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Undergraduate research as a capstone requirement

Hannah L. Callender, James P. Solazzo and Elizabeth Wilcox





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(Communicated by Darren A. Narayan)

If a mathematics department has a capstone course, how does undergraduate research figure into that capstone requirement? What challenges are involved when instituting undergraduate research as part of the capstone experience? These were the central questions for discussion in the undergraduate research as a capstone requirement breakout session at the 2012 Trends in Undergraduate Research in the Mathematical Sciences conference. In short, there is not one design that will satisfy the needs and goals of every mathematics program, but a department seeking to implement undergraduate research as a capstone requirement may benefit from the experiences of other departments. This article discusses the common objectives of a capstone in mathematical sciences and presents several successful models that incorporate undergraduate research in a capstone experience. The challenges and questions associated with each model are also discussed.

## 1. Introduction

During the 2012 Trends in Undergraduate Research in Mathematics conference there were four breakout sessions focusing on ways to foster undergraduate research in mathematics and a discussion of the challenges involved in such an endeavor. One of these breakout sessions targeted the questions surrounding incorporating undergraduate research as a capstone requirement for mathematical sciences programs. Among the institutions represented in this particular discussion, there were several with successful models and others that were still exploring the challenges of creating such a capstone experience.

In light of the different institutions, goals, and available resources, participants agreed that there is no single design that would be optimal for all academic systems. The type of model a department implements for a capstone course is intricately related to the make-up of that department and the demands of the institution. In this article, we discuss some of the existing capstone experiences that incorporate undergraduate research and the challenges faced by departments following such

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models. Section 2 provides a list of common objectives for a capstone course in mathematics as well as several different models of capstone courses that include some aspect of undergraduate research. Section 3 discusses some of the questions and challenges that arise from implementing these capstone course designs.

## 2. Course objectives and different models

What course objectives should departments have for a capstone requirement? Although there was considerable variance from institution to institution, all of the capstone experiences discussed shared common threads. Here is a list of course objectives that the participants considered valuable in a capstone requirement:

- ◇ professionalism;
- ◇ exposure to mathematics outside of the traditional classroom setting;
- ◇ synthesis;
- ◇ a self-driven learning experience.

The curious reader may inquire, “These are valuable goals, but where does undergraduate research come into play?” Here we define undergraduate research as any experience in which the student takes a lead role in understanding how mathematicians actually do mathematics. In undergraduate research, students should learn the process by which a mathematician conducts research as well as gain experience in the delivery of their own findings. From this perspective, undergraduate research projects can definitely help students to meet the capstone course objectives listed here.

The participants representing institutions with existing capstone requirements including some form of undergraduate research described their program’s capstone experiences during the breakout session. The different capstone experiences can be divided into four main types:

- ◇ *Capstone topics courses*: Essentially a topics course taught by a single instructor with additional research components not typically required as part of a topics course. This type of capstone includes (at least) an expository research experience for all students in the form of a presentation or paper.
- ◇ *Capstone seminar courses*: A course similar to a seminar, with additional components that allow students to engage in research to an extent fitting their capabilities and interest.
- ◇ *Existing capstone experiences*: An experience within an existing course, meeting many of the capstone objectives outlined above. This type of course includes a research experience where students are recreating mathematics as though for the first time.

- ◇ *Research methods courses*: A course built around methods of mathematical research.

The implementation of these types of capstone experiences is varied, and here we provide several existing models. In parentheses we provide the size of the student body at the institution, whether the institution is public or private, number of undergraduate math majors, and the highest level of degree in mathematics offered at the institution.

- i. This capstone topics course is a three-credit course with one instructor who chooses topics or a theme for the course, for example, “the fundamental theorems of mathematics”. Students are responsible for group presentations at the end of the semester. This particular course serves a dual role in preparing students for the major field test administered by the Educational Testing Service. (*Public university with 19,000 students and around 130 math majors, graduating an average of 40 per year; highest degree in math is Ph.D.*)
- ii. This capstone topics course is also a three-credit course with one instructor. However, in this example, students prepare a literature review; the instructor suggests a list of problems for the students to research in the literature. Each student submits a written report during the semester. (*Public university with 5,000 students and 75 math majors, graduating around 8 per year; highest degree in math is Ph.D.*)
- iii. This capstone seminar course is a one-credit course, run by one faculty facilitator with each student working individually alongside a faculty mentor from the department. Students either choose to read an article from a journal such as *The College Mathematics Journal* or work on a topic that their mentor has deemed acceptable. Each student gives a 15-minute presentation accompanied with a 10-page report, creates a poster, and then develops the earlier work into a 20-minute talk accompanied with 20-page report. The department that runs this course is seriously considering making it a three-credit course. (*Public university with 8,000 students and 80–90 math majors, graduating 10–15 per year; highest degree in math is B.S.*)
- iv. This capstone seminar course is a proposed three-credit course that will span two semesters. In the first semester students will actively research within the literature, writing reviews of articles published in *Mathematics Magazine* and other undergraduate accessible journals. During the second semester students will work in groups on projects, writing a 7–10 page paper and delivering presentations to both the department and the broader mathematical community. (*Public university with 8,500 students and 25–30 math majors, graduating 9–16 per year; highest degree in math is B.S.*)

- v. This example of an existing capstone experience relies on real analysis and abstract algebra courses to serve as the capstone requirement. These classes are taught using the modified Moore method; students regularly present and develop results without a textbook or provided solutions. (*Private university with 3,300 students and 50–60 math majors, graduating 12–15 per year; highest degree in math is B.S.*)
- vi. This capstone course in research methods is a three-credit course introducing students to methods common among research approaches in the mathematical sciences. Students are exposed to a variety of mathematical techniques and topics. Students gain experience with computational methods through MATLAB programming, write a paper in L<sup>A</sup>T<sub>E</sub>X on a mathematical article, and also give an oral presentation on the article. The department is considering splitting the course into two courses: a two-credit course introducing students to research in mathematics and a one-credit seminar in mathematics. (*Public university with 9,100 students and around 75 math majors, graduating around 11 per year; highest degree in math is Ph.D.*)

Each model described above involves undergraduates in a research experience at some level; additionally these models allow students to produce posters or presentations that are worthy of the regional MAA meetings or even national events such as MathFest and the Joint Mathematics Meetings. Occasionally a faculty mentor may have the opportunity to guide an undergraduate through the process of actually publishing her result from the capstone experience.

### 3. Questions and challenges

In this section we pose several questions and examine some of the challenges that arise from requiring research as part of a capstone experience. These questions are grouped by topic: course logistics, student logistics and abilities, and faculty workload and evaluation.

It is important to keep in mind that the different types of capstone courses have been taught in many different institutions and there are several sources of advice and aid available. Faculty should not feel alone in facing the challenges described here — these challenges were experienced by nearly every member of the breakout session discussion, regardless of institution or program capstone requirement design (or lack thereof).

**Course logistics.** How can departments make time in their students' schedules for undergraduate research? Typically the curriculum for a standard mathematics major leaves little or no room for “extra” classes in the major requirements. Consequently, in order to implement a capstone requirement what might need to be removed from the program curriculum? Similarly, in some departments the senior-level math

courses are very rigorous and present a serious time commitment. How can we enable our students to manage the time necessary to be successful in both their course work as well as a capstone requirement?

If a department chooses to implement a capstone seminar course model, there are several questions that should be addressed. For example, does the course facilitator assign grades or is that the responsibility of the faculty mentors? How can we account for the difference between faculty mentor grading schemes? A partial answer to the latter question is for the capstone course to have a well defined set of student learning outcomes that take into consideration the varying abilities of the students. Even with a well defined rubric, one should prepare to encounter passionate discussions regarding grade assignments.

Some institutions have specific requirements that all departments must satisfy in their capstone course, or there may be a wide array of departmental requirements that must fit into the capstone course. For example, some institutions require students to explore the history, philosophy, and ethics of the discipline in a capstone course. Other institutions use a capstone course as a means of preparing students for seeking a career, for example, writing CVs and applying to graduate school. This limits the amount of time that can be spent learning new topics and engaging in research.

***Student logistics and abilities.*** How can we empower even our weakest students to succeed in a research experience? Unlike an REU, where the students have similar abilities and skill level, there may be considerable variance in a senior class of majors.

How much novel research, as opposed to expository or literature research, should be expected in a required capstone course? It seems that the best answer to this question is to be inclusive about what constitutes “research” — giving students a chance to grow, without setting unattainable requirements.

What are the basic requirements, with regards to research, for students in a capstone experience? What if a student does not complete even these basic requirements? Answering these questions is a challenge for all four types of capstone experience.

***Faculty workload and evaluation.*** Regarding the capstone seminar course model, how will teaching credit be awarded to the faculty mentors? These faculty members typically work one-on-one with students and yet their names are not listed as instructors for the course. From a faculty point of view this is a serious consideration, especially for untenured faculty members. It is possible that capstone seminar course mentors get no credit for their efforts, other than acknowledgment in their annual report. Some departments develop a system for tracking the number of capstone students a faculty member mentors over a period of time and eventually provides a

course release. Of course, such considerations depend on individual department values and goals.

These questions that seem specific to the faculty mentors of a capstone seminar course are actually along the same lines as those raised for faculty engaging in undergraduate research at any level. How can a department foster an environment where faculty members are inspired to work with undergraduate students on research? How can a department maintain or keep building the momentum of such research engagement from year to year? In a department where only a small fraction of faculty members are committed to research projects with students, faculty burn-out would be a serious concern.

Should all faculty members be equally responsible for mentoring students in undergraduate research — whether or not this research is part of a capstone requirement? To what extent is a department chair responsible for making sure that faculty are willing to work with students and that faculty members are not overextending themselves? Who will see to it that faculty mentors are actually meeting and working with the students they initially agreed to sponsor?

How do the different models affect student evaluations of teaching? This may be of particular concern for untenured faculty experimenting with an existing traditional course by teaching it as an existing capstone experience. Initially students may perceive the faculty member as taking shortcuts and pushing the work of class preparation onto the shoulders of students. A charismatic faculty member may be able to present the experiment in a positive light and reduce student resistance to a new teaching method. How to achieve success with this model semester after semester, however, is not clear.

#### **4. Conclusion**

Although there is no one way to best answer all of the questions posed or address all of the challenges, participants agreed that it seems necessary to have at least one faculty member who is passionate about undergraduate research in order to achieve success in incorporating undergraduate research as a capstone requirement. Ultimately, students working on problems need a faculty mentor to guide their continuing efforts and provide feedback on both papers and presentations. Additionally, a department needs to have some agreement on the value of undergraduate research as a capstone requirement so that teaching the course and/or mentoring students is both rewarded and encouraged.

There are always limitations to the amount of resources that can be spent on undergraduate research as a capstone requirement. No one model will work for all programs, and no one model would work for all students. The important thing is to provide the best opportunity possible given the resources available.



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
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Preface	245
DARREN A. NARAYAN	
Undergraduate research in mathematics with deaf and hard-of-hearing students: four perspectives	247
HENRY ADLER, BONNIE JACOB, KIM KURZ AND RAJA KUSHALNAGAR	
Challenges in promoting undergraduate research in the mathematical sciences	265
FERYAL ALAYONT, YULIYA BABENKO, CRAIG JACKSON AND ZSUZSANNA SZANISZLO	
Undergraduate research as a capstone requirement	273
HANNAH L. CALLENDER, JAMES P. SOLAZZO AND ELIZABETH WILCOX	
A decade of undergraduate research for all East Tennessee State University mathematics majors	281
ARIEL CINTRÓN-ARIAS AND ANANT GODBOLE	
The MAA undergraduate poster session 1991–2013	295
JOYATI DEBNATH AND JOSEPH A. GALLIAN	
Nonacademic careers, internships, and undergraduate research	303
MICHAEL DORFF	
REU design: broadening participation and promoting success	315
REBECCA GARCIA AND CINDY WYELS	
Papers, posters, and presentations as outlets for undergraduate research	327
APARNA HIGGINS, LEWIS LUDWIG AND BRIGITTE SERVATIUS	
ISU REU: diverse, research-intense, team-based	335
LESLIE HOGBEN	
AIM's Research Experiences for Undergraduate Faculty program	343
LESLIE HOGBEN AND ULRICA WILSON	
Institutional support for undergraduate research	355
KATHY HOKE, ALESSANDRA PANTANO, MAZEN ZARROUK AND AKLILU ZELEKE	
Experiences of working with undergraduate students on research during an academic year	363
JOBBY JACOB	
The role of graduate students in research experience for undergraduates programs	369
MICHAEL A. KARLS, DAVID MCCUNE, LARA PUDWELL AND AZADEH RAFIZADEH	
An unexpected discovery	373
ERIKA L. C. KING	
Alternative resources for funding and supporting undergraduate research	377
ZACHARY KUDLAK, ZEYNEP TEYMUROGLU AND CARL YERGER	
Academic year undergraduate research: the CURM model	383
TOR A. KWEMBE, KATHRYN LEONARD AND ANGEL R. PINEDA	
Information for faculty new to undergraduate research	395
CAYLA MCBEE AND VIOLETA VASILEVSKA	
Promoting REU participation from students in underrepresented groups	403
HEATHER M. RUSSELL AND HEATHER A. DYE	
The Center for Industrial Mathematics and Statistics at Worcester Polytechnic Institute	413
SUZANNE L. WEEKES	
Nontraditional undergraduate research problems from sports analytics and related fields	423
CARL R. YERGER	



1944-4176(2014)7:3;1-6