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Statistical analysis of diagnostic accuracy with applications to cricket

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In the sport of cricket, as with any other sport, spectators and officials would like the games to be as fair as possible. To this end, we evaluate methods used to determine the winner of interrupted games using statistical accuracy. In the traditional One Day International cricket matches, the current Duckworth–Lewis (DL) method and the discounted most productive overs (DMPO) method are each used for predicting the winner. However, with the growing popularity of shorter Twenty20 matches, a new Bhattacharya–Gill–Swartz (BGS) method has also been introduced. We created both classical and Bayesian intervals to estimate the true accuracy of each. Using past game data from 2005–2010, we compared the DL, DMPO and BGS methods using the new accuracy intervals and receiver operating characteristic (ROC) curves.

1. Introduction

This paper examines the accuracy of methods for predicting the winner of interrupted cricket matches. Cricket is a field game that was first seen in 16th century England and, within 200 years, became England's national sport. International games were being held by the 19th century and the popularity has only grown since then, making it now the second most popular sport in the world, next to association football. The International Cricket Council (ICC) is the governing body, currently with 104 members. It was formed in 1909 by England, Australia, and South Africa, who originally called it the Imperial Cricket Conference.

There are many variations of cricket; however, Test cricket, One Day International (ODI), and Twenty20 are the most common. Test cricket is the longest form of cricket and can last up to five days. One Day International is the most common, lasting roughly eight hours with a maximum of 50 overs in each of two innings. The

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newest form of cricket, Twenty20, has two innings with a maximum of 20 overs each, lasting only about 4 hours and growing in popularity.

Similarities between cricket and baseball can be found, even though the two games differ in many ways. Both sports have batters and pitchers, although in cricket the pitchers are called bowlers. A bowler does not pitch the ball like a baseball pitcher would; he bowls it with a stiff arm. In baseball, there are nine innings with the teams alternating offense and defense in each inning. In cricket, the word “innings” is both singular and plural, with most variations of cricket consisting of only two innings. Additionally, only one team bats per innings. Therefore, the second team’s goal is to beat the first team’s score before they lose 10 wickets or have been bowled all of their overs. Six bowls are equivalent to one over, so, for example, in ODI there are 50 overs in an innings or 300 bowls. Cricket also differs from baseball in that there are 11 players on a team, and a coin toss at the beginning determines who bats first. Lastly, the batsmen bat in pairs and score by running back and forth between the wickets or by hitting the ball outside the boundary. The 10 outs per innings are obtained primarily by knocking over the wickets.

As with any sport played outside, games can be interrupted due to rain or other bad weather. When this happens, a winner must be decided. However, it would be unfair to simply decide the winner based on current points, since one of the teams may have had a full innings to score and the other has not had this opportunity. There are several methods that can be used to declare a winner. The most popular is the Duckworth–Lewis method, which is used for 50-over games and can be scaled down for 20-over games. A new method for 20-over games is the Bhattacharya–Gill–Swartz method, which, while based on the Duckworth–Lewis method, is designed specifically for 20-over games instead of 50-over games. A third method, used for 50-over games, is the discounted most productive overs (DMPO) method, which tends to favor the first team batting. We hope to see an improvement for 20-over cricket games. These three methods are compared to determine the most accurate method for determining the winner of Twenty20 cricket games.

2. Target methods for interrupted matches

Due to the nature of cricket, interrupted matches often result from inclement weather. When a game has been stopped, and thus shortened, the team batting does not get their intended number of overs. Therefore, the winner of the game must be predicted by calculating the second team’s target score, which takes into account their unused overs. Since cricket is such a historic and popular game, several methods of predicting the winner have been developed.

The most popular method of choosing a winner in an interrupted game is the Duckworth–Lewis (DL) method [1998]. This method is currently the preferred

	Wickets Lost				
Overs Left	0	2	5	7	9
50	100	85.1	49	22	4.7
40	89.3	77.8	47.6	22	4.7
30	75.1	67.3	44.7	21.8	4.7
20	56.6	52.4	38.6	21.2	4.7
10	32.1	30.8	26.1	17.9	4.7

Table 1. DL resource table for ODI cricket.

method of the ICC and is based on the number of overs remaining in the game (u) and the amount of wickets that have been lost (w). This relationship follows an exponential decay model $Z(u, w) = Z_0(w)\big(1 - \exp\{-b(w)u\}\big)$, where $b(w)$ is the exponential decay constant and $Z_0(w)$ is the asymptotic average from the wickets remaining in unlimited overs. Table 1 is a shortened version of the DL table used for ODI cricket. To calculate the target score of a game that was interrupted after 10 overs were bowled and 5 wickets were lost, find the row indicating that there are 40 overs left and the column indicating there have been 5 wickets lost. The common entry cell provides the percentage of resources remaining, so the target score in this situation would be the first team's final score multiplied by $(1 - 0.476)$. If this calculated number, or "target score," is greater than the second team's score at the time the game was interrupted, the second team loses; otherwise, the second team wins. The original DL table is based on a game that consists of innings with 50 overs. Here we consider games with innings of 20 overs. Currently, the ICC rescales the table from 50 overs to allow for matches with 20 overs in an innings. Table 1 is an example of a partial DL table for matches with 50 overs.

The DL table for Twenty20 cricket is presented in Table 2.

The Bhattacharya–Gill–Swartz (BGS) method [Bhattacharya et al. 2010] is a new method that is similar to the DL method, but developed specifically for Twenty20 matches. This method calculates r_{uw}, the estimated percentage of resources remaining when u overs are available and w wickets have been taken. To impose necessary monotonicity constraints on rows and columns, BGS employs isotonic regression, minimizing $F = \sum\sum q_{uw}(r_{uw} - y_{uw})^2$ with respect to the matrix $Y = (y_{uw})$, for $u = 1, \ldots, 20$ and $w = 0, \ldots, 9$, where q_{uw} are weights based on sample variances. The resulting matrix, Y, gives an unsatisfactory table of values, as with the Duckworth–Lewis table, with some adjacent entries being the same. Through Gibbs sampling and a Bayesian model, a new table consisting of the estimated posterior means is offered by [Bhattacharya et al. 2010] as an alternative to the Duckworth–Lewis table for Twenty20 cricket. Due to the lack of games available, some entries in the BGS table were initially left empty, then later filled

	Wickets Lost									
Overs Left	0	1	2	3	4	5	6	7	8	9
20	100	96.8	92.6	86.7	78.8	68.2	54.4	37.5	21.3	8.3
19	96.1	93.3	89.2	83.9	76.7	66.6	53.5	37.3	21.0	8.3
18	92.2	89.6	85.9	81.1	74.2	65.0	52.7	36.9	21.0	8.3
17	88.2	85.7	82.5	77.9	71.7	63.3	51.6	36.6	21.0	8.3
16	84.1	81.8	79.0	74.7	69.1	61.3	50.4	36.2	20.8	8.3
15	79.9	77.9	75.3	71.6	66.4	59.2	49.1	35.7	20.8	8.3
14	75.4	73.7	71.4	68.0	63.4	56.9	47.7	35.2	20.8	8.3
13	71.0	69.4	67.3	64.5	60.4	54.4	46.1	34.5	20.7	8.3
12	66.4	65.0	63.3	60.6	57.1	51.9	44.3	33.6	20.5	8.3
11	61.7	60.4	59.0	56.7	53.7	49.1	42.4	32.7	20.3	8.3
10	56.7	55.8	54.4	52.7	50.0	46.1	40.3	31.6	20.1	8.3
9	51.8	51.1	49.8	48.4	46.1	42.8	37.8	30.2	19.8	8.3
8	46.6	45.9	45.1	43.8	42.0	39.4	35.2	28.6	19.3	8.3
7	41.3	40.8	40.1	39.2	37.8	35.5	32.2	26.9	18.6	8.3
6	35.9	35.5	35.0	34.3	33.2	31.4	29.0	24.6	17.8	8.1
5	30.4	30.0	29.7	29.2	28.4	27.2	25.3	22.1	16.6	8.1
4	24.6	24.4	24.2	23.9	23.3	22.4	21.2	18.9	14.8	8.0
3	18.7	18.6	18.4	18.2	18.0	17.5	16.8	15.4	12.7	7.4
2	12.7	12.5	12.5	12.4	12.4	12.0	11.7	11.0	9.7	6.5
1	6.4	6.4	6.4	6.4	6.4	6.2	6.2	6.0	5.7	4.4

Table 2. DL resource table for Twenty20 cricket.

with the corresponding DL entries. The resulting BGS table is presented in Table 3.

A third, less commonly applied, method offered for comparison is the discounted most productive overs (DMPO) method. This method offers a new way of calculating a target score for the second team after the game is interrupted. First, order the overs according to the number of runs per over, highest to lowest. The target score for Team 2 is then found by summing the same number of highest scoring overs of Team 1 and then discounted by 0.5% per over lost. This method tends to favor Team 1 for 50-over cricket games. The DMPO method will be considered herein to determine whether or not its performance is competitive with the DL and BGS methods for 20-over cricket games.

3. Large-sample confidence intervals for accuracy

To compare the performance of the DL, BGS and DMPO methods when predicting winners in 20-over cricket, we compared their predictions to actual final results for 120 recent uninterrupted Twenty20 matches from 2005–2010, obtained from

	Wickets Lost									
Overs Left	0	1	2	3	4	5	6	7	8	9
20	100	96.9	93.0	87.9	81.3	72.2	59.9	44.8	29.7	17.6
19	95.6	90.9	87.7	83.0	76.9	68.3	56.5	42.0	27.2	15.3
18	91.7	86.7	82.9	78.7	73.2	65.4	54.2	40.2	25.7	13.9
17	87.7	82.3	78.9	73.8	69.7	62.8	52.2	38.7	24.6	12.8
16	83.5	78.2	75.3	70.5	66.4	60.2	50.3	37.4	23.5	12.0
15	79.2	74.3	70.9	66.9	62.6	57.4	48.4	36.2	22.7	11.2
14	75.1	70.7	67.3	63.7	59.3	54.6	46.4	35.0	21.8	10.5
13	71.5	67.4	63.6	60.3	56.2	51.5	44.3	33.8	21.0	9.8
12	68.3	63.7	60.2	56.8	52.9	47.5	41.9	32.6	20.2	9.1
11	65.0	59.9	56.6	53.3	49.7	43.9	39.3	31.3	19.4	8.5
10	61.3	56.0	52.6	50.1	46.0	40.8	36.1	30.0	18.6	7.9
9	57.9	52.3	47.9	46.1	42.5	37.8	33.1	28.3	17.7	7.2
8	54.0	48.3	44.3	41.7	38.9	34.9	30.2	26.1	16.7	6.6
7	49.3	44.2	40.2	37.4	35.4	32.1	27.2	23.4	15.7	5.9
6	41.7	38.5	35.7	33.0	31.7	29.0	24.2	20.0	14.5	5.2
5	36.2	33.4	31.0	28.6	27.3	25.5	21.5	17.0	12.2	4.4
4	30.8	28.0	26.1	24.1	22.4	20.7	18.3	14.2	10.0	4.4
3	25.4	22.8	21.1	19.4	17.7	16.5	14.4	11.6	7.9	2.5
2	19.7	17.2	15.5	14.1	12.7	11.9	10.6	9.3	6.2	1.6
1	13.7	11.3	9.7	8.5	7.3	6.7	6.0	5.2	4.2	0.9

Table 3. BGS resource table for Twenty20 cricket.

www.cricket.org. While the DL, BGS and DMPO methods are sometimes used for cricket matches interrupted more than once before being stopped, we considered their performance for cricket matches that were hypothetically interrupted only once during the second team's bat. For comparison purposes, we considered the predicted outcomes of each game for three possible situations: interruption at 5 overs, 10 overs and 15 overs. Because the effect of team superiority differences should be kept at a minimum, only 10 countries were included in the sample of 120 cricket matches. These countries are Australia, Bangladesh, Pakistan, South Africa, West Indies, Sri Lanka, India, New Zealand, England and Zimbabwe.

We compared the performance of each method by estimating its overall accuracy (Ac). Here accuracy is the rate at which each method properly predicts a game's outcome. It is a weighted average of the method's sensitivity (Se) and specificity (Sp), where Se is the proportion of times it predicts Team 1 as the winner given Team 1 actually won and Sp is the proportion of times it predicts Team 1 to lose given Team 1 actually lost. These values are weighted using the prevalence (p), the proportion of times that Team 1 wins the game in reality. The formula for accuracy is

$$\mathrm{Ac} = \mathrm{Se}(p) + \mathrm{Sp}(1 - p). \tag{1}$$

While this and related measures are commonly employed to assess medical diagnostic tests, to date we are aware of no interval estimates for Ac. In order to compare the different methods for predicting the winner of Twenty20 cricket matches, we derived large-sample classical confidence intervals and Bayesian credible sets for Ac. We then calculated the resulting interval estimates for the DL, BGS and DMPO methods using the sample described above. To do so, we collected the final score of Team 1 along with Team 2's score and the number of wickets lost after 5 overs were played, after 10 overs were played, and after 15 overs were played. The target score was calculated for each team in the sample. This target score then determined the winner under the DL, BGS and DMPO methods. This allowed calculation of estimates for Se, Sp and p.

Formulation of the large-sample classical confidence interval for accuracy requires derivation of the variance using the delta method. The delta method addresses a random sample with $E(X^i) = \mu$ and a covariance matrix $E(X^i - \mu)(X^i - \mu)^T = \Sigma$. For a given function g with continuous first partial derivatives and specific value of μ for which $\tau^2 = \nabla^T g(\mu)\Sigma\nabla g(\mu) > 0$, we have $\sqrt{n}(g(\bar{X}) - g(\mu)) \to N(0, \tau^2)$ in distribution. In other words, the delta method allows us to say that accuracy is considered to be normally distributed for a large sample, with the variance derived from

$$\mathrm{var(Ac)} = \begin{bmatrix} \partial\mathrm{Ac}/\partial\mathrm{Se} \\ \partial\mathrm{Ac}/\partial\mathrm{Sp} \end{bmatrix}^T \begin{bmatrix} \mathrm{var(Se)} & 0 \\ 0 & \mathrm{var(Sp)} \end{bmatrix} \begin{bmatrix} \partial\mathrm{Ac}/\partial\mathrm{Se} \\ \partial\mathrm{Ac}/\partial\mathrm{Sp} \end{bmatrix}. \tag{2}$$

The resulting variance is as follows, where n_1 is the number of games Team 1 won and n_2 is the number of games Team 1 lost:

$$\mathrm{var(Ac)} = \frac{p^2 n_2 \mathrm{Se}(1 - \mathrm{Se}) + (1 - p)^2 n_1 \mathrm{Sp}(1 - \mathrm{Sp})}{n_1 n_2}. \tag{3}$$

The formula for the classical 95% confidence interval for accuracy is

$$\widehat{\mathrm{Ac}} \pm 1.96\sqrt{\widehat{\mathrm{var(Ac)}}}. \tag{4}$$

We then calculate this interval for the DL, BGS and DMPO methods for situations where the game was interrupted after 5 overs, 10 overs and 15 overs.

4. Bayesian credible sets for accuracy

The Bayesian credible set is another method that is used as a means for comparison along with the classical confidence interval. The Bayesian method takes into account the possibility of prior information. The prior information is combined with the data to yield posterior values. Here we assumed binomial data so that the number of games correctly predicted as wins for Team 1, x_{11}, and the number of games correctly predicted as losses, x_{22}, are distributed as

$$x_{11} \sim \text{binomial}(n_1, \text{Se}) \quad \text{and} \quad x_{22} \sim \text{binomial}(n_2, \text{Sp}).$$

We also assumed conjugate beta priors so that

$$\text{Se} \sim \text{beta}(\alpha_{\text{Se}}, \beta_{\text{Se}}) \quad \text{and} \quad \text{Sp} \sim \text{beta}(\alpha_{\text{Sp}}, \beta_{\text{Sp}}),$$

which yielded posterior distributions for Se and Sp of

$$\text{Se}\,|\,\boldsymbol{d} \sim \text{beta}(x_{11}+\alpha_{\text{Se}}, n_1-x_{11}+\beta_{\text{Se}}), \quad \text{Sp}\,|\,\boldsymbol{d} \sim \text{beta}(x_{22}+\alpha_{\text{Sp}}, n_2-x_{22}+\beta_{\text{Sp}}), \quad (5)$$

where $\boldsymbol{d} = (x_{11}, x_{22}, n_1, n_2)$.

We took p to be known and equal to 0.5, implying that there is no advantage to batting first. Likewise, no *a priori* information was available for Se and Sp so we let

$$\alpha_{\text{Se}} = \alpha_{\text{Sp}} = \beta_{\text{Se}} = \beta_{\text{Sp}} = 0.5.$$

Monte Carlo sampling from the posteriors in (5) provided 5000 posterior estimates of Se and of Sp. Plugging these values into (1) estimated the posterior distribution for Ac. The distribution was then used to determine the 95% credible set by determining the 2.5th and the 97.5th percentiles. Table 4 provides the calculated accuracies and the interval for each method evaluated. From the table, we see that DL method is slightly more accurate than BGS and that both DL and BGS methods are superior to DMPO.

	Confidence Interval		Credible Set	
	Estimated Accuracy	Interval	Posterior Accuracy	Interval
DL 5 overs	0.767	(0.692, 0.842)	0.764	(0.686, 0.834)
BGS 5 overs	0.741	(0.665, 0.818)	0.739	(0.659, 0.809)
DMPO 5 overs	0.540	(0.502, 0.579)	0.549	(0.514, 0.590)
DL 10 overs	0.850	(0.788, 0.913)	0.846	(0.777, 0.903)
BGS 10 overs	0.842	(0.779, 0.905)	0.839	(0.770, 0.894)
DMPO 10 overs	0.619	(0.574, 0.664)	0.621	(0.577, 0.679)
DL 15 overs	0.921	(0.872, 0.970)	0.916	(0.857, 0.956)
BGS 15 overs	0.903	(0.850, 0.956)	0.898	(0.835, 0.944)
DMPO 15 overs	0.597	(0.555, 0.640)	0.588	(0.549, 0.643)

Table 4. Classical and Bayesian interval estimates for accuracy.

5. ROC curve analysis of methods

Next we considered receiver operating characteristic (ROC) curves, employed by [Manage et al. 2010] for ODI cricket matches, as another method of comparison. ROC curves present a graphical plot of Se vs. $(1-\text{Sp})$ to determine the best method. A greater area under the curve (AUC) for one method implies that it has higher values of Se together with higher values of Sp and that it is better than other methods with lower areas. To obtain the Se and Sp values for the plot, we needed to classify the thresholds. Here we used the ranking system presented in [Manage et al. 2010]. The rankings are as follows:

IF (Revised target − Actual score of Team 2) < -10
THEN Rank = 1 (strongly negative),
IF $-10 \leq$ (Revised target − Actual score of Team 2) < -2
THEN Rank = 2 (negative),
IF $-2 \leq$ (Revised target − Actual score of Team 2) ≤ 2
THEN Rank = 3 (not clear),
IF $2 <$ (Revised target − Actual score of Team 2) ≤ 10
THEN Rank = 4 (positive),
IF (Revised target − Actual score of Team 2) > 10
THEN Rank = 5 (strongly positive).

Here, "positive" means a victory for Team 1; in other words, the more positive, the more likely that Team 1 will win the game.

This ranking system was used for the three methods evaluated. The data, after ranking, is presented in Tables 5, 6 and 7.

	Ranking (5 overs)						Ranking (10 overs)						Ranking (15 overs)					
	1	2	3	4	5	Tot.	1	2	3	4	5	Tot.	1	2	3	4	5	Tot.
T1 Wins	24	17	7	6	3	57	28	13	4	8	4	57	30	11	2	2	5	50
T2 Wins	5	10	10	9	29	63	0	3	7	16	35	63	0	3	4	11	44	62
Total	29	27	17	15	32	120	28	16	11	26	39	120	30	14	6	13	49	112

Table 5. DL rankings for games interrupted at 5, 10, and 15 overs.

	Ranking (5 overs)						Ranking (10 overs)						Ranking (15 overs)					
	1	2	3	4	5	Tot.	1	2	3	4	5	Tot.	1	2	3	4	5	Tot.
T1 Wins	20	14	4	12	7	57	26	13	4	2	12	57	28	9	5	1	7	50
T2 Wins	1	8	5	12	37	63	0	1	5	12	45	63	0	1	3	10	48	62
Total	21	22	9	24	44	120	26	14	9	14	57	120	28	10	8	11	55	112

Table 6. BGS rankings for games interrupted at 5, 10, and 15 overs.

	Ranking (5 overs)						Ranking (10 overs)						Ranking (15 overs)					
	1	2	3	4	5	Tot.	1	2	3	4	5	Tot.	1	2	3	4	5	Tot.
T1 Wins	2	3	2	9	41	57	5	3	4	3	42	57	4	3	1	15	27	50
T2 Wins	0	1	0	1	61	63	0	0	0	1	62	63	0	0	0	1	61	62
Total	2	4	2	10	102	120	5	3	4	4	104	120	4	3	1	16	88	113

Table 7. DMPO rankings for games interrupted at 5, 10, and 15 overs.

	DL	BGS	DMPO
AUC at 5 overs	0.807	0.811	0.628
AUC at 10 overs	0.894	0.852	0.628
AUC at 15 overs	0.919	0.906	0.709

Table 8. AUC for DL, BGS, DMPO for interruptions at 5, 10 and 15 overs.

ROC curves can now be created for each situation. The situations considered here were hypothetical interruptions after 5 overs, 10 overs and 15 overs. The ROC curves are shown in Figure 1 on the next page. In these graphs, higher curves represent better models. We can see that the Duckworth–Lewis and Bhattacharya–Gill–Swartz methods are higher than the DMPO method. Thus, the DMPO does not perform as well when predicting the winner of Twenty20 cricket matches, as in the case of the ODI matches.

The associated AUC values were calculated using the trapezoidal method and are reported in Table 8. That table also supports the conclusion that the DL method is not significantly different from the BGS method when predicting the winner of Twenty20 cricket matches and both are superior to the DMPO method.

6. Conclusions

This article evaluated three methods of predicting the winner of interrupted Twenty20 cricket matches: the Duckworth–Lewis, discounted most productive overs, and Bhattacharya–Gill–Swartz methods. These methods were compared based on both accuracy and AUC determined from ROC curves. In order to estimate accuracy, formulas for both classical confidence intervals and Bayesian credible sets were derived. A classification threshold provided by [Manage et al. 2010] was used to create ROC curves.

Comparison of the Duckworth–Lewis and Bhattacharya–Gill–Swartz methods using accuracy showed that the posterior accuracies after stoppages at 5 overs, 10 overs and 15 overs in the game were slightly higher for the Duckworth–Lewis method in each situation. The interval estimates overlap, however, implying that the true accuracy rate could be equivalent for these two methods. The ROC curve

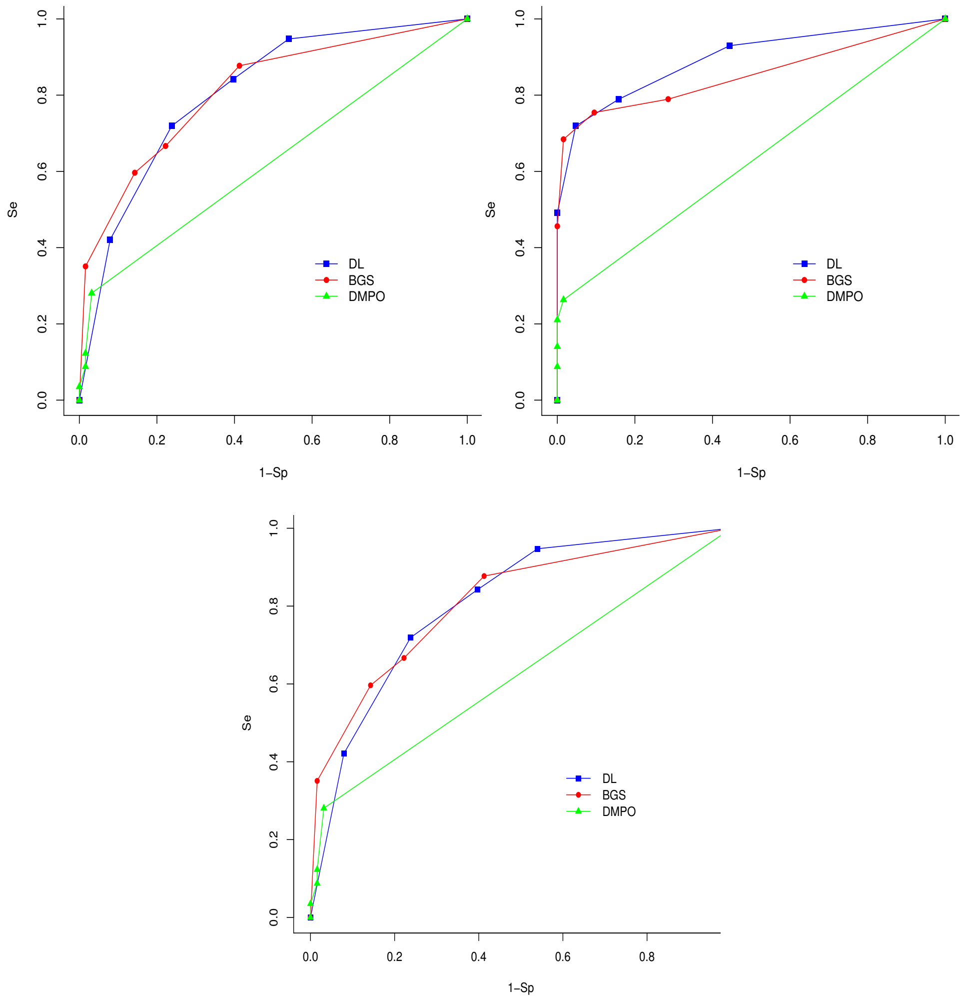

Figure 1. ROC curve for DL, BGS and DMPO for games interrupted at 5, 10, and 15 overs.

method showed similar results, with minimal difference between their AUCs. Thus, we conclude that the Duckworth–Lewis and Bhattacharya–Gill–Swartz methods have comparable success in predicting the winner of interrupted Twenty20 cricket matches. Both outperform the discounted most productive overs method.

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