^c.$$

In both cases, a solution is possible. See Figure 8(a) for the solution to Case 1(a); Case 1(b) is similar. (Notice that if $B^c \to A$ is replaced by $B \to A^c$ then the proof that the puzzle has a solution is the same.)

Case 2: If the second piece involving $|A|$ and $|B|$ is $A^c \to B$, then the other two pieces must be $B^c \to A$ and $B^c \to A^c$, which are the same pieces used in Case 1(a); see Figure 8, left. A similar argument works when the second piece is $A \to B^c$; see second diagram in Figure 8.



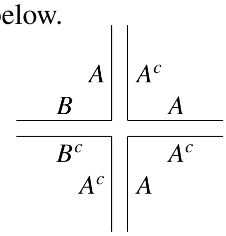
Case 1(a) Case 2 Case 3 Case 4

Figure 8. Solutions for puzzles with two repeated patterns.

Case 3: If the second piece is $A^c \to B^c$ then the other pieces must be $B^c \to A$ and $B \to A^c$ by conditions two and three, which together represent a solution depicted in the third diagram of Figure 8. If the second piece is $A \to B$ then the other two pieces are $B^c \to A^c$, which together with the first two pieces represent a solution similar to the solution for the puzzle with second piece $A^c \to B^c$.

Case 4: Finally, if the second piece is $B^c \to A^c$ then the other two pieces are either ($A \to A^c$ and $B^c \to B$) or ($A \to B$ and $B^c \to A^c$) or ($B^c \to A$ and $A^c \to B$) or ($A \to B^c$ and $B \to A^c$), all of which admit a solution similar to the previous solutions; the solution to the first is depicted in the rightmost part of Figure 8.

Finally, suppose that one of the patterns appears three times. Then we may assume $A \to B$ is a piece, since the two patterns A and B must be pseudoconnected. Assuming A is the piece repeated three times, there is one piece containing B^c as the tail and either A or A^c as the head. Since the remaining two pieces must contain two occurrences of A and two occurrences of A^c by the repetition-free assumption, this second piece must be $B^c \to A^c$. The remaining pieces must both be $A \to A^c$ since $A^c \to A$ violates condition 3. This collection of pieces can easily be arranged to produce a solution, shown below.



Finally assume that only one distinct pattern appears in the pieces represented by G. If $A \to A^c$ is one of the pieces, condition 3 implies that all other pieces must be of this form. Therefore any arrangement of the pieces represents a solution and our proof is complete. □

3. Backtracking

Brandt et. al [2002] use the method of backtracking to solve Scramble Squares puzzles algorithmically. Their procedure begins by labeling the 3×3 grid with the numbers 1 through 9 in the order shown in Figure 9. The numbers stand for the order in which pieces are inserted.

The pieces are then randomly numbered 1 through 9 as well and the orientation of each piece is numbered 0 to 3 since each piece can be rotated and placed in four different ways. The first step is to place a piece into position #1 with a settled orientation. The orientation of the piece at position #1 is set to avoid repetitions obtained by rotating the whole grid.

Next, another piece is placed at position #2 with orientation 0. If the edges match, one of the remaining pieces is chosen at random for position #3 with orientation 0. This process is repeated until a piece is placed in such a way that the edges don't match. If rotating this piece 90 (or 180 or 270) degrees clockwise causes the edges to match, then the process continues. Otherwise, this piece is removed (backtracking) and a different piece is selected. If none of the pieces under any rotation makes the edges match, the previous piece is rotated 90 degrees clockwise (or removed, if its orientation number is 3) and the process continues. This trial and error process continues until all nine pieces match perfectly in their positions.

Finding the middle piece. The backtracking process described above uses randomization to select the pieces involved and thus does not take any information from the puzzle into account. We introduce a procedure called *maximizing the center* that uses information about the puzzle to potentially improve the speed of the algorithm. In the following, we will assume for simplicity of exposition that there is only one solution to a given Scramble Squares puzzle.

Notice that all of the pictures on edges in the middle of the solved puzzle will be matched and thus will need a complement, while the edges facing out on the boundary of the solved puzzle will not need a complement. Therefore we seek a procedure which will select an initial middle piece which is most likely to have matches for all four of its edge pictures.

7	8	9
6	1	2
5	4	3

Figure 9. Order of placement of the pieces in the 3×3 grid.

Consider a picture A and its complement A^c. Let n_A be the number of times the picture A appears on a puzzle piece and let n_{A^c} be the number of times the picture A^c appears on a puzzle piece, called the *index* of that picture. Assume without loss of generality that $n_A \geq n_{A^c}$. If x is the number of times the pattern $|A|$ appears as a complete (matched) pattern in the solution, then the probability that an occurrence of the picture A will be matched in the solution is x/n_A, while the probability that an occurrence of the picture A^c will be matched in the solution is x/n_{A^c}. Since $x/n_{A^c} \geq x/n_A$, an arbitrary occurrence of picture A^c is more likely to be matched in the solution to the puzzle than an arbitrary occurrence of picture A. Therefore, it is reasonable to select as middle position candidates pieces whose pictures have lower indices, since all four sides of the middle piece must be matched. In fact, since a picture and its complement might both have a low index, an even better measure is to use the index of the complement of a picture. This value equals the number of pictures available to be matched to the picture, and thus higher values imply more potential matches are available. The following procedure provides a method for ordering the pieces so that the ones “most likely” to be in the middle are tested there first. Of course, there are examples of puzzles in which the last piece chosen by this procedure appears in the middle, so this method is not always faster than the original backtracking method. It would be interesting to determine how frequently this method does yield some improvement over previous backtracking methods.

(1) Count the number of times each picture occurs. This is the *index* of the picture.

(2) Assign a *value index* to each piece by summing the indices of the *complements* of the pictures appearing on the piece.

(3) Place the piece with the *highest* value index in the middle.

(4) Begin with the picture on this piece whose complement has the *lowest* index.

(5) Find all the pieces containing the complement of this picture and place the one with the lowest value index next to the picture.

(6) Next use the interior picture whose complement has the lowest index from the two placed pieces and repeat step (5). Repeat the process until a picture on the interior is reached which cannot be matched to any of the remaining pieces. If no such piece exists, then the algorithm has produced a solution.

(7) If such a piece exists, rotate this piece 90 degrees clockwise and repeat. If its orientation is 3, backtrack and replace the previous piece with another piece whose value index is greater than or equal to the value index of the previous piece.

(8) Continue the procedure until arriving at a solution.

The purpose of starting with the picture whose complement has the lowest index in Steps (4) and (5) is to ensure that the picture with the highest probability of failing to find a match is tested first. Ideally this will avoid testing many extra correct pictures before finding a side of the middle piece that cannot be matched. Again, this is not a perfect strategy because it is possible for the mismatched side to be one with a high index, but perhaps this will reduce the amount of time needed to arrive at a solution for certain puzzles. Further investigation is necessary to determine the efficiency of this approach.

4. Future directions and open questions

The use of graph theory and informed backtracking to solve Scramble Squares puzzles paves the way for many new and exciting research topics. We describe several potential directions the interested reader is encouraged to explore.

Puzzles with repetitions. Repetition occurs when one picture appears two or more times in one piece. However, in a specific 2×2 puzzle, the solution relies on the two adjacent sides used to match other pieces. Hence, when the same picture shows up on opposite sides of a piece, while the other pictures are distinct, the solution graph properties are the same as for puzzles with no repetition. However, when the same picture appears on two adjacent sides of one piece, represented by a loop in the recording graph, different conditions are required to find a solution. While some of the conditions are similar to those for the repetition-free case, the full necessary and sufficient conditions for puzzles containing repetitions are currently unknown.

Solutions to larger puzzles. This paper focuses on solutions to 2×2 Scramble Squares puzzles. Certainly these results could be extended to larger puzzles in an ad hoc manner, but an ideal solution would describe conditions on a subgraph of the overall recording graph so that the subgraph corresponds to a solution.

Uniqueness. Some Scramble Squares puzzles have multiple solutions. Is it possible to find conditions under which a puzzle has a unique solution? Perhaps there is a formula using the recording graph or on the puzzle itself that enumerates the number of solutions to a given Scramble Squares puzzle. This seems to be an extremely difficult problem, but perhaps a probabilistic approach would be more likely to yield results. Such an approach would look for the probability that an arbitrary puzzle has a unique solution. Calculations could be made toward this effort by placing restrictions on the number of patterns or the number of appearances of any given pattern and then counting the number of puzzles which exhibit such properties.

It is not difficult to find conditions which are necessary for a puzzle to have at least one solution. It would be useful to have sufficient conditions as well, ideally conditions which could be easily checked using a counting argument or by verifying properties of the recording graph.

Probability. The "maximizing the center" approach will not always be faster than the depth first backtracking approach. It is possible that for some puzzles the additional information taken into account through our approach does not decrease the total time needed to solve the puzzle. If a puzzle is unusual in the sense that its central piece has the smallest value of all the pieces, then the "maximizing the center" approach would actually force us to run through all of the possible center pieces before finding the correct center piece, thus potentially taking longer than a random backtracking process. It would be very useful, therefore, to determine the probability that, given a random Scramble Squares puzzle, our approach will actually improve upon the amount of time needed to determine a solution as compared to the random backtracking approach.

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masonsk@wfu.edu *Department of Mathematics, Wake Forest University, 127 Manchester Hall, Winston-Salem, NC 27109, United States*

mzmazhang@gmail.com *Davidson College, Davidson, NC 28035, United States*

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