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Preface

Darren A. Narayan

In 1999, the American Mathematical Society received funding from the National Security Agency (NSA) to hold a conference on Summer Undergraduate Research Programs in Washington, D.C. This conference turned out to be the first in a series of three conferences, all with the mission of expanding research opportunities for undergraduates. The second conference was entitled "Promoting Undergraduate Research in Mathematics" (PURM) and was held in 2006 in Chicago.

By 2012, there were more programs aimed at involving students in undergraduate research. In addition to approximately 75 REU federally funded sites, such as NSF-REU sites, there were many other programs that increased the number of opportunities for undergraduates. These included programs supported by the National Science Foundation (NSF) in which undergraduate research plays an important role: the Mentoring at Critical Transition Points (MCTP) Program, (which funded the Center for Undergraduate Research in Mathematics (CURM) at Brigham Young University and the Long Term Undergraduate Research Experiences Program (LURE)), and the Undergraduate Biology and Mathematics Program (UBM).

Other programs were designed to provide additional opportunities for undergraduates: the MAA NREUP Program (which focuses on engaging students from underrepresented groups), the Institute of Pure and Applied Mathematics (IPAM), the Mathematical Sciences Research Institute UP program (MRSI-UP), and the Council on Undergraduate Research (CUR). With this significant growth in programs, it was clear that a third conference to review best practices for undergraduate research in mathematics was needed. Thanks to the generous support from the NSA and the NSF, the 2012 Trends in Undergraduate Research in the Mathematical Sciences (TURMS) conference was held at the Westin O'Hare hotel in Chicago, October 26–28, 2012. The central goal was to bring together faculty from these programs to discuss current practices and exchange ideas to enhance undergraduate research in mathematics.

The 2012 conference featured a keynote address by Dr. Lloyd Douglas, a former NSF-REU program officer. This was followed by an address by Dr. Jennifer Pearl,

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Keywords: undergraduate research in mathematics.

the current program officer, and invited talks, panel discussions, and break-out sessions.

The conference organizers included Michael Dorff, Joseph Gallian, Aparna Higgins, Darren Narayan, and Ivelisse Rubio. Also assisting with the conference planning were Linda Braddy, Barbara Deuink, Olga Dixon, Barbara Johnson, Michael Pearson, Peter Smith, and Laura Witowsky. We were especially grateful to Floyd (Ben) Cole of the NSA and to Jennifer Pearl from the NSF.

This volume of conference proceedings appears as a special issue of the journal *Involve*, published by Mathematical Sciences Publishers. The purpose of the volume is to promote undergraduate research in mathematics beyond the conference. A printed version of this special issue is being distributed to all subscribing institutions and an electronic version will be available free to everyone.

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Undergraduate research in mathematics with deaf and hard-of-hearing students: four perspectives

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(Communicated by Darren A. Narayan)

Involving more deaf and hard-of-hearing students in undergraduate research is a step toward getting more such students into STEM (science, technology, engineering and mathematics) careers. Since evidence exists that undergraduate research improves retention, especially for some underrepresented groups that have low retention rates — as, for example, deaf and hard-of-hearing STEM majors — it is a particularly pertinent step to keep interested students in these career paths. Nunes and Moreno have suggested that deaf and hard-of-hearing students have the potential to pursue mathematics, but lack the resources. By involving more such individuals in undergraduate mathematics research, we can improve their success rates and promote mathematics research within the Deaf community.

Here we describe our experiences working both *with* and *as* deaf or hard-of-hearing students in research, as well as advice that stems from these experiences. Each of the authors is a faculty member at the National Technical Institute for the Deaf, a college of Rochester Institute of Technology, and holds a PhD in a scientific field. Three of the authors are deaf, and one (Jacob) is hearing. While this paper describes the experiences and opinions of individuals, and is not meant to be an all-inclusive handbook on how to do research with any deaf or hard-of-hearing student, we hope that it will be a helpful resource.

1. Introduction

Why we need deaf and hard-of-hearing undergraduates in mathematics research. Perhaps for many mathematicians, the question of why we should encourage more deaf and hard-of-hearing students to pursue mathematics is unnecessary to consider. The benefits are too obvious: first, math is great fun and good for the brain, so we should share it with everyone, and second, deaf and hard-of-hearing students have talent and perspectives that can further add to the existing body of mathematics literature.

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However, we would like to pause to describe some potential benefits. We argue that there are, in fact, especially strong arguments for involving deaf and hard-of-hearing undergraduates students in mathematics research. Certainly, an undergraduate research experience can help not only mathematics majors, but also other STEM majors in their future careers, both by simply appearing on the student's resume, and also by developing the student's research skills. In addition, there is evidence that for some underrepresented groups, undergraduate research experiences actually reduce attrition [UROP 2012] and increase the likelihood of students in those groups pursuing graduate school [Hathaway et al. 2002].

While there are examples of highly successful deaf and hard-of-hearing individuals in STEM fields, there are few deaf and hard-of-hearing mathematicians, leaving deaf and hard-of-hearing students few role models and potential mentors. Deaf students, starting in secondary school, are steered towards vocational or applied fields due to the belief that they cannot succeed in more rigorous, abstract fields. By including deaf and hard-of-hearing students in research, we will not only promote the individuals' careers, but also potentially influence the Deaf community by increasing awareness that mathematics research is an option, and that there are deaf and hard-of-hearing mathematicians out there. In the future, there will hopefully be deaf and hard-of-hearing mathematicians in all areas of mathematics, available to serve as mentors and role models to deaf and hard-of-hearing students who seek them; however, at this point there are few deaf and hard-of-hearing mathematicians, meaning that hearing mathematicians need to take on the roles of mentor and role model for deaf and hard-of-hearing students, to the best of our abilities.

For two of the deaf authors of this paper, a central reason that the authors ended up in academic research careers is because of hearing researchers who sought them out. Often, areas of research that are related to hearing loss, deafness, or accessibility for deaf people attract deaf and hard-of-hearing researchers, in part because researchers in these fields think to look for deaf and hard-of-hearing talent. As mathematicians, we should be motivated to look for deaf and hard-of-hearing talent as well.

Finally, we believe that deaf and hard-of-hearing students will be more likely to participate in mathematics research if they understand the benefits. The second author, who currently does mathematics research with undergraduate students, has found that curiosity draws many students to mathematics, but it is also a good idea to express to students how mathematics research can positively impact their future. Particularly for deaf and hard-of-hearing students in computer-related majors, of which there are many at RIT, mathematics research can be attractive to future employers and graduate school admissions. Depending on a faculty member's research interest and a student's major and career goals, explicitly describing the potential benefits of the research problem to the student's career may be a good move.

Description of this paper. In this paper, our goal is first to encourage faculty to pursue research with deaf and hard-of-hearing students, and second, to give some general advice for each part of the research process, including selecting students, in-person as well as written communication, supporting students at conferences, and potential pitfalls. To illustrate our suggestions, we include our own experiences, both from our own student research experiences and our experiences in leading student research.

A recurring theme in this paper is the fact that deaf and hard-of-hearing students are individuals. While we attempt to give helpful advice, each student is different. Faculty members will encounter deaf and hard-of-hearing students who are exceptions to each generalization we describe here, and to whom none of this advice applies. It is essential to keep this idea in mind when reading this paper and interacting with students. We hope, however, that we can provide some general advice to get started and to allow faculty to build a better working relationship between themselves and their deaf and hard-of-hearing students.

Three of the authors in this paper are deaf or hard-of-hearing faculty in scientific fields, while one, Bonnie Jacob, is a hearing mathematics faculty. We write the paper as a group, but in sections where one of us describes his or her personal experience, we will use the author's name along with italics to indicate that these are the words of one specific author.

2. Before research: recruiting your student and finding problems

Bonnie: *When we look for students for mathematics research, one important consideration is whether the student has the background to understand the question and to make any progress. As I've sought students to work on problems, however, I've realized that there is a great deal of talent at a lower course level than I would ordinarily look. While I recruited one rare deaf mathematics major who had already had some advanced courses such as abstract algebra, I now keep my eyes open for students who have had as little as a semester in elementary calculus or a single course in discrete mathematics. My current pair of students are just that: one has had two quarters of nontheoretical discrete mathematics, while the other has had two quarters of basic calculus.*

The reason that I started considering students with less course preparation than I would normally consider is simple: numbers. There are very few deaf and hard-of-hearing students taking upper-level mathematics courses, even here at NTID/RIT. However, quickly it became apparent that this approach has an added benefit: there is a great deal of undiscovered talent, I believe, in the beginning stages of college mathematics. I like to think of many of the students I encounter as "diamonds in the rough."

Many deaf students have not had the exposure to the possibility of going far in mathematics. First, deaf and hard-of-hearing students are often steered toward more applied rather than theoretical fields, under the often mistaken assumption that they do not have the aptitude to succeed in theoretical fields, and that these majors will not lead to viable careers for deaf students. This means that students with potential to look at more abstract problems have been kept in more applied, lower-level courses. Second, the Deaf community is a cultural group, whose members share values and exchange information. Since there are currently relatively few mathematicians who are also members of the Deaf community, there is a lack of influence from this perspective as well.

Bonnie: *When I consider whether a student would be a good fit for doing mathematics research with me, I look at a few factors. First, at this point I only consider students who have had at least one course in calculus or discrete mathematics, and who have done well in these courses. I also look at their basic mathematics skills such as manipulation of fractions and algebraic expressions. Many mathematics faculty may consider these abilities a given in their students, but when I'm picking students from a lower course level, I need to consider whether the student has these basic skills or not. The student also needs to be responsible, with a good work ethic: I will not consider a student who breaks appointments or shows up late. A strong sense of mathematical curiosity is essential: even among students with a good work ethic, some students are interested in simply getting a good grade, while others show interest in the problems. I look for students with evidence of being in the latter group. Finally, while this last criterion is somewhat intangible, I look for students who will work well in a group. Normally, students I have met with the other necessary characteristics satisfy this one as well, but I need to make sure that the student will make the meetings pleasant for everyone.*

My list of criteria for students is, of course, tied in with my research interests. One area I work on is graph theory. While it is not my main area of research, I enjoy it, and I find that it works well for students in a variety of levels. Students with less background can understand a problem if I choose it carefully; students with computer skills can write programs related to our problems; students with more background can often work on aspects of the problem that may be inaccessible to other students. When I worked with a mathematics major, however, we worked on a totally different problem. I had worked with Tyler Swob on material from an abstract algebra course, and found that, while abstract algebra is not his favorite topic, Tyler showed a high level of responsibility, motivation, and mathematical curiosity. I decided to seize the opportunity to work with Tyler, since deaf mathematics majors are uncommon, even at RIT. For Tyler, I chose a completely different sort of problem:

applications of wavelets to finance, because finance is a particular interest of Tyler's.

Since Tyler is a rare deaf mathematics major, with a tremendous number of interests, talents and activities, I hesitated to ask him about research, because I assumed he would already be doing research if he was interested. This was another learning moment for me: on a whim, I finally just asked, and Tyler jumped at the chance. No one had ever asked him to do research. In fact, he had approached one professor, but the faculty member could not work with him at the time, so Tyler stopped looking for a research mentor. The lesson here, I believe, is to be willing to jump in and ask any student you think would be a good fit to do research with you. Now, I do, and I'm surprised by how many students are interested.

The experience with Tyler was quite positive, I believe both for him and for myself as well. However, it was not sustainable: I could not wait for deaf or hard-of-hearing math majors to come along, select those who were a good fit, and design an entirely new project for each one. When I realized this, I changed my approach, and started looking at my pool of students who were taking discrete mathematics or calculus. If I find students who are a good fit, I approach them either by email or in person. Some do not respond at all, or are not interested. However, three of my students did respond positively.

While working with deaf and hard-of-hearing students on mathematics research can be a great experience, and we encourage mathematics faculty to just jump in, it is essential that you recruit students that you can believe in.

Henry: *In my case, there was Mrs. Lois Holland, my fourth grade teacher at the Lexington School for the Deaf, who believed that I would succeed in mainstreaming from that school to the New York public school system, in spite of the fact that everyone else at the school believed I would fail. There was Dr. Daniel Albert, an ophthalmologist who believed in me and hired me as a research technician at the Massachusetts Eye and Ear Infirmary. Then there was Dr. James Saunders, who was my PhD advisor at the University of Pennsylvania (UPenn).*

If we pick deaf students who are not a good fit for a research group or the project itself, we could be hurting more than we help.

Kim: *Often, deaf people feel like they are "tokens" in research projects — they need to feel like they are respected for their academic work.*

Therefore, while we should think outside the box and seek out students who may not have the course background we would normally expect, we need to limit our search to students who have the potential to succeed at research. A group from the City University of New York that involved high school and undergraduate deaf students in research describes multiple resources that they had that led to their successful collaboration, but mentions that research among deaf and hearing individuals can be successful without all of these resources, if certain important characteristics are present [Huenerfauth 2010]. Specifically, they describe, "The key elements that we feel are the most important are that the students feel they are making a real contribution to the project and that they feel that they have full access to the communication environment of the laboratory." We discuss communication in Section 3.

3. Doing research together

Just jump in. Bonnie: *I've been pleased by how well students have done once we start working. With Tyler Swob, the mathematics major, I spent a great deal of time planning the project, and preparing materials. This was primarily because we worked on the problem as more of an independent study than straight research, but also because the problem was from outside of my area. As the quarter progressed, I adjusted content as necessary, and we developed the project together, which involved a good deal of Matlab programming.*

I asked Katherine Fetcie to do research together after working with her on content from her second quarter of calculus. A sustainability major, Katherine is very pleasant to work with and highly motivated, always showing up prepared to work together. I soon noticed that she solved problems from her course quickly and easily, and is adept at seeing patterns. While Katherine and I tended to talk in English, she also showed an enthusiasm for learning American Sign Language (ASL), which is another important characteristic I look for in my students: a desire to communicate better with their peers. Katherine and I started working together one-on-one in the fall. The graph theory project I initially chose was a concept I thought was completely new, but turned out to be explored in the literature. Katherine managed to find some early results, but because much of the groundwork had already been done on the problem, I switched problems when we began work again in the winter quarter, this time with an additional student.

Daniel Saavedra is an information technology major who showed a strong work ethic and ability to grasp mathematical concepts when we worked on material from his discrete mathematics course. I asked him to join Katherine and me. While I would have liked the students to work individually, and had some concerns about disrupting the balance Katherine and I had, I chose to pair Daniel and Katherine together for a few reasons. First, I had no time specifically in my schedule to work with students on research, so managing two different problems and two different meetings each week would be more than I could reasonably manage. Also, both students are pleasant to work with, and I felt could benefit from each other's presence. Further, communication is smooth, because Katherine speaks English *and has worked at her ASL; Daniel is strong at both. I have also observed both students making concerted efforts to communicate with peers who have different communication styles.*

With both Katherine and Daniel, I gave a very basic introduction to the problem, which is related to zero forcing in graphs. I only define what is absolutely necessary, and then start asking them questions. This has been reasonably successful. However, around the same time I recruited Daniel and Katherine, I found another student who showed strong potential for research, and asked him if he would like to try a problem. We met only once before the student quit. The problem I showed to this student is also related to zero forcing, but in my preparation to talk with the student, I was not careful in figuring out how to succinctly describe some basic ideas. I suspect that the student struggled to understand the problem, which led to him not wanting to return. While I regret this, it was a good learning experience because it helped me better prepare for explaining zero forcing to Katherine and Daniel. I've learned that with students who have less mathematics background, providing the least information possible in the simplest manner possible is key.

To be successful in mentoring deaf and hard-of-hearing undergraduates in research, mentors need to be ready to take a risk and jump in, but setting up clear expectations with the student from the beginning is critical.

Henry: *Devastating consequences can occur if neither mentor nor student gets a good understanding of the other. For example, before arriving at UPenn, I was rejected by one medical school because the person who interviewed me wrongly thought that I only wanted to work with deaf patients. This last point emphasizes the importance of having a good understanding not only between the interviewer and interviewee but also between the mentor and his or her student, regardless of disability.*

Asking good questions. A recurring theme in this paper is that deaf and hard-of-hearing students are all different, that there is no way to generalize what any particular deaf or hard-of-hearing student's needs are. Because of this, it is important that faculty mentors are ready to ask questions. Even faculty who are deaf or hard-of-hearing themselves, or have worked extensively with deaf and hard-of-hearing students, may not know exactly what works for a particular student.

If you believe that knowing the answer to a particular question will help make research a more positive experience for the deaf or hard-of-hearing student, then by all means, ask. Of course there are rude questions, but common sense can be a good guide.

Bonnie: *One important piece of advice came from an ASL teacher, who explained to me that asking whether a deaf or hard-of-hearing person can lipread may be considered offensive. At the beginning, I respected this statement, but I didn't* *understand it. Recently, when I asked her to explain more, she pointed out that asking someone, "Do you lipread?" is a question that appraises the person who is being asked. Rather than putting the burden on the person being asked, it would be more polite to ask, "Is my communication clear to you?" or "Is my speech clear?" This puts the focus on effective communication, rather than on a deaf or hard-of-hearing person's ability or lack of ability to speech read.*

It is important to realize that you will make mistakes. However, if you ask what you do not know, and keep in mind that deaf and hard-of-hearing students are experts when it comes to working with them because they experience the frustrations of working with hearing people every day, you will certainly have a better experience than if you try to make all the decisions yourself without input from the student.

Communication.

In-person communication. Specific strategies for communicating effectively in person with a deaf or hard-of-hearing student depend on the student's communication style. We present some general advice as well as advice for specific situations here.

Kim: *One recommendation I have is that there should be an interpreter present anytime a hearing faculty wants to do research with deaf students who use sign language. Taking notes is not the same. Naturally, some deaf students may struggle with their English skills. Not everyone is bilingual – some deaf students may be stronger in one language than the other language. Some may be weak in both languages and some may be strong in both languages.*

We again return to the theme that deaf and hard-of-hearing students are all individuals, that there is no one-size-fits-all approach. Some students use ASL and never use their voices, while others don't sign at all; some can do both skillfully; some students are comfortable using their voices in certain situations and not others. No matter what your student's preferred communication mode is, it is important to have a plan for communication during meetings.

Some faculty members actually jump in and take ASL courses themselves, which is a wonderful step, but it is important to remember that ASL is a full language, and it will take a lot of time and effort (undoubtedly more time than it takes your student to complete a research paper) to learn ASL. Further, even if you become skilled at ASL, it is impossible to both function effectively as an interpreter for a student and participate in the meeting at the same time. Therefore, if you are planning on working with a manual student (a deaf student who primarily signs) on mathematics research, you will probably need an interpreter. There are instances when an interpreter does not show up, or is unavailable, or an unexpected meeting happens. In those cases, often writing back and forth, or using computers or cell phones to type back and forth, can be a temporary substitute, but this sort of communication is time-consuming, and is generally not a good long-term solution.

One-on-one meetings between a research mentor and deaf student are valuable, and a mentor's creativity and ability to express concepts visually can also be critical.

Henry: *In my opinion, the most meaningful means of support at either scholarly or professional level is not note-taking or sign language interpreting but one-to-one discussions, that is, one teacher to one student (or one employer to one employee) discussions. I do appreciate such support when it comes to discussion with my superiors, colleagues, and students. This is because it allows excellent opportunities for each to make sure that they have a good understanding of what the other is expected to learn or know, and what he or she is supposed to do. I can give a good example of such an opportunity — it came from a hearing medical student when we were working on a research project at UPenn. He used a simple visual gesture showing the relationship between the tectorial membrane and hair cell stereocilia, and this gave me a much needed comprehension of how the ear works.*

To facilitate communication no matter what mode of communication — ASL, spoken English, or something else — your student uses, meeting in a room that allows all group members to see each other clearly is a good idea. Also, incorporating as many visual examples as possible can also be beneficial, as can writing down frequently occurring specialized vocabulary or names as they turn up. The environment in these meetings is critical: no matter how good a student is at mathematics, if he cannot follow the meeting, it will be much more cumbersome for him to contribute substantially to the group.

Raja: *I loved reading while growing up so I was able to become fluent in English. Although I did not receive formal deaf-related accommodations in high school, my parents took steps to mitigate barriers. These steps included enrolling me in a small laboratory school that was attached to a teacher's college. My class cohort was small at around fifteen students, and did not change. I saw the same teachers throughout, so they were familiar, and I was able to speech read fairly well.*

For my undergraduate degree, I enrolled in a large university, and this was too abrupt a change. Although I got accommodations such as note takers and oral interpreters, I missed a lot of information and struggled. I moved to a small university with smaller classes, which immediately made a huge difference in classroom learning and inclusion. I started to learn sign language. For my masters, I enrolled at RIT and made sure to enroll in small, inclusive classes to maximize my learning and outcomes.

Written communication. Writing can be a great communication tool when a hearing faculty member is working with a deaf or hard-of-hearing student. It is essential to keep in mind, however, that for many deaf and hard-of-hearing students, English is not a first language; additionally, deaf and hard-of-hearing students do not have the benefit of hearing it every day, in contrast to their hearing peers. Written communication such as email between faculty and students is an important part of staying in touch in research; keeping emails succinct may be the best way to ensure that communication is clear. Beyond simple email communication, however, writing is major part of the research process, and may be one of the more difficult parts for many undergraduate students.

Henry: *I have never forgotten how ruthlessly my PhD advisor, Dr. James Saunders, criticized my first lab report with lots of comments about 24 years ago. I still have my original lab report, and yet it pains me to even remember that paper, much less look at it, but it has certainly inspired me to write well.*

The aforementioned lab report brings up another lesson that is relevant to mentoring deaf and hard-of-hearing students in mathematics. Mentors must show extra patience and willingness to teach their deaf or hard-of-hearing students how to write a paper properly. This simply cannot be learned overnight. The student must learn to act as though he or she were someone else reading the paper; and must make sure that it is simple and understandable enough so that an external reader could replicate the results without much help.

Therefore, providing extensive feedback on write-ups will probably be part of the territory when preparing for publication with your student. For most hearing students, of course, this is the case as well, since most of them have not experienced writing an article for submission in a peer-reviewed journal; for deaf students, however, the difficulty may be magnified.

Naturally, this is one more area where we cannot generalize: many deaf students, regardless of which communication mode they prefer, are quite comfortable with written English, and thoroughly enjoy writing. Some deaf and hard-of-hearing students have better writing skills than have most of their hearing peers. There are many students, however, who have the ability to do mathematics research, but will require much more intensive support with regard to their writing. A possible suggestion, for faculty members who can find the time, is to work specifically on reading journal articles with the student to expose her to the kind of language used in journal articles.

Kim: *One of the most successful strategies I found was a weekly journal club hosted by my research mentor. She would pick out a peer-reviewed article, have me read it and we would meet to discuss that article. She helped me develop critique skills, that is, the ability to recognize what was good about that research project and what that research project could have done to improve the results.*

While many faculty will not be able to start a formal journal club, assigning specific, manageable amounts of reading to students on a regular basis may potentially be helpful as students prepare to write up their results.

Issues beyond communication: fully supporting your student. When working with a deaf or hard-of-hearing student, overcoming the hurdle of communication alone is not sufficient to guarantee success. There is also the issue of managing attention.

Raja: *Though academic support can go a long way in improving deaf students' graduation rates, it is not sufficient, because classroom accommodations by themselves do not equalize access to information. Deaf students have to learn to effectively "manage and shift attention" among multiple information sources (e.g., interpreter, instructor, blackboard, slides), which remains an elusive goal. When this attention is poorly managed, loss of information is likely to occur, and cognitive effort is shifted towards managing lower level attention management at the expense of higher order thinking skills* [Mayer and Moreno 1998].

The cognitive effort that is focused on managing attention rather than learning can potentially result in poorer performance in academics and results in deaf students' withdrawal. This tends to decrease the overall number of deaf students. The ability to effectively engage, disengage and shift attention is important not just for learning in the classroom, but for working in technical and scientific jobs where deaf employees are expected to participate in meetings and group projects that involve dialogues. Deaf students must compete with hearing counterparts and be able to capture as well as understand high-level information during the lecture, thereby being able to engage in cognitive activities that would otherwise be out of their reach.

Most of us pick strong students to involve in research. Strong students — hearing or deaf — have developed strategies that work for them in the classroom. If you recruit a deaf student to work together on research, the student has undoubtedly found effective strategies for managing in the classroom. These same strategies may not be as effective during research.

Raja: *I did not have any problems with my professional coursework, as the same strategies in high school, undergraduate and graduate school worked. But these strategies did not work for my doctoral studies, which had two distinct phases. The first phase, to pass core courses and comprehensive exams with the aim of showing academic knowledge breadth, was a continuation of previous education. The second phase, becoming an independent researcher, was much harder. For a long time, I was the only deaf individual in my studies and work place, and became accustomed to working on my own with less collaboration with others.*

Although it was somewhat difficult to follow group conversations and many one-to-one conversations, I learned compensating strategies that served me well, such as mini-support groups and explicitly learning unwritten codes.

While these strategies served well in the work place and courses, they did not serve well towards the goal of becoming an independent researcher. The main reason was that for the first five years I did not meet any peers or mentors in my field who were deaf or could sign. Without peers or mentors, I was unable to transition from passing my classes to becoming an independent researcher, which is a core expectation in PhD studies.

While we cannot develop strategies on behalf of students to ensure their success in research, and we also cannot — as hearing faculty — replace a network of deaf and hard-of-hearing peers or mentors, we can be aware of these potential pitfalls and do what we can to make the transition easier. For example, during a research meeting, making the environment as deaf-friendly as possible will allow the student to manage his or her attention better; also, spending time mentoring the student one-on-one, and making contact with other deaf and hard-of-hearing mathematicians, or with mathematicians who can sign may alleviate some of the isolation or frustration a student may experience. In the next section, we will discuss strategies for helping students to develop the tools they will need outside their research group meetings to grow in research.

4. Outside the research meeting

A note on deaf culture. While many people are aware of sign languages such as American Sign Language, many of us have never heard of Deaf culture. A thorough description of Deaf culture, the American Deaf community, and other topics related to the difference between "deaf" and "Deaf" are beyond the scope of this paper. We refer interested readers to [Cohen 1994; Holcomb 2012; Padden and Humphries 2006] as a starting point.

However, being aware of the existence of Deaf culture may help avoid misunderstandings during research-related interactions. For example, if a faculty member sits in his office with the door closed, and does not have any kind of uncovered window between the hallway and the office, a deaf or hard-of-hearing student may have difficulty. Even if the student knocks, he may not be able to hear whether there is an answer or not. Therefore, uncovering a window on one's office door (if such a window exists), or leaving the door open whenever a student may visit can help the student have equal access to you.

Also, if a student depends on looking at you for communication, certain seating arrangements may be uncomfortable. It is better to sit in a way that allows the student to clearly see your face. If you sit side-by-side with the student rather than facing her, or with your back to a window that gives off a glare, the student may not be as comfortable.

The second author, as a hearing faculty member newly among deaf students, experienced culture shock in her first year working with deaf and hard-of-hearing students.

Bonnie: *At the beginning, I was surprised by students who would walk right into my office without hesitation, sometimes shutting the door behind them. One or two students even sat down in my chair on occasion, which, frankly, shocked me a bit, or hung their coats up on top of mine on the single hook on my wall. Of course, there are many students who do not do these things, but as a hearing person, I tend to notice the students whose habits differ from hearing etiquette.*

On one occasion, I was working in my office with a student who needed help on a calculus problem. He closed the door behind him when he entered, which was normal for him. However, when his friend tried to open the door to join us, the door did not open, because I have it set to automatically lock when I close it. The student sitting at the table in my office got up to let her in, but also changed my lock setting so that it would not automatically lock. I was actually somewhat alarmed at this, because if I hadn't noticed, I would have closed my door later and unknowingly failed to lock it. I asked the student to change it back, and not change my lock settings in the future. He complied quite pleasantly, and we moved on.

It is important to realize that Deaf culture differs from hearing culture in many ways. Be ready for some potential surprises, as well as flexible and understanding of ways you can benefit the student; however, if there are situations that you feel compromise your security or comfort, you should discuss your discomfort directly with the student.

Before and during conferences. Many of us attended our first mathematics conference as graduate students, and may have found the experience daunting. Undergraduates, of course, may be even less prepared, and a deaf or hard-of-hearing student often has more concerns. First, even if you normally work one-on-one with a hard-of-hearing student who seems to communicate fine with you in your office, there is a good chance that your student will require accommodations of some sort at a conference, whether captioning or an interpreter. Conferences can be noisy, a speaker may be very quiet or difficult to see, and the room layout is unpredictable. In particular, if your student is giving a talk or a poster, it is essential to plan ahead and make sure the student has the necessary tools.

In addition, for many mathematicians, the important part of the conference is the interactions we have with other mathematicians in hallways between or after talks. It would be a good idea to address your student's communication needs in these situations as well, whether it means having an interpreter available or choosing quiet, well-lit spaces for social events.

Grundy and McGinn [2008] describe the graduate research experience of the first author, Annabelle Grundy, who is hard-of-hearing. One topic that the authors discuss is Grundy's experience at conferences, pointing out that smaller conferences provided intimate settings that allowed Grundy to get to know speakers, who would then make their talks more inclusive, but larger conferences were overwhelming. When Grundy gave her thesis defense, the group selected a room with "strong lighting, close seating, and minimal background noise," and the committee was asked to write their questions on cards in addition to asking the questions orally. Taking such relevant steps based on a student's needs will reduce the student's stress during the talk, and increase the chance that the student will give a successful presentation.

Networking and the water coolers. As with the aspect of writing, deaf and hard-of-hearing undergraduate students will gain considerably from the research experience if faculty mentors devote conscious effort to helping their students develop a support network. Many undergraduates, including hearing undergraduates, struggle with networking, but for many deaf and hard-of-hearing students, this is one hurdle that is more daunting because of the inherent communication barrier.

Kim: *One of the most successful strategies I learned, as a young researcher, was to network as much as possible. This was a critical part that I didn't realize was very important especially as a professional. My mentor introduced me to some "big names" in our field at workshops, conferences, etc. Some of those introductions led to professional working relationships with these professionals and researchers which have been maintained to this day.*

Having a network of peers and mentors will increase the chances that students will find out about relevant conferences, fellowship or scholarship opportunities, and research opportunities. Simple knowledge of these opportunities can be the difference between a successful research experience and an unsuccessful one.

Raja: *In retrospect, during my early years of doctoral studies, being the first deaf student in the program inadvertently delayed my dissertation progress. My initial dissertation environment involved working in two labs at different universities. This arrangement made it hard to obtain interpreting services outside of my institution. In addition to my own work, I had to learn the ABCs involved in networking in both biomedicine and computer science. The reduced accessibility resulted in loss of valuable information in lab meetings as well as outside, such as informal chats by the water coolers. I was unaware of doctoral fellowship grants that were available to students in underserved populations. I was also unaware of* *which conferences I would benefit from attending related to my research work. It was a blessing in disguise that the external lab's closure forced me to change advisors. My advisor explained which conferences were relevant and their unwritten expectations. I also learned how to maximize networking opportunities at these conferences. This resulted in successful conference submissions and subsequent research collaborations.*

Leading the student's research is not the only way that you can ensure that a deaf or hard-of-hearing student has a chance to experience research. You may not have the time to work with a deaf student, or your research interests may not match the talents and interests of the student. In this case, it's a great idea to inform the student about different REUs, conferences, or programs that may be a good fit. Many undergraduates, hearing and deaf, are unaware of the research opportunities available for them, but a deaf mathematics major may be even less aware.

Bonnie: *In one case, I encountered a deaf mathematics major who was thinking about a summer research experience. I did not have the time or resources to do research with the student at that point, and her interests did not overlap mine. Further, since the student was an international student, she faced an additional constraint. I looked into programs that would accept international students. What struck me was that the student was surprised when I gave her a list of programs. "Why are you helping me?" she asked. It never occurred to her that a faculty member would be interested in helping her to find research opportunities.*

In many cases, students are surprised when I suggest research. Many students have no idea that these opportunities exist, or that they can get paid to participate in research.

Kim: *One day, I was working as a student development educator in the department of human development at the National Technical Institute for the Deaf (NTID); a faculty member approached me and told me about a summer internship opportunity at a research hospital. That faculty also happened to know the person who was coordinating the summer research internship program for deaf and hard-of-hearing students. What was also appealing about that summer program was that it was right in my home state! It was a great opportunity to participate in that program and at the same time, be with my family during the summer time. After I was convinced, I decided to apply to the summer internship program. A couple of months later, I learned that I had been accepted. Prior to beginning work in that summer internship program, I had no idea whether I would be interested in a research career or not.*

It is essential that we educate all deaf and hard-of-hearing mathematics students about the research opportunities that are available.

5. Further reading

We recommend resources for readers interested in particular topics that were mentioned in this article. These references represent only a starting point for further reading.

For general background information about Deaf culture, we refer readers to [Cohen 1994; Holcomb 2012; Padden and Humphries 2006]. For background about deaf and hard-of-hearing children and the learning of mathematics, see [Kritzer 2009; Lang and Pagliaro 2007; Nunes and Moreno 2002]. A reference on the education of deaf and hard-of-hearing children in general is [Marschark et al. 2001]. The articles [Lang et al. 1993] and [Marschark et al. 2008] present research into how deaf college students learn and perceptions among deaf college students and faculty, respectively. For reports on deaf students' transition from high school through employment, see [Walter 2010]. For a detailed synopsis on scientists with hearing loss from the 16th century to the early 1990s, see [Lang 1994].

A handful of articles describe involvement of deaf or hard-of-hearing students in research. [Grundy and McGinn 2008] describes a hard-of-hearing graduate student's experience; [Huenerfauth 2010] describes the experience of a group that involved deaf undergraduate and high school students in ASL animation research through the City University of New York; [MacDonald et al. 2002] describes James Madison University's undergraduate research program in chemistry, which involves deaf and hard-of-hearing students. Finally, for a description of some of the benefits that have been shown for members of underrepresented groups who do undergraduate research, see [UROP 2012; Hathaway et al. 2002].

6. Conclusion

The goal of this paper is to encourage mathematics faculty not only to seek out opportunities to work with deaf and hard-of-hearing students in research, but to become mentors for deaf and hard-of-hearing students in mathematics as well. We have presented recommendations based on our own experiences of working with deaf and hard-of-hearing undergraduates, and of being ourselves onetime deaf student researchers (as is the case with three of us).

There is, as we have mentioned, evidence to suggest that deaf and hard-of-hearing students will benefit tremendously from undergraduate research experiences in mathematics. As opportunities arise, we hope that faculty will become mentors and advocates for deaf and hard-of-hearing undergraduates in mathematics, and that the advice that we have presented here will make this process a more positive experience for students as well as faculty.

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Challenges in promoting undergraduate research in the mathematical sciences

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We describe the challenges in promoting undergraduate research in the mathematical sciences. The challenges are grouped in regards to the population that research is promoted to: students, faculty and administrators. For each category, we provide some suggestions for overcoming the challenges taking into account the variety of institutions involved.

1. Introduction

The benefits of implementing strong undergraduate research (UR) programs across the sciences have been investigated in some depth by several authors [Karukstis and Hensel 2010; Laursen et al. 2010; Lopatto 2009; Seymour et al. 2004]. However, the progress towards implementing robust UR programs in the mathematical sciences has been slower than it has been for the other sciences. It has been especially difficult to integrate research into the undergraduate mathematics curriculum. In improving the UR landscape in the mathematical sciences, each of the three constituents — students, faculty (including both faculty to serve as mentors and other mathematics faculty) and the administration — can play a significant role. To get support from these constituents, UR and its benefits has to be effectively promoted to them.

In this article, which grew out of the discussions during the "Challenges in Promoting Undergraduate Research" session at the TURMS conference, we discuss the challenges in promoting UR to each of the three audiences and offer some suggestions for how to overcome these challenges. Throughout, we use "undergraduate research" to mean the following definition, provided in [CUPM 2006]:

- The student is engaged in original work in pure or applied mathematics.
- The student understands and works on a problem of current research interest.

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- The activity simulates publishable mathematical work even if the outcome is not publishable.
- The topic addressed is significantly beyond the standard undergraduate curriculum.

This expands upon the definition by the Council of Undergraduate Research, which is "an inquiry or investigation conducted by an undergraduate student that makes an original intellectual or creative contribution to the discipline".

2. Challenges in promoting undergraduate research to faculty

Supervising a UR experience may feel like an enormous task for many faculty. In particular, coming up with appropriate topics and problems can be a major source of difficulty. Rarely can mathematics faculty simply give their students the problems that they are working on and expect a meaningful and substantial contribution. Doing so puts students in an untenable position. Instead, faculty should formulate problems that are both relevant to their research agenda and, at the same time, accessible to talented undergraduate students after a brief tutorial on relevant background material. Put another way, in discussions on UR it is important to keep in mind that the word "undergraduate" is an important qualifier.

However, keeping in mind the definition, a UR experience should be original work in pure or applied mathematics which the student understands. Discoveries that are new only for students cannot be classified as research. By keeping UR problems related to a faculty's main research interests, there is a much greater likelihood that the research will result in something of real value to both the faculty mentor and the student. The faculty mentor will be able to gauge student progress through the research project better and will know for sure whether the results are original research. For junior faculty, in particular, attending student presentations and reading papers written by students will help in developing a sense for the appropriate level of problems for students. A website with open problems appropriate for UR will also be useful in addressing this issue. If the level and the topic of the problems are appropriate, the faculty mentor will be able to get more value from the research experience.

Indeed, the value of UR for faculty is an extremely important concern. Faculty time is a precious commodity. Undergraduate students need effective mentoring for the duration of their project, and many faculty — especially pretenure — may not be able to justify spending this time if it takes away from duties that are perceived to have more value to their career advancement. In light of this, discussions on the institutional value of UR should be initiated at both the department and university level, and the outcome of these discussions should be formalized as much as possible with the appropriate university committees that decide on annual evaluation, promotion and tenure. These discussions can also include how the faculty and students involved in research will be supported, funded and recognized.

A department which values UR should consider this work as part of the department's regular workload and the faculty members involved in UR should be appropriately recognized for their contributions in this area. Ideally all faculty members, whether they are involved in UR themselves or not, will promote the benefits of UR: to students, to other faculty and staff within the university, and to the mathematical community outside of the university. If there is a critical mass of faculty members involved in and promoting UR to others, the department is more likely to have an overall positive UR atmosphere.

Even in cases where the value of UR is recognized by the department and the university, there are many challenges that faculty face in mentoring UR, including recruiting students for research projects, finding additional time and resources for mathematically underprepared students, and finding additional time and resources for teaching auxiliary skills, such as typing in LaTeX, giving presentations, etc.

In a department with a critical mass of faculty interested in UR, collaborating on certain aspects of the student preparation can alleviate the workload. For example, instead of individual faculty members teaching their own students LaTeX, students can participate in LaTeX workshops. Faculty can also collaborate on and support each other in grant writing activities.

3. Challenges in promoting undergraduate research to students

Students who participate in research are overwhelmingly excited about the experience. However, the overall student population is not generally enthusiastic or even well-informed about participating in research. Many students are unaware of the different levels and varieties of research options, or what these different options involve. Partly due to this unawareness, some students have the perception that they are not adequately prepared or talented enough to do research. This may also be because these students do not have confidence in their mathematical abilities, or because they do not envision enough benefits from a research experience. Students intending to go to graduate school in mathematical sciences are more likely to appreciate the benefits of UR. Yet, a research experience will also be extremely beneficial for preservice teachers and students intending to work in industry, and even for students intending to go to a professional school or nonmathematics students. Finally, nontraditional students with families or students who work close to full-time have a hard time fitting research into their regular schedule. So, what can we do to counter these issues in promoting UR to students?

A successful marketing campaign promoting UR to students will address most of the reasons contributing to low student interest. A variety of media are available to reach the students, including classroom time, social media, newsletters, student club events, and word-of-mouth. Faculty members can use some of their class time to provide information on research opportunities and specific research project topics. Research opportunities and information on previous student participants can be announced in print or online media targeting students. Examples include Facebook student groups, Twitter posts, web pages dedicated to UR opportunities, and newsletters and emails sent to students. Department seminars provide venues for students to present their own research and to learn about others' work. During seminars and student club meetings, students can also be informed about general logistical information such as what UR means, when students should apply, which materials are needed for an application, sample UR topics by the department faculty members, and other related information. Students especially appreciate receiving individualized information from faculty members and advisors during one-on-one conversations. A student will be more convinced of the benefit of research if multiple faculty members mention the opportunity. Additionally, a personal invitation and encouragement from the faculty supervisor of a specific project carries more weight for a student. Although we listed several suggestions, it is important that in each department the faculty study their students carefully and use marketing tools appropriate for their audience.

The success of a marketing campaign also depends on creating a community among students in which the positive messages about UR are reinforced through peers. Enthusiastic personal reports from students who participated in research presented through panels, seminars, student club meetings, newsletter articles, and other venues will strengthen the messages students receive from faculty and administrators. In smaller schools where these role models may not exist, students can participate in conference trips to network with students from other schools who participated in research. Faculty members also play a significant role in helping the student community value UR. When faculty members agree that UR is valuable, this value will be reflected in their interactions with students.

Interest in UR can also be increased by providing various perks to students. To help create a community of scholars within the department, conference and seminar attendance can be encouraged through extra credit or professional development credit in classes. Active faculty participation in these events will facilitate building a community of scholars among students. Research can be incorporated into the curriculum through optional or required independent studies, research/seminar courses, or capstone courses with research components. Receiving college credit for research will provide students with documentation of their work on their transcripts. For students with financial needs, making them aware of funded research opportunities will allow them to pursue research instead of having to work. Keeping the research project schedule flexible will help students with families to participate in the experience. Finally, for students going into teaching or industry jobs, faculty must strive to make it clear that research experiences are beneficial to every student. During a research experience, students sharpen their critical thinking, lifelong learning, communication, and problem-solving skills, all of which are highly sought by employers. Students also develop a close professional relationship with their faculty supervisor which will help them receive a better and more detailed recommendation letter from this faculty supervisor. These nongraduate school track students might be further motivated by research projects that focus on mathematics education, applications of mathematics or industrial problems.

4. Challenges in promoting undergraduate research to administrators

Administrative support is the key to overcoming the challenges related to promoting UR to faculty and students. Faculty participation in UR in both numbers and time will be higher if faculty's UR work is rewarded and encouraged by the administration. Similarly, students will be more motivated to participate in research if there is a clear articulation and endorsement of the benefits of UR by their institutions. Administrative support in the form of tangible funding is also critical for internally run UR programs. Finally, a certain degree of institutional support is also required for incorporating UR in the curriculum.

In order to increase administrative support for UR, the benefits of UR can be described in relation to the standard measures of university success, such as recruitment, retention, and job/graduate school placement. As one of the "high impact practices", UR is shown to have a positive effect on all of these measures, especially the retention and overall academic performance of minority and first-generation students [Barlow and Villarejo 2004; Ishiyama and Hopkins 2002]. These results will support the case for UR in the eyes of administrators. It would also be very helpful to obtain external support in the form of letters from alumni (reflecting on importance of the research experience in their careers), letters of support from industry as well as colleagues from other schools and departments. When communicating the importance of UR to administrators, the following points for each of the main measures can be made.

Recruitment. Prospective students look for a college experience that will be unique and exciting. Participating in research as an undergraduate is an attractive feature. It signals close interaction with faculty and individual attention paid to the students. Mentioning these opportunities in university advertising materials and in various communications with prospective students will help recruit highly qualified students to the university. Smaller institutions that can provide research opportunities to most of their students, possibly as part of their curriculum, can use these opportunities to lure students to their institution and to the mathematics major.

Retention. Undergraduate research provides students the opportunity to discover and/or nourish their passion for research, to receive individual mentoring from faculty members and to meet other people as excited as themselves about mathematics. Close relationships with faculty and other mathematics majors built early in a student's studies help students to stay in school and in the mathematics field. Through these relationships a student receives personal encouragement to continue in the mathematics field, and learns more about career opportunities for mathematics majors in academia, industry and government, and how to be successful in mathematics, all of which positively impact a student's interest and success in mathematics. The excitement and pride a student feels in the process of discovering mathematics also positively affects the students chances of continuing in mathematics and at the institution they are at.

Job placement. Mathematics graduates typically choose one of three career directions. They might go to graduate school, teach in K–12, or find employment in industry. A UR experience will help students get into better graduate schools. UR experiences make preservice teachers highly marketable in places where the job market for teachers is tight. Finally, employers outside of academia also highly value UR because it shows that the student has intellectual curiosity, can work with others, and has analytical skills as well as experience in problem solving.

Once the positive effects of UR on the important university success measures are demonstrated, it will be easier to get administrative support. The first and foremost type of support that faculty needs is the recognition of the UR work in the promotion, tenure and annual evaluation processes. Especially untenured faculty will be unmotivated in supervising UR projects if this work will not be valued in the tenure process. By the time the faculty member is tenured, it might be too late for this faculty member to be motivated to start a UR program from scratch. In addition to recognizing UR in personnel reviews, many tangible benefits can be provided to faculty members. If the administrators truly acknowledge the benefits of UR and the university has funding, they will be willing to provide funding to faculty and students engaged in research. If budget constraints do not allow monetary support, the administrators can provide other tangible benefits to faculty. Some examples of such support would be offering course release to faculty in exchange for supervising a certain number of projects/students and extra travel funds to faculty supervising UR projects.

In promoting UR to administrators, mathematics faculty also need to place special emphasis on describing differences of UR in mathematical sciences and the other disciplines. The CUPM report [CUPM 2006] has a section highlighting these differences and can be very useful during conversations with administrators.

In conclusion, most of the challenges of promoting UR arise from not having the benefits of UR widely known among all parties involved. Continuous administrative support will allow faculty to be more committed to and students to be more interested in UR. Many faculty already inherently believe that UR is beneficial to students, but administrative support, coupled with an awareness of benefits of UR, will help more faculty to be more invested in UR. On the student front, an awareness of UR benefits and opportunities will convince more students to pursue these opportunities, even to the extent that they become advocates of UR in the future when they become faculty members or university donors. A lot has already been accomplished in putting UR in mathematical sciences on the agendas of most departments and universities, but we still have more to do to expand the scope of UR to include more faculty and many more students.

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Undergraduate research as a capstone requirement

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If a mathematics department has a capstone course, how does undergraduate research figure into that capstone requirement? What challenges are involved when instituting undergraduate research as part of the capstone experience? These were the central questions for discussion in the undergraduate research as a capstone requirement breakout session at the 2012 Trends in Undergraduate Research in the Mathematical Sciences conference. In short, there is not one design that will satisfy the needs and goals of every mathematics program, but a department seeking to implement undergraduate research as a capstone requirement may benefit from the experiences of other departments. This article discusses the common objectives of a capstone in mathematical sciences and presents several successful models that incorporate undergraduate research in a capstone experience. The challenges and questions associated with each model are also discussed.

1. Introduction

During the 2012 Trends in Undergraduate Research in Mathematics conference there were four breakout sessions focusing on ways to foster undergraduate research in mathematics and a discussion of the challenges involved in such an endeavor. One of these breakout sessions targeted the questions surrounding incorporating undergraduate research as a capstone requirement for mathematical sciences programs. Among the institutions represented in this particular discussion, there were several with successful models and others that were still exploring the challenges of creating such a capstone experience.

In light of the different institutions, goals, and available resources, participants agreed that there is no single design that would be optimal for all academic systems. The type of model a department implements for a capstone course is intricately related to the make-up of that department and the demands of the institution. In this article, we discuss some of the existing capstone experiences that incorporate undergraduate research and the challenges faced by departments following such

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models. Section 2 provides a list of common objectives for a capstone course in mathematics as well as several different models of capstone courses that include some aspect of undergraduate research. Section 3 discusses some of the questions and challenges that arise from implementing these capstone course designs.

2. Course objectives and different models

What course objectives should departments have for a capstone requirement? Although there was considerable variance from institution to institution, all of the capstone experiences discussed shared common threads. Here is a list of course objectives that the participants considered valuable in a capstone requirement:

- professionalism;
- exposure to mathematics outside of the traditional classroom setting;
- synthesis;
- a self-driven learning experience.

The curious reader may inquire, "These are valuable goals, but where does undergraduate research come into play?" Here we define undergraduate research as any experience in which the student takes a lead role in understanding how mathematicians actually do mathematics. In undergraduate research, students should learn the process by which a mathematician conducts research as well as gain experience in the delivery of their own findings. From this perspective, undergraduate research projects can definitely help students to meet the capstone course objectives listed here.

The participants representing institutions with existing capstone requirements including some form of undergraduate research described their program's capstone experiences during the breakout session. The different capstone experiences can be divided into four main types:

- *Capstone topics courses*: Essentially a topics course taught by a single instructor with additional research components not typically required as part of a topics course. This type of capstone includes (at least) an expository research experience for all students in the form of a presentation or paper.
- *Capstone seminar courses*: A course similar to a seminar, with additional components that allow students to engage in research to an extent fitting their capabilities and interest.
- *Existing capstone experiences*: An experience within an existing course, meeting many of the capstone objectives outlined above. This type of course includes a research experience where students are recreating mathematics as though for the first time.

◇ *Research methods courses*: A course built around methods of mathematical research.

The implementation of these types of capstone experiences is varied, and here we provide several existing models. In parentheses we provide the size of the student body at the institution, whether the institution is public or private, number of undergraduate math majors, and the highest level of degree in mathematics offered at the institution.

i. This capstone topics course is a three-credit course with one instructor who chooses topics or a theme for the course, for example, "the fundamental theorems of mathematics". Students are responsible for group presentations at the end of the semester. This particular course serves a dual role in preparing students for the major field test administered by the Educational Testing Service. (*Public university with* 19,000 *students and around* 130 *math majors*, *graduating an average of* 40 *per year*; *highest degree in math is Ph.D.*)

ii. This capstone topics course is also a three-credit course with one instructor. However, in this example, students prepare a literature review; the instructor suggests a list of problems for the students to research in the literature. Each student submits a written report during the semester. (*Public university with* 5,000 *students and* 75 *math majors*, *graduating around* 8 *per year*; *highest degree in math is Ph.D.*)

iii. This capstone seminar course is a one-credit course, run by one faculty facilitator with each student working individually alongside a faculty mentor from the department. Students either choose to read an article from a journal such as *The College Mathematics Journal* or work on a topic that their mentor has deemed acceptable. Each student gives a 15-minute presentation accompanied with a 10-page report, creates a poster, and then develops the earlier work into a 20-minute talk accompanied with 20-page report. The department that runs this course is seriously considering making it a three-credit course. (*Public university with* 8,000 *students and* 80–90 *math majors*, *graduating* 10–15 *per year*; *highest degree in math is B.S.*)

iv. This capstone seminar course is a proposed three-credit course that will span two semesters. In the first semester students will actively research within the literature, writing reviews of articles published in *Mathematics Magazine* and other undergraduate accessible journals. During the second semester students will work in groups on projects, writing a 7–10 page paper and delivering presentations to both the department and the broader mathematical community. (*Public university with* 8,500 *students and* 25–30 *math majors*, *graduating* 9–16 *per year*; *highest degree in math is B.S.*)

v. This example of an existing capstone experience relies on real analysis and abstract algebra courses to serve as the capstone requirement. These classes are taught using the modified Moore method; students regularly present and develop results without a textbook or provided solutions. (*Private university with* 3,300 *students and* 50–60 *math majors*, *graduating* 12–15 *per year*; *highest degree in math is B.S.*)

vi. This capstone course in research methods is a three-credit course introducing students to methods common among research approaches in the mathematical sciences. Students are exposed to a variety of mathematical techniques and topics. Students gain experience with computational methods through MATLAB programming, write a paper in LaTeX on a mathematical article, and also give an oral presentation on the article. The department is considering splitting the course into two courses: a two-credit course introducing students to research in mathematics and a one-credit seminar in mathematics. (*Public university with* 9,100 *students and around* 75 *math majors*, *graduating around* 11 *per year*; *highest degree in math is Ph.D.*)

Each model described above involves undergraduates in a research experience at some level; additionally these models allow students to produce posters or presentations that are worthy of the regional MAA meetings or even national events such as MathFest and the Joint Mathematics Meetings. Occasionally a faculty mentor may have the opportunity to guide an undergraduate through the process of actually publishing her result from the capstone experience.

3. Questions and challenges

In this section we pose several questions and examine some of the challenges that arise from requiring research as part of a capstone experience. These questions are grouped by topic: course logistics, student logistics and abilities, and faculty workload and evaluation.

It is important to keep in mind that the different types of capstone courses have been taught in many different institutions and there are several sources of advice and aid available. Faculty should not feel alone in facing the challenges described here — these challenges were experienced by nearly every member of the breakout session discussion, regardless of institution or program capstone requirement design (or lack thereof).

Course logistics. How can departments make time in their students' schedules for undergraduate research? Typically the curriculum for a standard mathematics major leaves little or no room for "extra" classes in the major requirements. Consequently, in order to implement a capstone requirement what might need to be removed from the program curriculum? Similarly, in some departments the senior-level math courses are very rigorous and present a serious time commitment. How can we enable our students to manage the time necessary to be successful in both their course work as well as a capstone requirement?

If a department chooses to implement a capstone seminar course model, there are several questions that should be addressed. For example, does the course facilitator assign grades or is that the responsibility of the faculty mentors? How can we account for the difference between faculty mentor grading schemes? A partial answer to the latter question is for the capstone course to have a well defined set of student learning outcomes that take into consideration the varying abilities of the students. Even with a well defined rubric, one should prepare to encounter passionate discussions regarding grade assignments.

Some institutions have specific requirements that all departments must satisfy in their capstone course, or there may be a wide array of departmental requirements that must fit into the capstone course. For example, some institutions require students to explore the history, philosophy, and ethics of the discipline in a capstone course. Other institutions use a capstone course as a means of preparing students for seeking a career, for example, writing CVs and applying to graduate school. This limits the amount of time that can be spent learning new topics and engaging in research.

Student logistics and abilities. How can we empower even our weakest students to succeed in a research experience? Unlike an REU, where the students have similar abilities and skill level, there may be considerable variance in a senior class of majors.

How much novel research, as opposed to expository or literature research, should be expected in a required capstone course? It seems that the best answer to this question is to be inclusive about what constitutes “research” — giving students a chance to grow, without setting unattainable requirements.

What are the basic requirements, with regards to research, for students in a capstone experience? What if a student does not complete even these basic requirements? Answering these questions is a challenge for all four types of capstone experience.

Faculty workload and evaluation. Regarding the capstone seminar course model, how will teaching credit be awarded to the faculty mentors? These faculty members typically work one-on-one with students and yet their names are not listed as instructors for the course. From a faculty point of view this is a serious consideration, especially for untenured faculty members. It is possible that capstone seminar course mentors get no credit for their efforts, other than acknowledgment in their annual report. Some departments develop a system for tracking the number of capstone students a faculty member mentors over a period of time and eventually provides a course release. Of course, such considerations depend on individual department values and goals.

These questions that seem specific to the faculty mentors of a capstone seminar course are actually along the same lines as those raised for faculty engaging in undergraduate research at any level. How can a department foster an environment where faculty members are inspired to work with undergraduate students on research? How can a department maintain or keep building the momentum of such research engagement from year to year? In a department where only a small fraction of faculty members are committed to research projects with students, faculty burn-out would be a serious concern.

Should all faculty members be equally responsible for mentoring students in undergraduate research — whether or not this research is part of a capstone requirement? To what extent is a department chair responsible for making sure that faculty are willing to work with students and that faculty members are not overextending themselves? Who will see to it that faculty mentors are actually meeting and working with the students they initially agreed to sponsor?

How do the different models affect student evaluations of teaching? This may be of particular concern for untenured faculty experimenting with an existing traditional course by teaching it as an existing capstone experience. Initially students may perceive the faculty member as taking shortcuts and pushing the work of class preparation onto the shoulders of students. A charismatic faculty member may be able to present the experiment in a positive light and reduce student resistance to a new teaching method. How to achieve success with this model semester after semester, however, is not clear.

4. Conclusion

Although there is no one way to best answer all of the questions posed or address all of the challenges, participants agreed that it seems necessary to have at least one faculty member who is passionate about undergraduate research in order to achieve success in incorporating undergraduate research as a capstone requirement. Ultimately, students working on problems need a faculty mentor to guide their continuing efforts and provide feedback on both papers and presentations. Additionally, a department needs to have some agreement on the value of undergraduate research as a capstone requirement so that teaching the course and/or mentoring students is both rewarded and encouraged.

There are always limitations to the amount of resources that can be spent on undergraduate research as a capstone requirement. No one model will work for all programs, and no one model would work for all students. The important thing is to provide the best opportunity possible given the resources available.

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A decade of undergraduate research for all East Tennessee State University mathematics majors

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This paper offers a brief history of an experiment begun in 2002, namely, the institutionalization of undergraduate research (UGR) in the mathematical sciences as a semester-long requirement for *all* mathematics majors at East Tennessee State University, a public, regional school. We describe the early, middle, and later years of this ten-year journey; assessment methods; and other aspects. Technical aspects of the student projects are limited to those in the authors' fields of expertise, as captured by the MSC secondary classifications for this paper.

1. Introduction and overview

East Tennessee State University (ETSU) is a regional, state-supported institution with a current undergraduate and graduate enrollment of 15,133 students. ETSU is managed by the Tennessee Board of Regents, an education system that ranks number six in size at a national level. Over 100 years, ETSU's mission has evolved from serving initially as a normal school into a multifaceted university conferring undergraduate, graduate and professional degrees.

ETSU is located in the eastern part of Tennessee, adjoining the southern Appalachians. The main campus of ETSU is housed in Johnson City, TN, while satellite campuses operate across the tri-cities (Bristol, Johnson City, Kingsport). ETSU is located near the state lines of Tennessee, Virginia, and North Carolina, with some proximity to the state lines of Kentucky, West Virginia, and South Carolina. Approximately 47,000 alumni of ETSU reside within a 100 mile radius of Johnson City (out of 84,000 graduates since 1911). Approximately 70% of the currently enrolled students are from the three neighboring counties (Carter, Sullivan, Washington).

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As of 2011–12, the ETSU student population includes the following demographics: 86% white; 6% black; 3% Asian/Hawaiian/Pacific Islander; 2% Hispanic; 3% other (multiracial or with unknown race). This demographic composition is somewhat consistent with that of the tri-cities region, having a predominantly white population in this part of the state of Tennessee.

The undergraduate population of ETSU (12,539 as of 2011) has a five-year average of 44% male versus 56% female students. The age distribution of the undergraduate body includes three main age cohorts (with average percentages over 2007–11):

age 22 and younger:	64.80%
age 23–24:	10.60%
age 25 and older:	24.80%

The number of undergraduate mathematics majors was 60 in 2002, and has risen to about 100 (an average of 92 students over 2007–11) at the present time.

Faculty expertise in the Department of Mathematics and Statistics (as it is now officially known) was, and continues to be, in discrete mathematics, statistics, and applied mathematics, with undergraduate and graduate (MS) concentrations under consideration in statistics. Faculty research, as measured by publications (if not funded grants), has always been high; when the second author came to ETSU in 2000 and initiated a subscription to MathSciNet, both he and the agent at AMS were surprised to note that we were being charged the second-highest rate in the state, indicating the second-highest volume of published papers per faculty member per year in the state. Since then, we have attracted significant Infrastructure and Research funding from NSF, including the following grants: NSF-STEP ($1M), NSF-ATE (co-PI; $1.2M), NSF-GK12 (co-PI, $3M), NSF-Noyce ($1M), NSF-UBM (2 awards, $750K), NSF-IGMS ($50K), and NSF-REU-RET (4 awards, $1M). Two other REU programs exist on campus, in the biological sciences and physics and astronomy, and the campus-wide involvement of students in undergraduate research, including that by students in the Quillen College of Medicine and Gatton College of Pharmacy, is very high. Additionally, the Honors College of ETSU has an office dedicated to undergraduate research affairs, which fosters creative efforts advancing student knowledge. They promote efforts in composing music, sculpturing, laboratory bench research, and theoretical research. One of the accomplishments of this office is to keep up a number of research discovery positions, funded through a federal work-and-study program. All these components constitute backdrop and context for our experiment.

2. What we replaced

Until 2002, ETSU mathematics majors (in each of four tracks: mathematical sciences, mathematics education, statistics, and quantitative modeling) had always been required to take a two-credit-hour freshman seminar, MATH 1090, and a two-credit-hour junior seminar, MATH 3090. These courses were consistent with the recommendations in [MAA 2004]. Each of these classes was both oral and writing intensive, and blessed by the University Intensive Course Committee as satisfying the criteria for being labeled thus. They were taught by different faculty, who gave the classes a different slant at each offering, with the common thread being writing assignments and oral presentations. In 2000, Godbole came to ETSU as chair and taught the junior seminar for the first time in 2002 as an undergraduate research class. It made sense for him to do so, given his active involvement in undergraduate research through REU site direction, editorship of the *CUR Quarterly*, etc. It was, moreover, just intended to be a one-semester, one-time variation of a class that had seen many variations. Students in the small class of about 12 students worked in teams with no more than three students, and their work was impressive. In one of the partnerships, Jamie Howard, who went on to get a PhD in mathematics education, and Re'el Street, who obtained an MBA from Georgia Tech, worked on the "chicken king project", an area that has connections to Seymour's second neighborhood conjecture and kings in tournaments, both notions in graph theory. Their work was never published, but it is good enough to be accepted in a medium-level refereed journal. Even the not-so-motivated students in the class did creditable work, and each proved *something* original. It was enough to make one think, particularly while editing *CURQ* articles on how far the reach of undergraduate research could extend.

In 2003 we went to a 120-credit-hour curriculum, and massive curricular changes were made by the departmental curriculum committee headed by Jeff Knisley and Janice Huang. Among many changes:

- the four tracks assumed their present names;
- we eliminated a 6-hour programming (C++) requirement;
- we replaced the freshman seminar MATH 1090 by MATH 2090 (math computing, comprising, at the time, instruction in LaTeX and Maple); and
- we replaced MATH 3090 by MATH 4010 (undergraduate research), the class that we describe in this paper.

The timing of the last move was fortuitous; 2003 was still part of Godbole's honeymoon period, the first new external funding was being obtained, and the department said "yes" to the changes with little protest. The new courses stayed technology intensive (MATH 2090), and oral and writing intensive (MATH 4010), with instructors of record submitting annual reports to the overseeing university committee. Godbole's ability to suggest research projects in statistics/probability, education, discrete math, and analysis helped, and he was able to accommodate a wide variety of research interests among the students in the first two offerings of the class. He began to give talks at the Joint Mathematics Meetings with titles such as “Undergraduate research for all math majors: feasible or a pipe dream?”, which culminated, years later, in [Godbole 2011].

3. Early years

During the first three years that we offered MATH 4010 — namely, 2003–04, 2004–05, and 2005–06 — ETSU averaged 75 mathematics majors with an average time to graduation of between 5 and 6 years. We thus saw a class size of between 10 and 15 students each fall, since the class was initially offered once a year. The first year had a single advisor, Godbole, so that the instructor of record was the sole advisor. Everything moved smoothly at first; he just considered this an extension of the summer REU, with him directing between four and six groups, with the time-frame being dilated, and with the demands on students and expectations of scientific achievement being lesser.

A few students were also in the Honors Program, and we found that they had to also fulfill an Honors Thesis requirement. During the early years, such students were *occasionally* permitted by both parties to use their hard-earned MATH 4010 project as a “double-dipped” Honors project. This was soon discouraged and prohibited.

MATH 4010 became part of the Academic Quality Initiative (AQI) in 2004, a university-wide project launched in response to the 2003 site accreditation visit by the Southern Association of Colleges and Schools (SACS). AQI was part of the larger Quality Enhancement Plan (QEP) that each institution must submit to SACS as part of its reaffirmation process. It allowed for systematic outcomes-based assessment for several courses across campus, and we were happy to enter MATH 4010 as a participating course. As part of this scrutiny, we formalized the oral and writing requirements as follows: students would submit 5-, 10-, 15-, and 20-page reports at regularly scheduled intervals; the longer reports would be an enhancement of earlier versions. Also, submission of each report would be accompanied by oral presentations that would be attended by all students.

This worked well during the first and second years. Two more advisors were added in 2004–05; they would turn grades in to Godbole, who would submit them along with the grades of his own advisees. The course was offered twice in 2005–06, and even more advisors entered the fray. It became difficult to monitor whether or how the intensive requirements were enforced by other people, or perhaps Godbole just didn’t do his job well! The diversity of projects grew as advisors in graph theory, mathematics education, geometry, and statistics began to supervise projects. The students’ sense of ownership became evident through AQI assessment and folklore tales. AQI assessment was conducted by a committee chaired by Edith

Seier, and the committee reports were submitted to the University AQI committee. Aspects that were assessed were the oral presentation quality, quality of the final report, and scientific quality.

4. Middle years: the multiple mentors model

The three years 2006–07, 2007–08, 2008–09 were the years when the momentum of the course definitely lessened. Godbole did not enforce the writing/oral requirement too stringently. There were two other instructors of record: Teresa Haynes and newcomer Robert Beeler, who joined us in 2007. The tendency to let advisors "quietly do their own thing" grew, certainly after AQI ended in 2007. It became clear that we needed new blood.

5. New faculty, the reemergence of structure in the present time

Ariel Cintrón-Arias (henceforth Cintrón-A) joined the faculty ranks in 2009, and Michele Joyner came on board in 2010. Cintrón-A immediately became the MATH 4010 instructor of record on several occasions, together with Michele Joyner, who did the job once. Cintrón-A had been to MTBI in 1998 as an undergraduate participant, and had past experience at MTBI (at both Cornell and Arizona State) as a PhD student and at SAMSI-CRSC Undergraduate Modeling Workshop as a postdoc. He had worked extensively with Carlos Castillo-Chavez, who was his PhD advisor. Michele Joyner was H. T. Banks' PhD student and has mentored students at the NCSU REU. It was evident that our undergraduate research strengths had grown, and in new directions, as students began research in the mathematical modeling of complex phenomena such as epidemics, rumors, and antibiotic resistance. Here is an example of how the landscape changed: Cintrón-A ran a summer-fall undergrad research program in mathematical epidemiology; all his students participated in a poster session during the NSF-CBMS Regional Research Conference, alongside graduate students and postdocs from ASU, NCSU, Purdue, etc. His students attended/received offers from summer REUs at MBI-OSU, ASU, UMBC, and NCSU; Joyner's student went to the REU at NCSU. Over the last four years, Cintrón-A set in place a method of ensuring that course objectives are met. Technical reports have to be submitted by preassigned deadline dates, no matter who is the advisor; these still follow the 5-, 10-, 15-, 20-page format. Dates are set for oral presentations as well, and a final capstone talk is attended by all students and their advisors. Students receive LaTeX templates for their papers and copies of books/papers by Higham, Gallian, etc. on how to write mathematics well, how to give good talks, and so forth.

6. Expectations, goals, outcomes

To steal a line from the NSF-REU announcement, one of our goals has always been "to lead the participants from a relatively dependent status to as independent a status as their competence warrants." Correspondingly, we expect all students to produce work that is commensurate with their abilities, and even perhaps to recalibrate their understanding of their own abilities. This is even more critical at an across-the-board UGR than at a summer NSF-REU where often many students are quite independent to begin with. We find, in fact, that the same advisor often has two students who understand/undertake/accomplish similar projects at substantially different levels. Secondly, oral and written communication of mathematics is the most visible/fundamental common thread of the class, and our goal is to ensure that common criteria lead to all students satisfying our stated criteria. We are accountable to the University Intensive Course Committee, especially when SACS comes to town for a site visit. Most students get excited about the sense of ownership they feel, and we look upon quantifying this excitement as a valuable outcome. In a similar fashion, we consider poster presentations, talks at conferences, and possible publication as most desirable outcomes.

7. Replicability: pros, cons, specific issues

Ours still appears to be one of the few across-the-board UGR curriculum requirements in the US; substitutions are never made, and students cannot ask for a substitution because the class "isn't offered": MATH 4010 is now available each semester and sometimes in the summers too. How does one institute such a requirement at another school? Certainly, something has to be given up, as we did with the freshman and junior seminars, so that interested schools might start with the question of "what can we eliminate from the curriculum?" Likewise, our ETSU faculty had widespread UGR experience, and this helped enormously. A school interested in starting a UGR program for all its majors might thus either have the depth of faculty or be willing to invest in building such depth. Other questions we asked ourselves, and which are relevant anywhere: How do students juggle other classes with UGR? How do we empower even our weakest students? How do we give credit for teaching this class? For instructors of record? For advisors?

8. Students' accomplishments

8.1. ***Other programs and departmental involvement.*** This paper has been written from the viewpoint of the authors and details their experiences. However, the ETSU Department of Mathematics and Statistics has a rich and deep involvement with

Faculty member	Research area
Anahita Ayasoufi	thermodynamics
Robert Beeler	graph theory, number theory, combinatorics
Robert Gardner	combinatorics, graph theory, analysis
Teresa Haynes	graph theory
Michel Helfgott	mathematics education, mathematics history
Michele Joyner	mathematical modeling
Debra Knisley	graph theory, nucleotide analysis
Jeff Knisley	applied mathematics and analysis
Yali Liu	statistics
Yared Nigussie	graph theory
Rick Norwood	knot theory
George Poole	geometry and mathematics education
Jamie Price	mathematics education
Robert Price	statistics
Edith Seier	statistics
Daryl Stephens	mathematics education

Table 1. Faculty members who have supervised undergraduate research in MATH 4010.

undergraduate research, much of it funded. Table 1 displays faculty members who have supervised undergraduate research in MATH 4010.

In fact, the only tenure-track persons who have *not* directed undergraduate research projects are faculty without the terminal degree, or those who specialize in the teaching of developmental mathematics. Furthermore, Table 1 includes two persons who are not tenure-track/tenured! But this is not all. Over the years we have built an impressive record of funded activity that supports undergraduate research, as seen by the list below. Such cognate activity only enhances the quality of our MATH 4010 class:

- Robert Beeler and Jeff Knisley are on the leadership team for an NSF-Noyce grant in which students do UGR-like projects at various stages.
- Ariel Cintrón-Arias and Anant Godbole were PI and co-PI on an NSF-CBMS grant at which Cintrón-A's students gave poster presentations describing their UGR projects alongside graduate students and postdocs.
- Anant Godbole's NSF-REU site grant has run since 1991 with two years taken off. Six ETSU students have participated at various times, two of whom have published refereed papers.

- Anant Godbole was PI on an NSF-STEP grant in which 45 students did undergraduate research in quantitative biology, at ETSU and at external REUs. Two students participated in his summer REU with STEP support.
- Michele Joyner and Edith Seier are co-PIs on an NSF-UBM grant that features *long-term and sustained* UGR in quantitative biology.
- Debra Knisley ran an NSF-SUMMA REU program for three years, targeted at underrepresented students.
- International agreements such as the ESTU-NCUT Bridge Program with the North China University of Technology were attractive to the Chinese administrators primarily because of the ETSU UGR class. Likewise, a Cameroonian student began doing UGR "long distance" with ETSU faculty, and eventually got an MS degree from ETSU and PhD from Duke; and
- Jeff Knisley was PI on an early round NSF-UBM grant that was based on UGR in quantitative biology (see [Karsai et al. 2011]).

8.2. ***Godbole and discrete math/probability.*** The second author has supervised the research of about 30 students in MATH 4010 over the years, in combinatorics, graph theory, probability, and statistics. Six novel sequences have been submitted to *The Online Encyclopedia of Integer Sequences*, one of which was labeled "nice" by Neil Sloane. Five refereed papers have appeared in *Congressus Numerantium (2), The Mathematical Scientist, Journal of Combinatorial Designs*, and *Lecture Notes of the London Mathematical Society*. Several other students' work is definitely publishable. Most students' work ended at the modest original contributions made in their technical reports.

8.3. ***Cintrón-Arias and mathematical epidemiology.*** From July 25–29, 2011, a regional lecture series was hosted by ETSU. This event promoted some mathematical methods employed to better understand the underlying dynamics of epidemics. The lecture series was led by two principal lecturers who are leaders in the field of compartmental models of infectious diseases: Fred Brauer and Carlos Castillo-Chavez. In what can be considered a celebration of the tenth anniversary of their textbook *Mathematical Models in Population Biology and Epidemiology*, both Brauer and Castillo-Chavez delivered ten lectures, from introductory formulations of epidemic models through survey topics of current interest, including illustrations relating to influenza, HIV, rotavirus, and tuberculosis.

The Conference Board of the Mathematical Sciences funded this lecture series through a research grant by the National Science Foundation, with Godbole and Cintrón-A as principal investigators. A total of 59 participants attended this CBMS lecture series known as *Mathematical Epidemiology with Applications*, of which 14 were undergraduate students.

In an effort to anticipate the specialized technical level of these lectures, Cintrón-A organized a mini-REU prior to this CBMS conference. Four weeks before the lecture series began, Cintrón-A launched a special session of MATH 4010 (with some additional students joining but enrolled in an independent study), focusing on infectious disease modeling. During the first three weeks, students attended lectures and worked in class activities that included numerical simulations. The following topics were studied:

(1) single-outbreak SIR (susceptible-infective-recovered) model;
(2) final epidemic size;
(3) equilibrium points and stability conditions;
(4) SIR model with vital dynamics;
(5) interpretation of basic reproductive number;
(6) the effect of quarantine in the dynamics of an SIR model;
(7) reduction of state variables and nondimensionalization;
(8) ordinary least square estimation of parameters from longitudinal prevalence data;
(9) stochastic epidemic models;
(10) payoff matrices and stable strategies of evolutionary games;
(11) prisoner's dilemma and hawk-dove games;
(12) replicator equations of evolutionary games;
(13) next-generation operator approach.

This special session of MATH 4010 required students to give a poster presentation at the CBMS lecture series with preliminary results of their research project, thus fulfilling oral-intensive requirements. To complete writing-intensive requirements, students continued working on their manuscript through Fall 2011. Table 2 shows the titles and authors of the posters presented by MATH 4010 students.

Four technical reports originated from these six posters; below we include the abstracts of these reports.

(a) Jordan Angel and Sam Peters, *Game theory analysis of vaccination uptake and risk perception*. In populations with voluntary vaccination policies, it is possible that the vaccination coverage achieved is not the coverage level that would be optimal for the population. We use a game theory model in combination with epidemic modeling to analyze what differences exist in vaccination uptake when individuals act in self-interest compared to the coverage that is best for the group. We define player strategies that reflect individuals acting in self-interest as opposed to the group's interest. Our results characterize the difference in vaccination uptake and

Authors	Title
J. Angel and S. Peters	*Game theory analysis of vaccination coverage with epidemic modeling*
C. Brewer and J. Lunsford	*Prevalence of infection in seasonally forced compartmental models*
S. Cameron	*Prisoner's dilemma implementation on Watts–Strogatz networks and real networks*
S. Peters	*Game theory and evolutionary dynamics*
B. Roland and C. Shimberg	*Analysis of influenza-like illness outbreaks at ETSU*
C. Shimberg	*Parameter selection for ordinary least square estimation of contact processes*

Table 2. Titles and authors of the posters presented by MATH 4010 students.

the increased cost to a population due to suboptimal coverage. Additionally, we will investigate a model when individuals vaccinate based on perceived risk of vaccination versus a perceived risk of infection.

(b) Chris Brewer and Jessica Lunsford, *Seasonal infection modeling: a look at the different parameters and their effects upon the prevalence of infection.* Seasonal infection modeling is used to describe the qualitative behavior of infection during various seasonal cycles within a fixed population. We use ordinary differential equations to achieve an understanding of the behavior of each aspect of infection and transmission that is assigned to each equation. We carry out with graphical data based upon calculated sensitivity equations that are compared over time for each of the parameters involved in the model.

(c) Sharon Cameron, *A study of prisoner's dilemma on real social networks*. Prisoner's dilemma is a game theory model used to describe altruistic behavior seen in various populations. Biologically, prisoner's dilemma is important in describing why a seemingly unsuccessful strategy does persist and spread throughout a population, although it seems to not benefit the player. Spatial prisoner's dilemma brings to light certain requirements that must be met in order for the cooperating or altruistic strategy to spread throughout the population with social network structure. Using MATLAB to simulate both the network and the prisoner's dilemma game, results have been obtained that support these requirements. In addition, a snapshot of the California Institute of Technology Facebook social network (as of 2005) is employed as a representation of a real life network.

(d) Byron Roland and Caleb Shimberg, *Analysis of influenza-like illness outbreaks at ETSU*. During the course of seven months, data was collected from the student health clinic located on the East Tennessee State University campus. The clinic reported the number of patients with influenza-like illnesses seen by each nurse in the facility. A model for the spread of this influenza-like illness was proposed using a basic single-outbreak SIR model. The differential equations defining the SIR model were solved numerically using a built-in MATLAB function called `ode45` (based off an explicit Runge–Kutta(4,5) integration method). In this system of equations a fourth was created to report the incidence rate of the influenza-like illness. This incidence rate is what we are going to fit to our data in order to determine transmission rates, reproductive rates, and recovery rates. In this project there are two models, one with constant parameters and another with a time-dependent transmission rate. In the second model, we will use another MATLAB function called `Pchip` (piecewise cubic Hermite interpolating polynomial) to interpolate the values of transmission rate over the time of the data. This interpolation allows us to vary the number of interpolating values of transmission and to explore subintervals for major shifts.

From October 21–22, 2011, six of these students attended the Undergraduate Research Conference at the Interface of Biology and Mathematics, organized by the National Institute for Mathematical and Biological Synthesis. Four of them gave oral presentations, while two students presented posters.

Because of the interest sparked by the CBMS conference and MATH 4010, some students from this group joined a continued discussion on infectious disease modeling during the spring 2012 term. In this term, Cintrón-A and Godbole led a seminar titled *Theory of Networks & Epidemics*, with a diverse attendance including undergraduate and graduate students (from mathematics and sociology) as well as faculty members (from biology and mathematics). In this seminar attendees took turns in leading discussions centered around network analysis and disease transmission. Additionally, this seminar had three external speakers: J. Rivera, *Spreading speed, traveling waves and linear determinacy for STD Models*; E. Shim, *A game dynamic model for vaccine behavior*; J. Medlock, *Optimizing influenza vaccine allocation*.

Due to the momentum generated by the CBMS conference, the undergraduate research experience, and the follow-up seminar, there were two MATH 4010 students from the 2011 class who decided to apply to external REUs. They successfully landed several offers. Mrs. Jessica Lunsford decided to attend the Mathematical and Theoretical Biology Institute (MTBI) of Arizona State University (June 12–August 1, 2012). Mr. Jordan Angel participated in the Interdisciplinary Program in High Performance Computing, hosted by the University of Maryland, Baltimore County (June 12–August 10, 2012).

When interviewed about the role that MATH 4010 played in their external REU participation, these students gave their impressions.

Mrs. Jessica Lunsford: "The summer coursework for MATH 4010 was by far the hardest class I have ever taken. I spent most of the class dazed, confused, and generally lost. However, when it came to the project, I was able to discover some of the material on my own. This made me proud of my work. Something that generally doesn't come from rote computation, as my prior experiences have asked of me.

To me, my participation in MTBI only sealed the deal. I wanted to go so that I could learn as much as humanly possible from a person who has made their career by blazing new trails in this field, Dr. Carlos Castillo-Chavez. What I got was exponentially more than I expected. We worked in groups and learned enough mathematics to make our projects viable. Upon completion of our classroom portion, we were able to work in small, independent groups on a project of our choosing. My group tossed around project ideas for a little while and settled on studying a genetic disorder called Friedreich's ataxia. From that we learned about cell biology and the importance of different proteins involved in iron regulation. From that knowledge, we were able to build and improve upon an existing model such that our research culminated in a publishable, viable, and robust model of cellular iron homeostasis as it pertains to protein levels influenced by Friedreich's ataxia."

Mr. Jordan Angel: "The [REU] program is structured so that the first three weeks are spent completing a 3 credit course on parallel computing. During this time, many potential project mentors visit and pitch projects to student teams. My team and I chose to work with Richard Murphy of Sandia National Labs and David Mountain of NSA (National Security Agency). The project they pitched was to implement a memory access benchmark on UMBC's computing cluster and explain our results based on the machine architecture. We presented our results at UMBC's Summer Undergraduate Research Fest (SURF), an undergraduate research conference that hosts talks and posters for students completing research over the summer at UMBC.

The structure was similar to my MATH 4010 experience. Both started with an intense introduction in a lecture setting, then unique projects were taken on by teams. This is how the REU site advertised itself and was one reason I chose to apply. Both experiences have helped me to understand more clearly what mathematics I am interested in pursuing in graduate school."

Based on his supervising experience during the Summer and Fall 2011, Cintrón-A is currently putting together proposals for external funding, with a slightly different focus of undergraduate research while maintaining some the key tipping points of training and mentoring MATH 4010 students. He is submitting proposals to the Center for Undergraduate Research in Mathematics and to the Tennessee Board of Regents under Access and Diversity Initiative Grants.

9. Closing remarks

The overachievers tend to use MATH 4010 to jump into a Research Experience for Undergraduates outside of ETSU, or to solidify ideas for a project that crossed their mind while they were participating at an external REU. Eventually the combination of MATH 4010 and an external REU fuels and facilitates plans of graduate studies.

For the average students, this tends to be their only research experience during their undergraduate studies. Sometimes these average students struggle in certain areas of mathematics, and they are not very proficient. However, despite some nonlinearities in mathematical background this course serves them well from the standpoint of technical writing and public speaking.

For example, for many of our majors with a mathematics education track (who may very well be overachievers but tend to join the workforce immediately upon graduation and do not pursue graduate studies), writing a 15-page paper on mathematics and giving oral presentations falls within the day-to-day demands of their future job (writing lesson plans and giving lectures, another form of public speaking).

Our department continues to be challenged in accommodating students who enroll in MATH 4010, and we are currently revising the prerequisites (several core courses at the sophomore level). Another challenge for our department is the evaluation metric, that is, how to give grades, especially when there are multiple professors mentoring students with projects in distinct areas of mathematics.

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The MAA undergraduate poster session 1991–2013

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(Communicated by Darren A. Narayan)

We provide a historical report of the undergraduate poster session at the annual joint meetings of the American Mathematical Society and the Mathematical Association of America from its inception in 1991 until 2013. We also provide data on the number of undergraduates attending the joint meetings and the number giving talks.

From a humble beginning in 1991 with 12 posters involving fewer than 20 students, no judges, and no prizes, the Mathematical Association of America (MAA) undergraduate poster session at the Joint Mathematics Meetings (JMM) has grown to 304 posters representing the work of 487 students, 234 judges, and 50 prizes in 2013. The idea for the poster session came from the newly formed MAA subcommittee of the Committee on the Undergraduate Program in Mathematics (CUPM) on research by undergraduates chaired by Lester Senechal. Because of the small number of participants in 1991, there were no follow-up poster sessions in 1992 or 1993. John Greever organized the 1994 and 1995 sessions, followed by Judy Palagallo in 1996, Mario Martelli in 1997, and Palagallo again in 1998. Participation doubled in 1999 when Martelli and Aparna Higgins were organizers and doubled again in 2000 with Martelli as organizer. From 1999 until 2005 Higgins was host of the award ceremony.

Figure 1. Aparna Higgins announcing the winners.

MSC2010: 00A99, 00B25.

Figure 2. Judges entering scores.

In 2000 and the next few years thereafter, the number of students desiring to participate exceeded the capacity of the space available at the convention site. In 2003 floor space was so tight that the poster session was spread over two time slots. Martelli continued as the organizer through 2006, with Diana Thomas joining him that year. Thomas served as organizer from 2007 to 2010, followed by Joyati Debnath in 2011.

Starting in 2008 the poster session got a boost when the MAA received a multiyear NSF grant for nearly $500,000 to support student travel to the joint meetings.

As the number of participants grew over the years, so did the number of prize sponsors. For the poster session in 1994 the three judges pitched in $50 each toward three prizes of $75, $50 and $25. Starting in 1995 the Council on Undergraduate Research (CUR) began contributing $300 for three prizes at $100 each. By around 2000 there were fifteen $100 prizes, with the AMS, the MAA, and CUR all contributing. Also around that time the AMS and MAA began providing refreshments for all attendees. The number of prizes more than doubled in 2004 with additional sponsorship from the Educational Advancement Foundation. The number reached 37 in 2008, and by 2010 the list of prize sponsors had swelled to include the Center for Undergraduate Research in Mathematics, Educational Advancement Foundation,

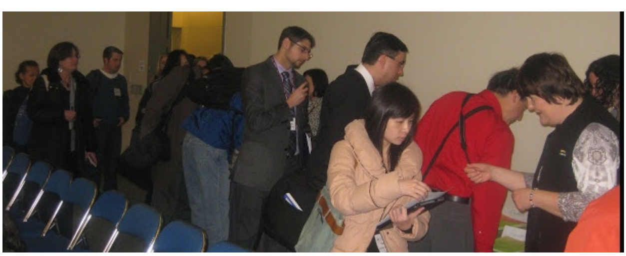

Figure 3. Judges registration.

Brigham Young University Mathematics Department, NSF, SIAM, AMS, MAA, AWM, and Pi Mu Epsilon.

As a chair of the MAA subcommittee on research by undergraduates, Michael Dorff gave out 33 one-hundred-dollar bills for 33 awards in 2010 and 35 for prizes in 2011. The feeling was that winners would enjoy receiving the $100 bills at the awards ceremony rather than receiving a check from the MAA weeks later. Regrettably, it was decided that because of the logistics involved with so many sponsors and the difficulty of finding funding, beginning in 2012 prize-winners would receive certificates instead.

Offering a large number of prizes at poster sessions with hundreds of participants would be impractical without the availability of many dedicated people willing to devote an entire afternoon to the difficult task of judging the posters. The establishment of Project NExT in 1994 provided the perfect source for an enthusiastic pool of judges. Every time there was a shortage of judges the Project NExT listservs were used to fill the gap. The 2004 poster session was the first at which there were judges who were themselves participants in the poster sessions as students. Moreover, two of the three were Project NExT fellows.

It is no coincidence that the rise in popularity of the poster session mirrors the increase in the number of ways in which the joint meetings became more attractive for undergraduates. When the first poster session was held in 1991, the JMM

Figure 4. Undergraduate poster session, JMM 2012.

Figure 5. Angel Pineda training judges, JMM 2013.

registration form did not even have a category for undergraduates. Seventy-one students registered for the inaugural undergraduate student category in 1993. At the same time the MAA subcommittee initiated the first poster session in 1991, it also organized a three-part special session for research talks by undergraduates featuring 22 speakers representing the work of 54 students. The Association for Women in Mathematics gave the first Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman at the 1991 JMM as well. In 1994 there was a four-part special session for research by undergraduates at joint meetings with 38 talks representing the work of 68 students. The first AMS–MAA Morgan Prize for Outstanding Research by an Undergraduate was given at the 1995 meetings. In 1996 the JMM program listing began identifying speakers as undergraduates. There was not a designated session for research talks by undergraduates that year, but six spoke at contributed paper sessions, four of whom were from the same REU.

By 2009 student participants in the poster session were able to meet electronically to organize social events and share tips on preparing posters. That same year the MAA had a Facebook site for the poster session participants. In 2012 a video about the poster session was posted on YouTube. (See Resources for the URL.)

To give a sense of the demographics of the 304 posters at the 2013 JMM we provide the following data: 487 students from 281 institutions, 46 states, Puerto Rico, and foreign countries; 220 male and 267 female; 184 from REUs; 7 high school students; 24 sophomores; 128 juniors; 317 seniors; 20 first-year graduate students (the research was done as undergraduates); 1 post-baccalaureate student; 234 judges; and 50 winning posters.

Students say that the most important benefits from participating in the JMM is networking, mingling with other undergraduates, making friends, going to social events, finding out what other students are doing and which graduate schools others are applying to, and discussing mathematics. Students enjoy being among the thousands of mathematicians from across the United States at the meeting and feel special being there. Students who enter the poster session and give talks find it stimulating.

Figure 6. Judges writing comments.

Of course there are many factors that have contributed to popularity of the poster session as a venue for undergraduates to present their research. By the mid to late 1990s the number of REU programs had grown substantially, and the MAA was offering an annual JMM minicourse for faculty and Project NExT an annual four-hour course on how to involve undergraduates in research.

Even before the first poster session in 1991, activities for undergraduates at the joints meetings were being created. Richard Neal started the JMM Problem Solving Competition in 1987. In 1988 Neal and his wife Araceli established the first JMM

Figure 7. Annie Baer waiting for the students to pick up award certificates, JMM 2013.

Student Hospitality Center, serving as hosts for many years. There, undergraduates hang around, do mathematical puzzles, read copies of *Math Horizons*, attend meet-the-speaker events, and enjoy refreshments at the student reception. Over 300 students visited the center at the 2013 JMM.

The first meeting of the MAA Committee on Student Chapters, chaired by Howard Anton, was held at the 1991 JMM. In 1994 that committee, under the new name of the MAA Committee on Undergraduate Student Activities and Chapters (CUSAC), began sponsoring the poster session, the Hospitality Center, sessions for student talks, social opportunities for undergraduates at the JMM, and established an annual invited 50-minute lecture for undergraduates.

Members of CUSAC and MAA staff are continually improving the poster session experience. Michael O'Leary created software for avoiding conflicts of interest in assigning judges and automating the scoring of the posters. He handled the assigning of judges and scoring for the 2007–2010 sessions. Students now receive feedback about their abstracts and presentations, and registration for students and judges has been streamlined.

Posters are judged on three criteria: mathematical content, answers to questions, and poster design, with two to four judges assigned to each poster. Posters by individuals, teams, or REU participants are not judged differently.

The JMM has become a meeting that offers many exciting options and opportunities for undergraduates and is a meeting at which they feel welcome. In 2013, for the first time, undergraduate students at the JMM outnumbered graduate students 929 to 917. Undergraduates comprised 16.4% of the mathematicians registered. From the prestigious invited 50-minute lectures to the 10-minute contributed papers and posters by undergraduates, vertical integration of the mathematics research community is taking place. The expansion of the mathematics research community to include undergraduates is the culmination of the efforts of hundreds of dedicated people over two decades. We thank them for their efforts.

We conclude with Tables 2–3, which provide data about the participation of undergraduates at the JMM.

Year	Number	Year	Number	Year	Number
1996	6	2002	38	2008	62
1997	23	2003	50	2009	94
1998	22	2004	38	2010	95
1999	23	2005	51	2011	137
2000	35	2006	43	2012	152
2001	15	2007	64	2013	187

Table 1. Number of talks by undergraduates at the JMM.

Year	Number	Site	Organizer/miscellaneous data
1991	12	San Francisco	Greever
1992	None	Baltimore	None
1993	None	San Antonio	None
1994	19	Cincinnati	Greever
1995	17	San Francisco	Greever
1996	32	Orlando	Palagallo
1997	13	San Diego	Martelli
1998	36	Baltimore	Palagallo
1999	68	San Antonio	Higgins and Martelli
2000	140	Washington D.C.	Martelli
2001	148	New Orleans	Martelli
2002	185	San Diego	Martelli
2003	200	Baltimore	Martelli; (2 parts) 15 prizes
2004	110	Phoenix	Martelli; 115 judges, 32 prizes
2005	120	Atlanta	Martelli
2006	130	San Antonio	Martelli and Thomas
2007	175	New Orleans	Thomas
2008	170	San Diego	Thomas; 260 students, 200 judges, 37 prizes
2009	220	Washington D.C.	Thomas; 300 students, 250 judges, 33 prizes
2010	241	San Francisco	Thomas; 367 students, 173 judges
2011	265	New Orleans	Debnath; 300 students, 179 judges
2012	313	Boston	Debnath; 525 students, 189 judges
2013	304	San Diego	Debnath; 487 students, 234 judges, 50 prizes

Table 2. Poster session data.

Year	Number	Year	Number	Year	Number
1993	71	2000	275	2007	476
1994	153	2001	276	2008	527
1995	125	2002	300	2009	650
1996	141	2003	377	2010	683
1997	109	2004	292	2011	759
1998	176	2005	361	2012	945
1999	236	2006	377	2013	929

Table 3. Undergraduate attendance at the JMM.

Resources

- Mario Martelli, Report on the undergraduate student poster session, http://www.maa.org/news/student_poster.html

- Diana Thomas, The undergraduate poster session, http://www.maa.org/pubs/febmar09pg31-32.pdf
- Article about 2004 poster session. http://www.ams.org/meetings/in-cooperation-with/ugradposters-04
- 2008 prize winners http://www.maa.org/students/CUSAC/poster_session.html
- 2009 prize winners http://www.maa.org/students/undergrad/poster09.html
- 2010 prizes winners http://www.maa.org/students/undergrad/poster10.html
- 2010 poster abstracts http://www.maa.org/news/postsess10.pdf
- 2011 poster winners http://www.maa.org/students/undergrad/poster11.html
- 2011 poster abstracts http://www.maa.org/students/undergrad/DRAFT_StudentPosterAbstractBook.pdf
- 2012 prize winners http://www.maa.org/students/undergrad/pastwinners.html
- 2012 poster abstracts http://www.maa.org/students/PosterSessionAbstractsJMM12.pdf
- 2012 YouTube video http://www.youtube.com/watch?v=KjdAYSPTVYU

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Nonacademic careers, internships, and undergraduate research

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The benefits for students who do undergraduate research are mainly thought of in terms of graduate school success and opportunities for future careers as professors. These benefits also help students who go into business, industry, or government. Faculty mentors are often unaware of careers and internships in business, industry, or government. In this paper, some of these opportunities will be presented so that professors can better direct students to them as they are mentoring students. Much of this information has been obtained while organizing the summer internship program at Brigham Young University's Department of Mathematics, the "Careers in Math" speaker series funded by NSF grant DUE-1019594, and our academic-year undergraduate research program, which involves about 75 mathematics majors a year in original research.

1. The benefits of undergraduate research

Reports have shown that there are significant benefits for students who participate in undergraduate research in a science, technology, engineering, and mathematics (STEM) field [Bowen et al. 2009; Hathaway 2002; Hunter et al. 2006; Ishiyama 2001; Russell 2006; Seymour et al. 2004; Sharp et al. 2000; Summers and Hrabowski 2006]. These benefits can be summarized to include gains in knowledge and skills, academic achievement and educational attainment, professional growth and advancement, and personal growth [Osborn and Karukstis 2009]. With respect to mathematics, the MAA CUPM Subcommittee on Research by Undergraduates produced a 2006 report entitled "Mathematics research by undergraduates: costs and benefits to faculty and the institution" [MAASUB 2006]. The report states:

> Students receive tremendous benefit from this activity. Students get to be involved in a significant mathematics project under close supervision

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> by a professor. They gain experience with independent learning, a skill that will prepare them for research in graduate school as well as prepare them to be productive members of a company. They get control over their education in ways that are impossible to duplicate in the classroom environment. Students come out of this experience significantly enriched in their understanding of modern mathematics. Presentation of the results in written and oral formats improves the communication skills of the student.

Research projects for undergraduates can help them prepare for graduate school in mathematics. One way this is accomplished is by preparing students to make the transition from structured coursework to open-ended research. Although coursework provides foundational background in mathematical content, it is a different skill than the one used doing research which requires longer periods of work on a single problem without many inherent clues on how to prove results.

When we discuss undergraduate research, we often limit our thinking to students who will be attending graduate school and who eventually will becoming professors. However, many of the benefits mentioned above can help students who go into business, industry, or government. Often, professors do not know about careers and internships in business, industry, or government. In this paper, we will discuss some of these opportunities so that professors can better direct students to these possibilities as they are mentoring students.

2. "Careers in Math" speaker series

Seven years ago, we discovered something puzzling in the mathematics department at Brigham Young University (BYU). We had students who were excellent in mathematics but did not want to be mathematics majors. We asked them why. Their response was that they did not want to be a teacher or a professor. We told them that there are nonteaching careers using mathematics such as being a cryptographer or an actuary. Yet, later we found that these students still did not end up being mathematics majors. So, we decided we needed a more effective way to get the message to students that there are nonteaching careers for mathematics majors. To do this, we created a "Careers in Math" speaker series which has run for the past five years. For the speaker series we bring in 5–7 speakers who have a strong background in mathematics and who can show how mathematics can be used in business, industry, and government. The idea is to show students that mathematics is used in many careers and that taking mathematics courses is beneficial. Speakers come from various fields such as engineering, programming, operations research, finance, medical fields, actuarial sciences, government agencies, law, and even moviemaking.

When I mention our speaker series to mathematics faculty at other institutions, one question that is often asked is "How do you find speakers?" When I started the series, I knew only about a couple of nonteaching careers in mathematics, and I knew only one mathematician who worked in a nonteaching career — one of my Master's students who worked at the National Security Agency. So, first, I obtained a copy of our department's alumni list. I picked a few alumni who seemed like good candidates, and blindly emailed them. I explained what the speaker series was, that we were doing this to create awareness of career opportunities for mathematics students, and invited them to be one of our speakers. I was amazed by the positive response from alumni whom I did not know but who were eager to talk about mathematics and their careers. From there, things just grew. I mention to colleagues I meet at conferences what I am doing, and occasionally they will tell me of a contact who would be a good candidate as a speaker. Now I have more recommendations for speakers than I have spots.

Another question that I am asked is "How do you fund this?" Funding has been an unstructured aspect based upon the successful model of starting small and promoting successes to help you grow. When I first proposed the idea, the department chair liked it and was able to find some departmental funds to help us start the speaker series. The first year our expenses were small since I used local speakers and alumni who were visiting campus. However, we did arrange one main speaker whom we knew would give an excellent presentation. I made sure that a lot of students would be attending (offers of extra credit in classes and free donuts work like magic). Also, I invited the dean. The presentation was fabulous and the dean was so impressed that he offered some additional funds to support the speaker series for the next year. During the second year, we received an internal grant from the university to support internships among majors and adapted our speaker series by bringing in speakers from organizations that had summer internships for students. Also, we learned that some larger organizations have internal recruiters who visit campuses to give a presentation and will pay their own expenses. The next year, one of our previous speakers was involved in an NSF TUES grant proposal to promote applications of mathematics in the real world. This led to our speaker series becoming part of a collaborative NSF TUES grant funding us for $30,000 for several years (NSF grant DUE-1019594).

The speaker series has been a tremendous success. We are doing many activities to encourage more students to take mathematics courses and become mathematics majors, including the WeUseMath.org web site, the BYU IMPACT (Interdisciplinary Mentoring Program in Analysis, Computation, and Theory) lab that funds students to work on problems provided by business and industrial partners, the promotion of paid summer internships for our mathematics majors, and a new applied and computational mathematics emphasis connected to industry for our mathematics majors. Since we have started emphasizing career options in mathematics, we have had an 89% increase in the number of mathematics majors at BYU and this has resulted in the mathematics department receiving several new faculty positions.

3. What the employers have said about hiring mathematics majors

I have asked speakers and employers why they want to hire mathematics students. What do you think their response is? This is an important question. Think about it for a moment before you look at the answer. Mathematics majors should also think about the answer to this question. In fact, when I talk to students about this topic, I often ask them to tell me their answer. Why is it important for students to think about this? Because employers will not hire students just because they are mathematics majors. Instead, students need to convince their potential employer to hire them. Having thought about the answers to the question "Why hire a mathematics major?" provides students with some excellent talking points in an interview with an employer. So, what are the reasons the speakers and employers have given? They have said that they want to hire mathematics majors, because of the students' problem solving skills, attention to detail, ability to abstract, methodical approach, and the different perspective they bring to problem-solving.

Paying attention to detail is a characteristic that I did not think of at first. A BYU mathematics major was looking for a job and had interviews with five companies. In two of those interviews, she was asked a question similar to the following: "Suppose you have a clock with an hour hand and a minute hand, and the time was 1:25. What is the angle between the hands?" At first, someone would have to know that there is 360° all the way around the clock and so each five-minute interval represents 30°. Then some people would say that since there are 4 five-minute intervals between the 1 and the 5 on the clock, the answer would be $4 \times 30°$ or 120°. But that is not correct. Why? Because as the minute hand moves, the hour hand also moves. So, at 1:25 the hour hand is not pointing at 1 but is somewhere between 1 and 2. This is the type of detail that many mathematics majors are good at noticing.

Likewise, offering a different perspective on how to solve a problem is not an attribute that many mathematics students mention. Industrial firms and businesses create groups to work on problems. These groups often consist of people with different backgrounds such as programmers, scientists, engineers, statisticians, and mathematicians. These are effective, because each person brings a different perspective to solving a problem. It makes sense. The way an engineer approaches a problem is different from the way a mathematician does. Using these different perspectives provides a better result. As an aside, a solution to a problem in industry can be very different than a solution to an academic problem. In industry, the group is usually given a problem with a timeline and is told to come up with the best possible solution within that time frame. They are not necessarily looking for an exact answer but the best possible approximation given the constraints. Once the time period is over, the group will move on to another problem. They do not have the academic luxury of exploring nooks and crannies of a problem unless that is part of the group's task.

Just having these skills mentioned above is not enough to get a job. Students have asked "What should we do to better prepare ourselves for these careers?" Speakers and employers have recommended that students should

- know how to program,
- develop good communication skills (i.e., speaking and writing),
- have some background in some other STEM field such as statistics, computer science, or biology, and
- have experience working intensively on a hard problem whose solution is unknown (i.e., do an undergraduate research project or a summer internship).

Knowing how to program is an essential skill for a mathematics student who is interested in a career in industry. If a student does not know how to program, it will be extremely difficult for them to get a mathematics-oriented job. Most of the people in industry whom I talked with say that it is not too important which programming language students know. Instead, they say that having the experience of knowing how to program is what is most significant. If students have that experience, then they can more easily learn the programming language that the company needs. Students do not need to take a lot of programming courses, but they must be able to demonstrate their abilities to program. One way this can be done is during the job interview. One of BYU's graduating mathematics majors had been asked in at least two job interviews to describe how she would write a program to do a specific task such as taking a large set of random numbers and listing them from least to greatest, or determining which three-digit integers in a random set are prime.

Good skills in speaking and writing are also important. Often careers in business and industry require employees to work in groups that try to analyze a situation and find a solution. Good communication skills are helpful for success. Also, there may be times when an employee will need to explain a project to an executive who has little or no background in mathematics or convince an executive that the employee's group has an important project to which the company should allocate resources. The employee may be initially asked to give a 30-minute presentation but then, due to unforeseen circumstances, may have to shorten the presentation to 10 or even 5 minutes. A great skill is to be able to present the technical ideas of a project in a way that a nontechnical person can understand, while being flexible enough to give the highlights in a short 2-minute elevator speech or expand it with significant details for a longer 30-minute presentation. For more thoughts on this, see the book *The Persuasive Wizard* [Givens 2011]. This fits well into an undergraduate research experience since students can learn these skills as they explain their research to others through seminar and conference presentations and through posters and research reports/papers. I especially think of the MAA's student poster session at the Joint Meetings as a useful training stage for such a skill.

Studying advanced undergraduate mathematics and solving a homework problem from the textbook is often difficult for students. But it takes a different kind of difficulty to work on an open-ended research problem in an undergraduate research setting or in an internship. Such situations require longer periods of work on a single problem without knowing if a solution is possible and if it is, not having many inherent clues on how to obtain that solution. Yet, this is very similar to the way problems are approached and tackled in industry. Skills obtained and experience learned in working intensively on a hard problem whose solution is unknown are great preparations for careers in industry.

4. Careers for mathematics majors

Which careers employ students who study mathematics and have the additional skills mentioned above? While there are numerous careers, let me share some examples from engineering, programming, operations research, data mining and analytics, finance, medical fields, government laboratories and agencies, and computer graphics. Additional careers can be found on web sites such as WeUseMath.org.

Engineering. When I first starting talking with employers, I was intrigued that engineering firms such as Raytheon, Boeing, and General Dynamics would employ mathematicians. I had naively thought that they would want to hire only engineers. But their recruiters pointed out that they form groups consisting of people with different backgrounds — engineers, programmers, statisticians, and mathematicians. This allows for different perspectives to be used in solving a problem. We have several alumni who work for Raytheon. These engineering firms often work on projects related to aerospace and defense. For example, Raytheon is working on a real "iron man suit"; check out some video clips on YouTube by searching for the terms *Raytheon* and *exoskeleton*. Recently, I talked with recruiters from Raytheon and from Boeing. They wanted to hire students who have a graduate degree in mathematics, but they were not getting many applicants.

Programming. As mentioned above, programming is an essential skill for any mathematics student who wants a mathematical career in industry or business. If a student has this skill, it is not difficult to find a job as a programmer. There are national companies such as: Epic, which creates software for medical groups and hospitals; FAST Enterprises, which provides software and technology consulting services for government agencies; and SirsiDynix, which develop technologies for university and community libraries. These are three examples of companies that are currently recruiting mathematics majors as programmers. There are many other companies from small start-ups to large international companies that are constantly in need of programmers.

Operations research. It applies analytical and mathematical methods to help make better decisions. It deals with such questions as "What is the optimal way to schedule a set of tasks?" and "What is the most efficient way to arrange the flow of traffic?" Examples of scheduling problems include the scheduling of a sport team's games, the restocking of large businesses such as Walmart, and the scheduling of surgery in hospitals (i.e., arranging physicians, patients, nurses, and operating rooms). Transportation problems include stop light synchronization, the evacuation plan for a building or a stadium in case of a terrorism threat, and efficient delivery routes for UPS or airlines. Eric Murphy does operations research for the government. I met him at an AMS regional conference. He has a PhD in complex analysis and works in the Pentagon advising the Joint Chiefs of Staff on how to best move supplies and troops in and out of foreign countries such as Afghanistan. Sommer Gentry does operations research as a research associate for the John Hopkins University School of Medicine and as a mathematics professor at the US Naval Academy. A few years ago she made national news when she teamed up with her husband, a surgeon, to use operations research to find a more efficient way to match kidney donations with recipients.

Data mining and analytics. Data mining deals with analyzing extremely large amounts of data. Think of all the data being created on the internet. In 2011, it was estimated that the Library of Congress had collected about 235 terabytes of data. Google estimates that the internet holds about 5,000,000 terabytes of data — 20,000 times as much as the Library of Congress. And it is growing. Data mining is a very hot area. Google uses data mining to improve its internet searching techniques, Netflix uses it to provide recommended new movies based upon a user's ratings of previously watched movies (read about the "Netflix Prize"), professional sports teams use it to improve their chances of winning (think of the movie *Moneyball*). For more examples, check out Richard De Veaux's presentation on data mining at the MAA's Distinguished Lecture Series at the Carriage House on April 20, 2012.

Finance. Last year I attended a STEM career fair. I talked to representatives from 15 companies who were specifically advertising to hire mathematics majors. Three of the companies were in finance — Goldman Sachs (global investment banking), RBS (global banking and markets), and Capital One (banking and financial analyst).

The RBS representative mentioned that one reason they hired mathematics majors was for their attention to detail. Also, all three of these banking firms have summer internships for students. In spring 2013, Kurt Overley will be speaking in our "Careers in Math" speaker series. Kurt has a PhD in applied mathematics and has worked in investment banking for the past twenty years as a pioneer in the hedge fund derivative industry.

Medical fields. As medical fields become attuned to data, mathematics can provide a solid foundation to begin the study of medicine. Helen Moore who has a PhD in geometric analysis is an example. Originally, Helen's interest did not lie in the medical field, but after attending some conferences and workshops related to mathematics and medicine, she became interested. Now, she works for Pharsight, a pharmaceutical company, as a senior scientist. When a pharmaceutical company develops a new drug, they must determine safe and effective dosages to prescribe to patients. Helen uses control theory and mathematical modeling to do this. Michael Cannon is a BYU alumnus with a BS in mathematics and a PhD in epidemiology. He works for the Center for Disease Control and Prevention (CDC) doing research on the prevention of birth defects.

Government laboratories and agencies. The National Security Agency (NSA) is the largest employer of mathematicians in the United States. They use mathematicians to work on cryptographic problems and complex algorithms, and I have had three alumni who work at NSA speak at BYU. There are also US government laboratories that hire students with a strong mathematics background. For example, Robert Berry was a Master's student of mine who now works at Sandia National Labs on energy problems, and Carol Meyers, who has a BA in mathematics and a PhD in operations research, works at Lawrence Livermore National Labs on problems related to nuclear disarmament and emergency disaster preparedness.

Computer graphics. I have to admit that when I first heard that mathematics was being used in movies, I thought it was a gimmick to devise some application of mathematics in today's society. Now, I see how wrong I was. Doug Roble at Digital Domain Productions, Inc. has mentioned that the top 17 money making movies from *Avatar* to *Toy Story* used mathematics in their creation. Tony DeRose, a research scientist at Pixar Animation Studios, gives talks on how mathematics has changed Hollywood. Ramus Tamstorf at Walt Disney Animation Studios, Adam Sidwell, an independent Creature Technical Director, and Doug Roble have all voiced this same message. They deal with issues of making the movement of animated characters, the light shading on characters, the flow of water, and the crashing of objects seem realistic. For example, early on, animated characters were triangulated with a grid. With the vertices of the grid points, a character could be moved using matrices. However, this approach requires a lot of computation — which translates into money and time. Tony DeRose's group devised a technique to enclose an animated character in a cage that requires about 75 times fewer grid points. They then developed mathematical algorithms so that when they moved the cage, the character also moved in a realistic way.

5. Internships for mathematics majors

Internships are a great way to help students who want to go into a career in business or industry. A good internship for a student is like doing undergraduate research with problems from business and industry instead of problems from academia. Mathematics majors from our department who have participated in summer internships at a given company have typically found salaried employment with those same firms after graduation. But just like undergraduate research, students need guidance in finding internship possibilities. Basically, we have found that there are two ways to find internships. First, look at opportunities with companies and organizations on the national level. These include internships with companies and firms at the national level that deal in finance, technology, science, aerospace and defense, and actuarial science. In addition, there are internships at government facilities such as the National Security Agency, national laboratories, and NASA. Here is a short list of some internship opportunities at the national level for mathematics students along with a current link to details about the internship.

- Government
  - National Security Agency: nsa.gov/careers/opportunities_4_u/students/index.shtml
  - Lawrence Livermore National Laboratory: internships.llnl.gov/
  - Los Alamos National Laboratory: lanl.gov/education/undergrad/internships.shtml
  - Argonne National Laboratory: dep.anl.gov/p_undergrad/spring.htm
  - National Aeronautics and Space Administration (NASA): usrp.usra.edu/
  - Naval Surface Warfare Center, Dahlgren Division NSWCDD: navsea.navy.mil/nswc/dahlgren/RECRUIT/studop.aspx
- Finance
  - Goldman Sachs: http://www.goldmansachs.com/careers/students-and-graduates/index.html
  - Bank of America: http://campus.bankofamerica.com/
  - Citigroup: internships.about.com/od/banks/p/citiinternships.htm
- Computers and Technology
  - IBM: www-03.ibm.com/employment/us/un_interns_coops.shtml

  - Microsoft: research.microsoft.com/en-us/jobs/intern/default.aspx
  - PARC, a Xerox Company: parc.com/about/careers/internship_program.html
- Science
  - SAIC: saic.com/career/students/interns.html
  - Metron: metsci.com/Default.aspx?tabid=254
  - Phillips: philips.com/about/careers/student/internships/index.page
  - Mitre: mitre.org/employment/student_opps.html
- Aerospace and Defense
  - Raytheon: jobs.raytheon.com/career-paths/campus-recruiting/internships-coops
  - Boeing: boeing.com/careers/collegecareers/
  - Aerospace Corporation: aerospace.org/careers/internships/
- Actuary
  - Prudential: http://goo.gl/iPXytD

Some web sites with more internship information are:

- siam.org/careers/internships.php
- ams.org/employment/internships.html

Second, there are regional and local internships. Also, developing connections with alumni who work at businesses or industries that offer internships are a valuable resource. We have found that these alumni can guide a student along the process and become professional mentors for them.

6. Conclusion

The world is becoming more mathematically oriented, and there are a lot of exciting careers for people who understand mathematics. Let your undergraduate students know about these careers. Remember that undergraduate research projects are not only beneficial for students who go into academia, they can also be a great preparation for mathematics students who plan to go into business, industry, or government.

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REU design: broadening participation and promoting success

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This article summarizes the authors' presentations on the panel "Working with Students from Underrepresented Groups" as part of the MAA's Trends in Undergraduate Research in Mathematics Conference held in Chicago, in October 2012. We highlight effective aspects of our own successful programs that emphasize working with students from underrepresented groups. We discuss specific issues of program design that one might beneficially consider when planning to work with students from underrepresented groups and provide examples of ways in which the authors have addressed these concerns.

1. Research experiences for undergraduates and the STEM crisis

Much has been written and presented about the "STEM crisis" in the United States — the increasingly large gap between the number of highly educated science, technology, engineering and mathematics professionals needed to sustain our workforce and the number current educational practices will produce. For instance, mathematical workforce and relevant demographic trends are discussed in [Cortez et al. 2007]; a brief update is provided in [Dye and Russell 2014]. The concern is not new: for decades, calls to address this slow-moving crisis have made clear that developing talent within all segments of our population constitutes our best hope of competing economically.[1]

Two other arguments for diversifying our STEM student and professional populations are relevant. One is the fairness aspect: each individual should have the opportunity to realize his/her potential, with no limits imposed by race or ethnicity, gender, disability status, socioeconomic status, or family educational background.

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[1]For example, Widnall [1988] cites studies from the 1970s and 1980s in analyzing gender differences in experiences and perceptions of graduate school and PhD attainment in STEM fields; Jackson [2002] refers to the decades-long growth of the gap between STEM professional production and needs.

Another broadens the economic argument beyond the mere quantity of STEM professionals to the value of bringing diverse perspectives to bear on complex problems.

> *There are large and complex problems to solve. Many of these vital, exciting, and challenging problems are characterized by increasing complexity, ambiguity, uncertainty, and rapidly changing conditions. Solutions to these problems require the best minds and facilities to work together* [CEOSE 2004, p. 1].

Similarly, the National Science Foundation's Mathematical and Physical Sciences Broadening Participation Working Group refers to research indicating that diversity is a "powerful contributor to the attainment of effective solutions." They write "we see improving our nation's economic outlook by providing research teams with the distinct competitive advantage of a diverse workforce as an equally compelling reason to demand improvements in broadening participation" [DeSimone et al. 2010].

Research Experiences for Undergraduates (REUs) can play a significant role in enhancing diversity in the United States' STEM workforce by seeking out talented students from groups traditionally underrepresented in STEM fields and providing the benefits known to accrue from REU participation. Yet a recurring conversation among REU-engaged faculty centers on the dearth of competitive applications by students from underrepresented groups to many REUs. Advice regarding recruiting such students has remained fairly consistent in recent years [Dye and Russell 2014; Cortez et al. 2007; Vélez 2011] and has been effectively practiced by multiple REUs. However, reaching a target student with information about an REU is not sufficient if a student decides that REU is not for her, or even more damaging, that REUs in general are not for him. The concomitant challenge lies in designing an REU that both attracts and supports students from underrepresented groups. To be clear, by "students from underrepresented groups" (S-URGs), we will mean not only underrepresented minorities[2] and women, but also first-generation college students, community college students and transfers, and low-income students.

What are the considerations relevant to designing such an REU? One guide for a wider range of programs offers eight design principles gleaned from examining programs that successfully broadened participation by S-URGs [BEST 2004]:

- Institutional leadership
- Targeted recruitment
- Engaged faculty

[2]As defined by the federal government: African-Americans, Hispanics, Native Americans, and Pacific Islanders.

- Personal attention
- Peer support
- Enriched research experience
- Bridging to the next level
- Continuous evaluation

We propose a series of questions loosely corresponding to these principles that REU planners may consider when designing an REU to attract and support S-URGs. We then highlight aspects of two programs explicitly designed to broaden participation while intentionally constructing heterogeneous research groups. We offer these not as one-size-fits-all solutions but as our attempts, within the contexts present, to provide pathways to success in the mathematical sciences for S-URGs.

2. REU design: questions to consider

We offer here many questions that REU planners might productively consider in designing an REU to attract and support S-URGs. These lists are not exhaustive: other questions common to all REUs, for example, about finding good research topics, may be found elsewhere. While our first set of questions concerns goals and philosophy, some later questions (e.g., about forming heterogeneous groups) reflect the goals and philosophies of the authors.

- Why are the *faculty planners* hosting or considering hosting an REU?
  - What do they wish to accomplish through this REU?
  - What is the philosophy behind these goals?
- What is the *institutional environment* for the REU?
  - Do the relevant administrators value the goals of the faculty planners?
  - What type of support (staff, financial, logistical) is available?
  - Are physical spaces — for working, for living — conducive to the goals?
  - Will faculty engagement be valued in tenure and promotion decisions?
- In addition to the faculty planners, what *faculty* will be engaged in the work of the REU? (From this point on, we use "faculty" to include REU planners/ directors as well as research mentors.)
  - Do all faculty share the goals and philosophy of the planners?
  - Do the faculty value working with undergraduates highly?
  - Do the faculty view broadening participation in the mathematical sciences as an important undertaking?
  - Are the faculty engaged in working with S-URGs?
  - Are the faculty engaged in or willing to engage themselves in learning about

underrepresentation in the mathematical sciences: its history, ongoing barriers to participation, individual and cultural challenges, etc.?

- How will *students* be recruited and selected for the REU?
  - How will recruiting materials and processes attract the types of students desired?
  - How will the application process identify mathematical potential? Sufficient mathematical background for the research projects? Ability to work well with others? Motivation and work ethic? Openness to the possibility of pursuing a graduate degree in the mathematical sciences?
  - How will the faculty determine which students are likely to benefit to the greatest extent possible from their participation in the REU?
  - How will the selection and notification process ensure heterogeneous groups?
- What will the *activities* of the REU be?
  - What tone should be set during pre-REU communications? How will this be accomplished?
  - What activities will ensure that all students feel equally welcome and supported by faculty and peers?
  - How will the faculty provide each student sufficient personal attention for that student's mathematical and personal growth?
  - How will the mathematical activities be structured so as to ensure that all students have an enriched research experience?
  - What professional skills will students need to develop? How will this be accomplished?
  - Will REU activities help students manage the frustration inherent in mathematical research? Will they help students build confidence in their abilities?
- What sort of *follow-up* will there be after the on-site REU?
  - How will the REU be evaluated? How will lessons learned from the evaluation be incorporated to improve the REU?
  - How will student be supported through the process of disseminating their results?
  - How will students be supported as they consider their next REU, their post-graduation plans, etc.?

3. REU design: our approaches

The CI and PURE REUs*: *an overview. The CSU Channel Islands REU began in 2010. It has supported 14-15 students in three research groups each summer.[3]

[3] 12 students are supported by the National Science Foundation through grant DMS-1005140; private donor funding supports additional students

While welcoming all student applicants, it particularly seeks S-URGs, with a special emphasis on students who are native Spanish speakers or first-generation college students. At least one faculty mentor each summer is Spanish-English bilingual; at least one student is from a partner university in Mexico: these two features and several other aspects address the goal of creating an especially welcoming program for native Spanish speakers while enhancing the international perspectives and cross-cultural competency of the entire REU group. Students spend the bulk of their time working on research; skills workshops, distinguished visitors and colloquia, meals, and social outings are also included in the activities.

The Pacific Undergraduate Research Experience in Mathematics (PURE Math) is a combined summer program for undergraduates in mathematics.[4] It began in 2011 and was developed to bring the summer research experience to the people of the U.S. Pacific Islands. It is a collaborative project between the University of Hawai'i at Hilo and Sam Houston State University and is housed on the campus of the University of Hawai'i at Hilo. Through the experience of mathematics research (an 8-week Residents Program) and advanced mathematical training (a concurrent 5-week Interns Program), participants receive valuable mentoring towards

(1) the preparation for upper-level theoretical coursework in mathematics,
(2) the development of the necessary framework for continued academic success in science, technology, engineering and mathematics (STEM) disciplines,
(3) the consideration for further training in STEM graduate schools.

PURE Math and the CI REU share a primary goal for our student participants: for the students to conduct original mathematical research leading to conference presentations and publications. In so doing we wish to raise their levels of mathematical maturity and confidence while fostering an enthusiasm for mathematics. We intend to create and maintain research communities of mathematicians. We seek to improve the participants' abilities to communicate mathematics visually, orally, and in written form. We incorporate S-URGs and non-S-URGs into a heterogeneous group with shared intellectual goals. We want students to leave the program feeling excited and prepared to perform graduate-level mathematics in an academic or industrial setting.

Participants.

Faculty. All faculty engaged with our REUs consider working with undergraduates to be an essential component of their professional lives, are research active with components of their research conducive to undergraduate research, and are

[4]Funding is provided by the National Science Foundation under grants DMS-1045147 and DMS1045082 and through National Security Agency under grant H98230-12-1-0252

committed to broadening participation in the mathematical sciences. We mix experienced research mentors with early career mathematicians and include copious pre-REU planning and conversations with ongoing mid-REU meetings to engage in mutual professional development around the topics of undergraduate research and working with underserved populations. Mentor expectations are made explicit before mentors commit to the REUs: expectations include far more in-person work with individual students and research groups than seems to be the norm at most REUs.

Students. Students are selected for the CI REU by determining who ranks high on the first two criteria below, then considering the extent to which those students meet the remaining criteria:

- demonstrates mathematical talent and/or potential
- demonstrates mastery of sufficient mathematical background for selected projects
- attends an institution with limited opportunity for undergraduate research (typically a non-PhD-granting institution)
- is a member of a group underrepresented in mathematics
- is a first-generation college student
- either has considered a mathematical or scientific career, or should be encouraged to do so

(Not all criteria have to be satisfied.) Research mentors and project directors work together in an effort to create a heterogeneous community as well as heterogeneous research groups. As our REU emphasizes collaboration and a supportive-to-all atmosphere, faculty also seek indications that students are eager to contribute to this atmosphere in their essay responses and letters of recommendation.

A similar process is involved in selecting students for PURE Math. Particular attention is paid to the level of mathematical maturity so as to place students in the appropriate program. Building and maintaining an idyllic and genuinely collaborative community of young researchers that have access to appropriate levels of support is critical to the success of each summer. It is crucial that participants share a common experience in their logistical and academic environment, that there is a clear and common set of reasonably high expectations, and that there is diversity and balance among the group in terms of ability, experience and background. For these reasons, the selection process often involves directly contacting the candidate and the faculty who have written letters of recommendation to discuss the candidates' applications in greater depth.

Atmosphere and activities. As Ricardo Cortez observes [2007], "It is important to create the 'right' atmosphere right away".

Setting the tone: pre-REU. We send messages in the pre-REU stage through the following mechanisms.

- Webpages and recruiting materials are crafted to give students a sense of the programs and the atmospheres they seek to create.
  - All CI materials are presented in both English and Spanish; at least one faculty mentor is Spanish-English bilingual; a webpage entitled "What's research?" (see faculty.csuci.edu/cynthia.wyels/REU/) mentions the frustration (and fun) inherent in mathematical research and emphasizes the community and collaborative aspects of research in our REU; expectations of and benefits for students are spelled out. Goals listed on the webpage include "get participants excited about doing mathematical research", "create a learning community", and "help participants develop the confidence to succeed in ongoing mathematical studies".
  - The PURE Math webpage provides information regarding the upcoming summer (see www2.hawaii.edu/ pure/PURE_Math/Welcome.html), and the previous years' projects and reports are available for both programs as well. Photos and video montages are also available on the website. Students are directed to the application form available through mathprograms.org.
- Predecision communications with students are critical to helping program directors, research mentors, and students make good choices. Both PURE Math and the CI REU provide students with explicit expectations. These include working hard on every task, participating fully in all program activities, listening to peers and faculty, sharing ideas, providing helpful suggestions, asking questions, giving constructive criticism when possible, and having fun "stretching their brains in the company of like-minded people". Students are reminded that their efforts will be the determining factor in their outcomes. Similarly, PURE Math and the CI REU ask students to sign a contract to accept their offer of admittance: a sample pledge is "to approach the mathematical learning and all people involved in the program in a spirit of openness and collaboration".
- Prearrival communication continues to set the tone for students who will be participating. Both community building and mathematical formation takes place here: mentors provide articles along with tailored reading guides, and students are asked to use social media to introduce themselves to the group. Even logistical information can reinforce expectations about open-mindedness regarding other cultures: for example, PURE Math includes information on Native Hawai'ian culture to help students interact respectfully during their stay.

Establishing the desired environment: the first week. During the first week of each REU, several activities are carefully chosen to promote social cohesion, identity exploration, and team building. All such activities include debriefing and often further conversations initiated by students. Team building activities are used to help students develop effective collaboration skills and to help them understand the benefits of peer support. Discussions tie these activities to various ways in which mathematical research is undertaken, including the necessity of stepping away from others for private work at times. During identity exploration activities, students are led to self-assess who they are and who they are becoming as people, learners, and mathematicians. For example, one identity exploration (which doubles as an exercise in writing with precision) consists of providing students a list of 15 to 20 "common values" and asking them to write about the three they value most. Experience shows that this exercise also turns into group bonding: in spite of their diverse backgrounds, they discover they share similar values, with family repeatedly being highly valued among students. The goals are for them to better understand themselves and use that insight both to learn and do mathematics more effectively, and to better understand and work with others. After exploring their own cultural/familial backgrounds and individual identities, they're asked to identify strengths they can build on as mathematics students and researchers; at the same time, they consider potential hurdles and strategies to overcome them. Through these types of activities, we make clear that we expect diversity to enhance the work of the REUs. Heterogeneity (overall, and within each research group) offers each person the opportunity to enhance his/her "intercultural competence, cognitive complexity, ability to work in diverse groups, and capacity to take seriously the perspectives of others".[5]

Building competence and confidence: weeks 2–8. Effective student research requires a structured but flexible setting that allows faculty mentors to meet students at their current levels and draw them up to a place where they can engage productively with challenging problems. Initially, faculty mentors are heavily engaged with the students' mathematical learning, helping them understand the readings sent pre-REU and the possible research topics. Faculty may direct students to create and work out examples, they may present material, and they provide students extensive opportunities to ask questions. The goal of the mathematical activities during the first week is for each research group to understand their REU topic and several relevant problems, and to be able to present these coherently to the whole REU. As our REUs progress, faculty intentionally help students make the transition from classroom (directed) mathematics to independent research. They talk about the differences and what research "is", they model the thinking necessary, they give the students larger open-ended tasks and questions, and lead the students to develop their own avenues of investigation. Faculty continually nudge students forward on

[5]From the "Inclusion" core principle of AAC&U's "Making Excellence Inclusive" initiative, as cited in [O'Neill 2009].

this path, while being available to support as appropriate. Throughout, REU faculty take joint responsibility for monitoring students' progress, while staying attuned to group and individual dynamics.

REU activities common to many REUs include invited speakers, group meals, ethics components, skills development, student presentations, and social outings. We advocate these activities, while considering how they can best further the goals of our REUs. In seeking speakers, we invite accomplished mathematicians from academia and industry who are also dynamic speakers; we seek the same sort of diversity of background in our speakers as we do in our students. We ask speakers to share their pathways — especially hurdles and hard lessons learned, in addition to their mathematics. The more students are aware of different pathways taken — leaving school, pursuing different degrees and careers, and hurdles (dropping out of school, failing quals, being told "you're not good enough", dealing with a sense of not belonging) — the more they recognize that there are multiple paths to success. Exposing students to successful researchers and scientists who have taken very different and often difficult paths in the younger stages of their lives allows students to see their own potential to succeed. They are able to relate to someone who has achieved tremendous success, thereby permitting themselves to take on the same challenges and face the same fears, but with a higher degree of confidence due to an "existence proof" that it can be done. Social outings also build confidence: they include trips in which some of the S-URGs become cultural experts for the rest of the group; the same is true of the menus at some group meals. We take advantage of the locations of our REUs whenever possible to support subgroups of our S-URGs who may often feel out-of-place in majority-culture and language institutions.

As our REUs intentionally admit students at critical transition points in their mathematical education, we carefully plan a series of workshops[6] for students to develop professional skills. These are hands-on workshops; follow-up is done within research groups and between individual students and any faculty. Peer support is modeled and encouraged at all times, especially during students' weekly presentations of their work-to-date; these practice presentations also help students develop their presentation skills and confidence speaking before groups. The faculty also model the goal of ongoing learning through our informal interactions, planned meetings, and reliance on one another to support all our students.

[6] Topics for workshops include: reading mathematical literature; proof techniques (illustrations chosen from research topics or background for research areas); using MathSciNet; abstracts triage; mathematical software; LaTeX; preparing visuals for presentations; giving oral presentations; preparing posters; presenting posters; upcoming conference options (includes deadlines, funding, etc.); graduate school: what is it, how to prepare, apply, choose; careers in mathematics: how to learn more and seek positions; writing abstracts; writing mathematics; math chats (discussing research informally, e.g., at conferences and in interviews).

Onward and upward: post-REU. Our REUs structure our time together to ensure that students leave the summer program ready to give a 10–15 minute conference presentation and a poster presentation of their work. During the REU, faculty help students identify conferences appropriate to their goals at which to present,[7] and provide guidance in applying for travel funding. Students also write up their results, going through several drafts with faculty and peer feedback. When appropriate, faculty mentors oversee submission to journals. Faculty continue mentoring students post-REU both in person at conferences and through e-mail and social media. In addition to writing letters of recommendation for students' next research experience and/or graduate admissions, faculty contact individual students with opportunities, respond to questions, and celebrate students' successes.

Student outcomes. As our programs are relatively young, participants are still undergraduates or just beginning postgraduate lives. Yet the outcomes summarized below are promising.

PURE Math. 2011: 5 of the 12 participants in the Interns Program have graduated. Of these 5, 3 are going on to graduate programs (in math education, mathematics and physics) all with full funding. Of the 7 who are still undergraduates, 3 will graduate in 2013 and are applying to graduate programs. All 3 have spent their subsequent summers at other REU programs.

2012: There were 12 participants in the Residents Program and 12 in the Interns Program; all are still undergraduates at this time. 6 will be graduating in 2013; all 6 are applying to graduate programs.

CI REU. 2010: 12 of the 14 participants have graduated. Of these 12, 9 applied to graduate programs; all were accepted with full funding.

2011: 10 of the 15 participants have graduated. Of these 12, 7 applied to graduate programs; all were accepted with full funding. 2 of the 4 graduating in 2013 participated in other REUs during Summer 2012; these 2 are applying to graduate school for Fall 2013 admission.

2012: All of the 15 participants are undergraduates at this time. Of the 9 graduating in 2013, at least 7 are applying to graduate programs.

4. Final considerations

Designing a new REU, garnering the necessary institutional support, and acquiring external funding can be daunting. Multiple-research-group, externally funded

[7]SACNAS, the National Alliance's Field of Dreams Conference, and the Nebraska Conference for Undergraduate Women in Mathematics are all excellent venues for S-URGs and others; the Joint Mathematics Meetings also provides many opportunities for REU students. Regional MAA meetings provide student-friendly and more affordable options.

summer REUs are far from the only avenue for mentoring undergraduate research. The Center for Undergraduate Research in Mathematics (curm.byu.edu) not only offers funding for academic-year research groups but also provides mentoring and advice to help faculty grow in this undertaking. The National REU Program (see maa.org/nreup) is geared toward one-faculty, one-research-group REUs with minority students from the faculty member's institution. Individual institutions may provide teaching credit or internal grants for engaging undergraduates in research. Energetic faculty may simply invite a few students to engage in research for the pure joy and learning involved! These and other methods to facilitate undergraduate research can benefit from consciously designing the effort to attract and support S-URGs.

In the end, the benefits of broadening participation are themselves broad. To quote from [Hartline and Poston 2009], for students, "especially when the students are visibly different from the faculty and the majority of their peers, caring and resourceful mentors can help illuminate the possible pathways, develop the mentee's talents, and encourage them to transition successfully into the next career phase". For faculty, "insights gained from students with different perspectives can take research projects in unanticipated but exciting and worthwhile new directions". We attest from personal experience that working with talented S-URGs who did not initially view themselves as potential mathematicians is professionally and personally fulfilling. And finally, the mathematical community and society at large benefit from capturing individuals from all segments of our population, bringing their numbers and diverse perspectives to bear on the complex problems yet to solve.

Acknowledgements

We thank the TURMS organizers for recognizing the importance of broadening participation in the mathematics sciences and including this topic. Their doing so provided the authors the opportunity to meet and reflect more deeply on our efforts to broaden participation, and led to many thought-provoking conversations with TURMS attendees during and following the conference. The authors also wish to thank colleagues who have been instrumental to our thinking about this work: at CI, Kathryn Leonard, Geoff Buhl, Jorge Garcia, Fernando Barrera Mora, Ben Itza Ortiz and Heather Russell; in PURE Math, Roberto Pelayo, Brian Loft, Efren Ruiz, Luis García-Puente and John Little. Finally, we thank our students for sharing their goals and dreams, and for continuing to teach us how to work with them!

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Papers, posters, and presentations as outlets for undergraduate research

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(Communicated by Darren A. Narayan)

Presented in this paper are the findings of the panel entitled *Outlets for undergraduate research* as delivered at the Trends in Undergraduate Research in Mathematical Sciences (TURMS) in Chicago on October 27, 2012. We specifically discuss venues and best practices for student papers, posters, and presentations.

1. Introduction

With the dramatic increase in the number of undergraduate students conducting research, there are more opportunities than ever for students to share their work with others. We focus on preparing students for three such outlets: papers, posters, and presentations. While most readers are familiar with a wide variety of opportunities for student presentations, we also provide a number of venues for posters and written publications. Additional information on these topics can be found in [Gallian and Higgins 2007].

2. Papers: Brigitte Servatius

Print publication is to date the most lasting outlet for (student) research. Our times put immense pressure on researchers to publish their work. Publications in refereed journals are considered more important, because quality is typically better, and publications reviewed on MathSciNet are of even greater weight. Another easy measure of impact is the number of citations an article receives. However, citations usually take longer to appear than the span of an undergraduate career.

To prepare an article for publication it is important to choose an appropriate journal. Most student researchers need help from their advisor in doing this, because students do not normally read research journals. To foster student research and encourage students to publish their work, it is a good idea to have students read published papers early on. Journals such as the *Pi Mu Epsilon Journal* or *The*

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College Mathematics Journal, whose target readership is an undergrad math major or a faculty member at a primarily undergraduate institution, could be kept in a student lounge. A problem-solving group can be used for good writing exercises. Solving problems from the current issue of a journal is a good exercise in reading, writing, and using math. Members of a problem-solving group act simultaneously as researchers and peer reviewers and there is a natural time pressure set by the deadline for submission of solutions. The *Pi Mu Epsilon Journal*, for example, publishes student solutions to its posted problems. Such a published solution is a very nice and fast first publication. Students can also submit problems for solution to appear in the problems column of a journal. Again, this is an excellent mathematical writing exercise and fast publication.

Advisors of student research may gain valuable experience by serving as referees for research in their specific field and for student research in a broader context. Refereeing is, without doubt, additional work. Since blind or even double-blind refereeing is customary it is a bit tricky for young faculty to have this activity counted appropriately among annual activities toward tenure. However, most journals will send a "thank you" that can be used as documentation of your service to the profession.

Preparing research results for publication is a serious step beyond obtaining the results. The introduction should be directed to the main readership of the journal. If it is a journal for undergraduate students, the introduction should be of broad interest and also understandable to the mathematically inclined advanced undergrad. If it is a research journal in topology, a metric space need not be defined. The introduction and bibliography are of fundamental importance. The journal editor will select a referee whose research interest matches the topic of the paper. The editor will therefore scan the introduction as well as the bibliography for ideas for referees. Misleading introductions as well as missing or wrong citations can have a devastating effect even before the paper is read. Before a paper is sent to a journal it should be meticulously proofread. It is not the responsibility of a referee to check proofs or to correct spelling and grammar. Once the editor has sent the paper to the referees, no changes or corrections are possible until the editor receives the referees' reports and asks for specific changes. Never send off a paper before you think it is absolutely perfect. Once the paper is ready, make sure you are aware of the journal's instructions for authors. Some journals require paper submission and want your manuscript in triplicate; some journals accept email submission of files of a certain format; some journals require web submission; and some allow more than one option. In all cases, you are expected to write a cover letter. Make sure it is dated, contains your permanent address and email, the title of your paper, and the name of the journal in which you want your paper to appear. Some editors are editing several different journals simultaneously. They may receive a flood of mail, and might misplace your article if this information is not apparent. If possible, also put that on the cover page. If your journal has double-blind review, put this information on top of a cover page with author information removed.

MathSciNet is a great research tool. It enables even a novice undergrad to find publications using a set of key words. These hits produce a reading list. The reviews together with the paper abstracts will give a good idea of the usefulness of the paper toward the research goal. The citations point toward other venues. The number of hits a particular key word produces gives a first idea about the importance of the concept. Writing reviews for MathSciNet is about as time consuming as refereeing, but is easier to highlight on your annual report. I recommend it highly to potential advisors of student research.

3. Posters: Aparna Higgins

A poster session is an efficient way for a large number of researchers to showcase their work simultaneously. Many institutions have poster sessions associated with an annual celebration of student work, and there are regional opportunities for poster sessions as well. For example, the Mathematical Association of America (MAA) section meetings frequently have poster sessions for undergraduates, and several of the MAA–National Science Foundation (NSF) sponsored Regional Undergraduate Mathematics Conferences hold similar such sessions. The MAA website (www.maa.org) provides information on both of these opportunities.

National meetings of the professional societies in math also organize poster sessions; the biggest of these is at the Joint Mathematics Meetings (JMM), held annually in January. Started in the early 1990s, the poster session had a dozen to fifteen posters. By the late 1990s, there were forty posters. Sixty posters were presented in 2000, eighty posters were presented in 2002, and over 250 posters were presented in 2012. The 2013 JMM in San Diego had over 320 posters. More information on the subject of posters can be found in [Martelli 2002].

The poster session at the JMM is always exciting — the room is humming with energy from enthusiastic students explaining the results of their research to faculty, some of whom are judges for the posters. Under an initiative of Joe Gallian, MAA President 2007–2008, travel support is available to students who present posters (see maa.org/programs/students/meetings-conferences/student-travel-grants). For many years, several monetary prizes were awarded based on judging conducted by attending faculty. In recent years, the monetary prizes have been discontinued, but faculty continue to judge posters in an organized and formal manner, providing valuable feedback to the student researchers on their work and presentations. The poster session is always in need of judges; see http://www.maa.org/students/undergrad/judges.html.

It is important for research directors to help students understand the differences between preparing posters, presentations and papers for publication. Some advice on this matter can be found in [Hammarling and Higham 2013]. Additionally, here are some suggestions that I make to students who are preparing posters under my direction.

- Keep in mind who your audience is, and how little time they will be at your poster.
- It is very helpful for your audience if your poster indicates clearly what kind of objects are being studied (groups or manifolds or graphs or knots, for example), and what aspect of these objects is being studied (for example, a particular invariant or a product of two of these objects or a mapping between two of these objects). It helps the viewer if the poster presenter explains why this work may be interesting (for example, the sharpening or realization of a previously known bound).
- The font size used should be large enough to be legible from two or three feet away.
- The poster should contain the title of the project, the names of the researchers and their affiliations, the name of the program that sponsored this research (as in an REU), and the names of the advisors of the project, if they have not already been included as coresearchers.
- The poster should contain many pictures to help illustrate concepts. It is helpful to have both examples and nonexamples listed so that the contrast is evident and helps to clarify any definitions used.
- If abbreviations for techniques or objects are used, it is helpful to indicate, in a footnote, say, what the abbreviation stands for.
- If a proof is provided, it is better to provide broad strokes of the idea of the proof, rather than all the technical details. If the result is proved by breaking down the problem into many cases, it suffices to present the proof of only one case.
- Students should create a two-minute-or-less guided tour of the work that the poster illustrates. It is best to script this ahead of time and practice it.
- Have a handout ready with the names and email addresses of the researchers and the name of the organization where the work was done, name and date of the meeting, and a couple of the main results. This handout is a useful tool for networking, and can be given to anyone who is interested in the work presented by the student.

Posters can serve as vehicles of dissemination of results, but they can also serve as ways of getting feedback on partial results.

4. Presentations: Lew Ludwig

For over ten years I have been instructing students on ways to improve their oral communication skills. Not only has this work made for a number of successful student presentations, some winning national awards, but it has also prepared my students for life beyond my classroom. Whether my students attend grad school or join the work force, communication skills are a critical asset towards their success. Indeed, in the National Association of Colleges and Employers (NACE) *Job Outlook 2013* report [NACE 2013], employers rank "ability to verbally communicate with persons inside and outside the organization" as the number one candidate skill/quality they seek in future hires. As we prepare our math majors to present their senior project or summer research for that sectional meeting or department colloquium, we need to keep in mind that we are not only helping them convey their work to others, but we are helping them develop a life skill that will reach far beyond that one presentation experience.

In addition to preparing students to present their research, I have taught seven sections of the Technical Communication class at Denison University, where students improve their oral communication skills by delivering three mathematical presentations. It is safe to say that through these experiences, and by attending a number of regional and national presentations, I have seen hundreds of student presentations. Interestingly, when a particularly strong or weak talk stands out in my memory, it is not the student's name I recall, but that of the school. We might bear in mind that not only are we better preparing our students for the work force; through proper training of our students in oral communication skills, we are bettering the name of our own institutions.

Many readers will be familiar with Joe Gallian's *Advice on giving a good PowerPoint presentation* article through the MAA website [2006]. In this article, Joe provides a detailed checklist of dos and don'ts for delivering a good presentation including things like font size, use of color, refraining from animations, etc. This article served as a springboard for my NSF-funded website *Technically Speaking* (techspeaking.denison.edu), which provides a video on how to give a good presentation. While such resources have been a great help to students, I find that there are some finer points students often overlook in their presentations, and these can prevent a good presentation from being a great one. Here are a few things to consider.

Know the material well so your audience doesn't have to. During a recent tenure-track search to replace a retiring colleague, my colleague Michael Westmoreland commented how he placed a good deal of emphasis on the research statement. Coming from a national liberal arts college with a high emphasis on teaching, I initially found his focus misplaced, but he further explained. He argued that if someone really knew her research well enough, she could explain it to a nonexpert audience, much in the same way that quality instructors should be able to explain difficult mathematical concepts at a level appropriate for their students. So in a sense, my colleague was using the research statement to inform him about a candidate's ability to teach.

I think the same can be said for student presentations. If a student has a good understanding of the material contained in the presentation, then he should be able to deliver the concepts to a variety of audiences. Often students make too many assumptions about the audience. They automatically think everyone knows about the mosaic number of a knot, the chromatic number of a graph, or the genus of a surface. The student should know the audience and their limitations. It helps to use simple examples or even props to convey technical ideas. In a 10–15 minute talk, it is enough to convey the main ideas, without all the nitty-gritty details.

Play it again Sam. Why do the refrains of songs or jingles stick in our heads? Not only are they catchy, but they are repeated numerous times — the very definition of refrain. Often I see students deliver a well crafted presentation, but they do not repeat the important things. Everything is given the same weight, it is said once. Just like that jingle, students need to repeat the important parts of their message. This can be done when the idea is first introduced or throughout the presentation or both. If something is important, it bears repeating. When the audience hears something repeatedly, it will begin to take notice. A good gauge of understanding is whether an audience member can state the three most important things from the presentation. Don't forget, if it is important, repeat it.

Read less, talk more. Giving a presentation before a live audience can be nerve-racking. It is natural to get nervous and possibly forget what you want to say. To overcome this, many students print everything, or most everything, they want to say on their slides. Some go so far as to script complete sentences or even paragraphs. While this may lessen the student's anxiety, it is counterproductive for the audience. Any cognitive psychologist will tell you we cannot multitask. We cannot text and drive. We cannot tweet and study. And we surely can't listen to someone speak and read. It is human nature to start reading along with the speaker. More often then not, the audience will begin to read ahead in the written material, thus nullifying the effectiveness of the speaker, who would have done just as well to provide a handout and sit down.

To prevent your student's "talk" from becoming a "read," be sure she uses only keywords or short phrases on the slides. These notes should be cues to the presenter of what needs to be said, not a substitute for listening to the speaker. Most mathematicians are "thrifty" by nature. Encourage your students to embrace their inner mathematician and be thrifty with the numbers of words per slide. I tell my student each word cost 25 cents.

I offer one more word of advice, as a continuation with the "talk more" theme above. The more often students present in front of a group, the more comfortable they will become. If you plan to have a student present a mathematical topic at a conference, consider having them give a few short warm-up talks just to get used to standing before an audience. This could be as simple as a two-minute presentation on their favorite mathematician or why they chose to attend their school. The content is not as important as shaking out the nerves and gaining confidence. Once this is established, then it is time to focus on the details in Gallian's article or on the *Technically Speaking* website.

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ISU REU: diverse, research-intense, team-based

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This article describes the Iowa State University (ISU) mathematics REU. The emphasis is on how certain choices made have shaped the ISU REU. The ISU REU draws a diverse group of students from a broad spectrum of colleges and universities nationwide. It is research-intense, with no course or workshop component, and results are disseminated through publications and presentations at conferences. Students in the REU work in teams with graduate students and faculty.

1. Introduction

The Iowa State University (ISU) Mathematics Department frequently involves students, both undergraduate and graduate, in a variety of research projects during both the summer and the academic year. This article focuses on the summer REU program at ISU, here called the ISU REU, directed by Justin Peters and Leslie Hogben, and usually managed by Hogben. In addition to Hogben and Peters, Wolfgang Kliemann and Sung-Yell Song regularly mentor REU groups; a total of 14 current ISU Mathematics faculty have served as mentors, as have another 7 former or visiting faculty. This program has operated seven of the past ten summers, and we will again offer the ISU REU in the summer of 2013. More information can be found on our website at orion.math.iastate.edu/reu.

The diversity of undergraduate research experiences — summer or academic year, all-research or combination of workshop/background class and research, research project teams or individual research projects, nationwide or local — serves students well, but each program must make choices. In this article the focus is on how certain choices made at ISU for our summer undergraduate research program, namely to

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be a diverse national, research-intense, team-based REU, have shaped the ISU REU (Sections 2, 3, and 4, respectively). The most recent ISU REU (2011) is discussed in Section 5, and Section 6 expresses the author's concerns about the future of the national REU program; the remainder of this introduction discusses funding sources.

The ISU REU has had three major funding sources: (1) two NSF REU site grants (covering summer 2004, 2005, 2006, and 2009, 2011, 2013); (2) the NSF-funded Alliance grant (covering summers 2003, 2004, 2005, 2009, 2010, 2011); and (3) the ISU Mathematics Department (funding students in 2003, funding graduate research assistants during the other six summers, and having faculty donate time all seven summers of the program). In addition to students being funded by these three main sources, a few students have been funded by faculty grants or ISU programs, and several ISU students have been accepted as volunteers in the ISU REU (sometimes with their expenses paid by ISU Mathematics Department). The funding source affects the pool of eligible students. For example, the 2003 REU, in which many of the students were supported by the ISU Mathematics Department, had 7 of 14 students from ISU and 4 others from central Iowa, and Alliance funding was restricted to students at certain schools.

While NSF REU site grants are well known, and it is not uncommon for colleges or universities to support their own students in summer research, some description of the Alliance is needed here. The NSF-funded Alliance for the Production of African American PhDs in the Mathematical Sciences was a partnership between ISU, the University of Iowa, the University of Northern Iowa, and four historically black colleges and universities (HBCUs). It supported Alliance scholars at the partner HBCUs both financially and with mentoring during the academic year, and involved them in REUs at the three Iowa regents universities during the summer. The ISU REU has integrated all students in research projects regardless of funding source, and has used an all-research model. UI integrated Alliance-funded students with students from their NSF VIGRE REU, and both UI and UNI used a workshop and research model. All the Alliance students in REUs would get together at either UI or ISU at the beginning of the summer, and at the other institution at the end of the summer, where they would present their research.

Between 2006 and 2009 this Alliance merged with other programs that had successfully mentored under-represented minority students to pursue graduate degrees in the mathematical sciences and became the National Alliance for Doctoral Studies in the Mathematical Sciences. This change substantially expanded the activities of the Alliance, but the three Iowa regents universities continued to operate the (Iowa) Alliance REU in much the same manner. However, three of the changes in the Alliance and in the (Iowa) Alliance REU had major positive impacts on the ISU REU: (1) The expansion of the Alliance greatly expanded the pool of students eligible to be supported by Alliance funding. (2) The symposium at the end of the summer was expanded to include all mathematics and statistics students in the REUs at ISU, UI, and UNI. (3) At ISU and UI, all mathematical sciences REU students are now housed together regardless of funding source.

2. Working with students from diverse backgrounds

The ISU REU has involved 106 students over seven summers, with 9–21 students per summer.[1] Of these students, 26% are under-represented minorities as defined by the NSF (predominantly African Americans and Latinos/as, in roughly equal numbers), and 40% are women. Half of our students come from nondoctoral colleges and universities, and more than 3/4 attend(ed) a college or university that is *not* among the top 25 national universities or top 25 liberal arts colleges as ranked by *US News & World Report*. While all REUs are selective, the ISU REU is reaching a relatively broad spectrum of students.

We are working to diversify the pool of graduate student and faculty mentors: 32% of the graduate RAs are women and 11% are under-represented minorities; the corresponding percentages for faculty are 25% and 4%.[2]

More than half of the ISU REU students (and more than half of the under-represented minority students) who had graduated at the time we collected our most recent data have enrolled in graduate school.[3]

ISU REU students have won major national fellowships, such as NSF Graduate Research Fellowships and National Defense Science and Engineering Fellowships. The success of our students in publishing and presenting their research is discussed in Sections 3 and 5.

Changes in the Alliance program (discussed in Section 1), and changes at ISU have had a significant positive impact on the ISU REU in recent years. Although we have always believed in integrating all ISU REU students in research projects regardless of funding source, this integration was only partially successful prior to 2009, leading some project groups to split into subgroups and in some cases into different projects because of substantial differences in student background. One source of difficulty was that the number of places available at ISU, UI, and UNI for Alliance REU students sometimes exceeded the number of qualified, interested

[1] Two of the seven summer programs included students doing research in statistics as part of the living group, but their data is not included here because Statistics is a separate department at ISU and their research supervision model was quite different from ours (and varied over time).

[2] A faculty mentor is counted according to the number of summer programs in which s/he is a mentor, but is not counted extra if s/he mentors multiple projects in a single summer.

[3] This is a combination of relatively complete data for NSF REU site grant-funded students in the years 2004–2006, and one year after data combined with anecdotal data for the other years and for students from other funding sources.

students at the four partner HBCUs. The ISU REU struggled during these years to recruit Alliance students with the background to benefit from our all-research program. The broadening of the pool of Alliance-eligible students[4] has facilitated the recruitment of a diverse group of students with the necessary background for an all-research REU. Another factor that improved the pool of applicants is several new diversity initiatives undertaken by the ISU Mathematics Department starting in 2008. In particular, a partnership with the University of Puerto Rico, Rio Piedras campus has been helpful in recruiting students. Since 2009 the research groups have worked more smoothly and none has split (except in a planned manner, such as dividing a group of 6 into two groups of 3 with related projects). The expansion of the Alliance symposium to include all mathematics and statistics REU students at ISU, UI, and UNI, and the common housing of ISU REU students have helped the ISU REU to become more cohesive.

All of the standard measures of REU success (students enrolling in graduate school, students winning fellowships and awards, students publishing research, etc.) have increased since 2009. And perhaps more importantly, since 2009 our under-represented students (and our Alliance-funded students) have by any of these measures been just as successful as our majority students and our NSF REU site grant-funded students.

The diversity of students' cultural backgrounds has enriched the REU. For example, in 2009 ISU REU students organized a weekly Spanish class that was well attended by students, RAs and faculty. Dinners with ethnic dishes have also been organized.

3. Research and publications

The ISU REU program has had research projects in a wide range of areas, depending on faculty involved and their research interests. This flexibility has allowed the program to evolve over time. Nonetheless, there are several areas (corresponding to the interests of long-term faculty mentors) that have been offered repeatedly: these include combinatorial matrix theory, dynamical systems, and algebraic graph theory.

Iowa State University is a public land-grant university with more than 30,000 students, is a member of the American Association of Universities (an association of 62 leading research universities), and is classified by the Carnegie Commission as a research university with very high research activity (RU/VH). As such, research is the primary focus for most tenured and tenure-track faculty, especially during the summer, and we volunteer our time to do research. Many of us particularly

[4] Although any student nominated by an Alliance mentor is eligible, almost all Alliance-funded students continue to be under-represented minorities; other Alliance-funded students have faced health or financial crises.

enjoy doing research with bright enthusiastic young people, such as REU students or graduate students, but research remains a major goal.

We publish results in a wide variety of journals, ranging from standard research journals to undergraduate journals. As all but one ISU REU paper have faculty and graduate students as coauthors, and we want the researchers in the area to be aware of the results, *Involve*, with its articles being reviewed on MathSciNet and joint student-faculty authorship encouraged, is an excellent journal in which to publish REU research results. A list of ISU REU research publications can be found on our website at orion.math.iastate.edu/reu/REUpubs.html.

Over the course of seven ISU REU programs, there have been 43 projects; 19 papers, involving students in 20 of those projects, have appeared or been accepted.[5] Of the 106 REU students, 50 are coauthors of ISU REU research papers. Overall, 47% of ISU REU students are coauthors on published papers. Since 2009, there have been 14 research projects and 9 papers; 58% of the students are coauthors (and two papers are still under review).

4. Research teams and vertical integration

Throughout the ten-year life of this REU, research groups have been used to introduce undergraduates to research. While this is not unusual in REUs (in some cases out of economic necessity), the research group approach informs much of what we do with students at ISU, both undergraduate and graduate. Prior to starting the ISU REU, several faculty in the department, including the author, had used research groups of graduate students, postdocs, and faculty, both for their own students and to introduce new students to a research area.

For the ISU REU, research groups are not just the most practical way to offer our program, but we have found that they also the most effective. In many cases the students learn from each other. In the first few years we sometimes divided a group and gave students individual projects, but have not done so since 2007.

While all of our REU students are bright and enthusiastic, we engage a broad spectrum of students with a diverse range of backgrounds in research. Our REU students have typically completed junior level mathematical coursework,[6] although is some cases we take exceptional students who have completed only their sophomore mathematics courses.

The (paid) graduate student research assistants (RAs) play a critical role in the ISU REU. It would not be possible to offer the ISU REU without graduate RAs.

[5] In a few cases results from two summers were combined in one paper or a single project produced two papers.

[6] The student's mathematical age is more relevant here than the year of college the student has completed, as many of our REU students are accelerated, having completed junior level courses after two years of college.

As none of the faculty are paid any salary to serve as research mentors in the ISU REU[7] and most are on nine-month contracts without summer support, such a faculty volunteer needs the flexibility to travel to research conferences during the eight-week REU; the graduate RA supervises the undergraduates in the absence of the faculty member. Faculty members focus their efforts on research (and in some cases on mentoring the students for graduate school and mathematical careers), but the graduate RAs are involved in all aspects of the REU. They are actively engaged in research with the students and faculty, are available most of each work day to answer student questions, provide support for LaTeX, assist with mathematical software, work with the students on the preparation of final papers, presentations, and posters, and are wonderful mentors who are trusted by the students to provide the “inside story” about life in graduate school.

Postdoctoral associates have served as faculty mentors in this REU, and for the purposes of this article their data have been combined with faculty.

We have also used research groups (both academic year and summer) to create a supportive environment for faculty at undergraduate colleges to continue involvement in research, and several such faculty are coauthors on ISU REU publications. Such faculty also contribute substantially to mentoring students.

Our experience with research groups and the REU led us to develop an Early Graduate Research (EGR) course, which is offered in the spring semester. ISU’s EGR course is a model that can be replicated at research universities for graduate students (but not for undergraduates due to minimum enrollments), and possibly by undergraduate colleges that have lower undergraduate course enrollment requirements.

5. The 2011 ISU REU

The combination of these changes and our growing experience led to a highly successful 2011 REU.

One student won an NSF Graduate Research Fellowship and another won a National Defense Science and Engineering Graduate Fellowship. At least ten of fifteen students who graduated in 2012 enrolled in graduate school in an NSF discipline, most in doctoral programs in the mathematical sciences. Schools they are attending include Harvard, Cornell, SUNY Stonybrook, Georgia Tech, Arizona State and Iowa State.

Four of the five research projects have published (or had accepted) papers reporting their results, and the fifth group has its paper under review — we are aiming for 100% publication from this REU.

[7] In a few of the REU summer programs, one faculty member was paid a small (e.g., two-week) salary to manage the program; there is no funding in the current NSF REU-site grant for this.

Figure 1. The 2011 ISU REU in front of Carver Hall.

Fifteen of the eighteen students attended the 2011 SACNAS National Conference in San Jose and presented 5 posters. Fourteen students attended the 2012 Joint Mathematics Meetings in Boston and presented 6 posters.[8] At each of these conferences, ISU REU students won poster presentation awards.

6. The future of REUs

Involving undergraduates in research in the mathematical sciences, and specifically summer REU programs, has proved highly successful nationally in enhancing student interest in graduate school in the mathematical sciences. REUs are playing an important role in expanding the mathematical workforce and strengthening the next generation of mathematical scientists, and federal support, especially the NSF REU program, has played an essential role in this.

The author was disturbed by statements at the Trends in Undergraduate Research in Mathematical Sciences (TURMS) conference that the NSF is considering changing the funding model for REUs, expecting REU programs to be redesigned to continue without federal funding. It should be pointed out that the ISU Mathematics Department already provides a substantial contribution to the direct funding needed to operate the ISU REU, in the form of most of the graduate student research assistantships. Furthermore, if the value of the faculty time that is volunteered is included, the ISU Mathematics Department is providing more than one third of the entire cost of the current ISU REU that benefits students nationally.

The highly successful ISU REU, which has contributed significantly to both broadening the mathematical workforce and producing new mathematics, cannot continue in its present form without outside funding. Of course alternatives are possible — as noted earlier, we do research with students in a variety of settings — but an entirely locally funded alternative would understandably be for ISU students

[8]Not all students who attended presented a poster, and some presented other research (the number of posters given is for ISU REU research only).

only, and would therefore be substantially different from the ISU REU described in this article.

National impact requires national funding.

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AIM's Research Experiences for Undergraduate Faculty program

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(Communicated by Darren A. Narayan)

The Research Experiences for Undergraduate Faculty (REUF) program of the American Institute of Mathematics prepares faculty to engage in research with undergraduate students, encourages long-term research collaborations among some of its faculty, and builds a network of faculty who supervise undergraduate research. Participants of each REUF workshop are faculty members from undergraduate colleges interested in mentoring students in research mathematics at their home institutions. During a workshop, senior mathematicians with experience supervising undergraduate research present open problems suitable for undergraduates. The REUF program also includes several follow-up activities.

1. Overview

Participating in research in mathematics as an undergraduate can be a pivotal experience that contributes to a student's decision to pursue a career in the mathematical sciences. More than 60% of undergraduate mathematics degrees are awarded by colleges and universities that do not have doctoral programs. Faculty at primarily undergraduate institutions typically have more teaching responsibilities than their counterparts at research universities. Such faculty often have limited time to invest in their own research, and engaging in research with students can be challenging. The Research Experiences for Undergraduate Faculty (REUF) program addresses these issues with a series of activities designed to

- prepare faculty to engage in research with undergraduate students at their home institutions,
- involve some faculty in long-term research collaborations,

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- provide faculty the opportunity to have a research experience investigating open questions in the mathematical sciences, and thereby rekindle or further fuel a love of doing original mathematics, and
- establish a network of faculty at primarily undergraduate institutions together with faculty at research universities who support collaboration and undergraduate research.

The REUF program was developed by the American Institute of Mathematics (AIM) under the direction of Leslie Hogben, Roselyn Williams, and Ulrica Wilson. The REUF program continues to grow and develop, and in 2011 began a partnership with the Institute for Experimental and Computational Research in Mathematics (ICERM).

The core activity of REUF is a series of annual workshops. Each workshop involves a new group of faculty members who investigate problems suitable for subsequent research with undergraduates; each participant receives full funding for travel and local expenses. Four senior mathematicians (here called group leaders) who have experience supervising undergraduate research lead the research activities at each workshop. The organizers (Hogben, Williams, and Wilson) and Brianna Donaldson, AIM's Director of Special Projects, manage the program and facilitate the large group discussions. Several types of activities provide support beyond the workshop:

- The continuation of a research project begun at the workshop through a small research group meeting approximately a year after the workshop.
- Regular discussion gatherings of REUF participants (from any prior REUF workshop) who are attending national meetings such as Joint Mathematics Meetings or MAA MathFest.
- An online discussion group (the REUF Forum) and a REUF Resources webpage for participants.

More information about the REUF program can be found at reuf.aimath.org.

2. Recruitment

About half the participants for each annual REUF workshop are invited by the organizers and the other half are selected through an open application process. Recruitment for REUF participants targets faculty who want to direct undergraduate research projects, especially those in departments that serve a substantial percentage of undergraduates underrepresented in the mathematical sciences, or faculty who are themselves a member of an underrepresented group. No previous experience with undergraduate research is required. Participants come from colleges and universities throughout the United States. In selecting applicants, in addition to broadening the mathematical workforce, preference may be given to groups of two or three applicants at the same college or in close geographic proximity. Such circumstances provide local support for continuing research activities after the workshop. Fliers advertising the workshop are distributed through email, at professional mathematics meetings, on the AIM web page, and through professional organization newsletters and websites.

3. The workshop

The five-day workshop takes place in the summer and includes approximately 22 participants and four group leaders. Table 1 gives a typical schedule of activities for the week.

Participants are mathematicians at primarily undergraduate institutions interested in conducting research with undergraduates. The group leaders are mathematicians with an established record of excellence in doing research with undergraduates. Prior to a workshop, each of the four group leaders chooses a topic/problem that will be investigated by participants during the course of the workshop. A sampling of previous research topics is given in Section 3.2. A small amount of background reading material for each of the four topics is provided to every participant before the workshop begins. This allows participants to become more familiar with the general

Monday	morning:	Introductions followed by presentations of topics and open problems to be investigated during workshop.
	afternoon:	Divide into groups and begin working.
Tuesday	morning:	Introduction to *Sage* and work in research groups.
	afternoon:	Work in research groups.
Wednesday	morning:	Preliminary reports from each group; break out into research groups.
	afternoon:	Short discussion on publishing undergraduate research, including an introduction to the research journal *Involve*; participants work in research groups.
Thursday	morning:	Full group discussion: topics related to undergraduate research.
	afternoon:	Participants work in research groups.
Friday	morning:	Participants work in research groups and plan activities to continue project.
	afternoon:	Groups present final reports.

Table 1. Typical schedule of a REUF workshop.

Figure 1. REUF research group at AIM in 2008.

area of all four topics prior to the workshop without committing to a specific topic. Typically, participants will work in a mathematical area outside their primary field of expertise. This creates a dynamic similar to that of most undergraduate research experiences and allows the group leader to model best practices in supervising research projects with students.

On day 1, each group leader presents a 30-minute overview of his/her topic including a list of open problems, and subsequently the participants break up into groups — a working research group for each topic/group leader. Most of the week is designated for groups to make progress on their research problem (see Figure 1). On day 2 or 3, groups give very brief oral reports on the progress/plan for the remainder of the week; and on the last day a more detailed final report is presented including plans for continuing the work started. Along with the progress reports, there are also a few other full group sessions scheduled throughout the week, including an introduction to *Sage* — a free open source mathematics software system. Often undergraduate research includes experimentation and construction of examples using computer software. Since it is free and available for download, the *Sage* computer mathematics system is extremely accessible to students, so it was selected for use in the workshop. Participants receive a quick introduction and are encouraged (but not required) to practice using the software throughout the week as it relates to their problem. There is also a full group session scheduled during the workshop to discuss topics related to supervising undergraduate research.

Figure 2. REUF full group discussion at ICERM in 2012.

Some of the topics that have been covered in the group discussion on undergraduate research are listed in Section 3.1. Research groups are also encouraged to plan activities to continue the research started during the workshop; more information on continuation activities is given in Section 4.

3.1. ***Group discussion.*** A goal of the REUF program is for participants to increase their capacity to supervise undergraduate research as part of an academic year experience and/or a summer program. To this end, we schedule a group discussion (see Figure 2) on topics related to undergraduate research. The discussion may include attributes of a good undergraduate research problem and/or practical issues related to a productive undergraduate research experience, including how to structure these experiences during the academic year versus the summer. There is an introduction to some of the resources available to support faculty involvement in undergraduate research, and this information is also shared through the REUF Forum and on the REUF Resources website, which are discussed in Section 4.3.

The group discussion varies with participant interest, and begins by surveying the participants for the issues/questions that concern them the most. Each question to be discussed is written on the board, and then the moderator goes through each topic soliciting comments from all participants. Discussion topics from four REUF workshops have included:

- Characteristics of a good undergraduate research problem.
- Sources for good research problems for undergraduates.
- Mentoring undergraduate research.
- Fostering student collaborations.
- Selecting/recruiting students.
- Recruiting underrepresented students.

- Establishing expectations.
- Getting students to use LaTeX.
- Getting students to write-up results.
- Publishing student results.
- Student incentives: graduation requirement, course credit, compensation, etc.
- Faculty incentives: concurrent teaching credit, accrued teaching credit, compensation, etc.
- Balancing supervising undergraduate research with your own research agenda.
- Building a team of faculty on campus to supervise undergraduate research projects.
- Funding for summer research programs.

3.2. ***Research topics.*** The research problems for the workshop are chosen by the group leaders and are expected to be suitable for research with undergraduates. Some of the problems at each workshop are also suitable for faculty research. Problems have been chosen in algebra, linear algebra, graph theory, operator theory, and number theory. Examples of previous topics include:

- The linear algebra of the *Lights Out!* game.
- The distribution of eigenvalues of the $n \times n$ unitary matrices given certain restrictions on the eigenvalues.
- The relationship between large gaps between zeros of the Riemann zeta function and large values of the zeta function on the critical line.
- Sphere-of-influence graphs.
- Minimum rank of a graph.
- Cyclotomy using representation theory.
- Groups of perfect shuffles.
- Exponential graph domination.
- Structure of symmetric k-varieties.
- Symmetry of numerical range.
- Prime and coprime labeling of graphs.
- Dessins d'enfant graphs.

In response to participant feedback, REUF 2013 will include one or more group leaders who will focus on more applied topics.

4. Continuation activities

The workshop only starts the process of involvement with undergraduate research or faculty collaborations in new research areas. Continuing the work initiated is necessary to realize the full effectiveness of the workshop. As such, time is designated during the workshop for participants to develop a plan to continue workshop activities. Such continuation can be facilitated by electronic dialog, but in-person meetings are usually necessary.

For example, participants at the same institutions or in close proximity to each other will be encouraged to make plans together for undergraduate research. Those who wish to continue their research collaboration will have the opportunity to begin preparing an application for a small research group meeting at AIM (see Section 4.1) and/or identify professional meetings that several group members are attending and make plans to work there. AIM directors and workshop organizers are available to consult with participants throughout the workshop to provide information about AIM programs and other funding sources.

4.1. ***Small research groups.*** Each year participants in the REUF workshop who wish to continue work on their research project at a level needed for publication are invited to apply for REUF funding to go to AIM for an additional week of research that takes place a year or more after the REUF workshop. This an attractive opportunity for participants eager to continue their work started during the workshop. It is modeled on the AIM SQuaREs program, in which a small group comes to AIM for two or more weeks a year apart to collaborate on a long-term research project. During the interval between the REUF workshop and the small research group meeting, group members are expected to continue work and communicate electronically and/or through personal contact, for example, at professional meetings.

4.2. ***REUF at meetings.*** We have experimented with various strategies to bring REUF alumni together to provide additional support to faculty participants who are actively mentoring undergraduate student research. We have tried to bring together participants from the most recent REUF workshop for a day-long meeting at a national conference such as Joint Mathematics Meetings (JMM) or MAA MathFest for group discussions on their experience supervising undergraduate research: What have participants done? What worked? What were the challenges? We have found this costly and the majority of participants are not able to attend.

We now host shorter gatherings of all REUF alumni at national meetings such as JMM and MathFest. By including participants from several REUF workshops, we have a larger group with a broad perspective, including faculty with varying amounts of experience mentoring student research, and are building a network supporting undergraduate research. The first of these gatherings was held at MathFest 2012 and produced a good discussion of successes and challenges of doing research with undergraduates.

At such national meetings many REUF alumni bring their undergraduates. One of the suggestions from MathFest 2012 that we are implementing at JMM 2013 is to gather information about undergraduate talks/posters by students of REUF alumni and disseminate it to all REUF alumni attending the meeting.

4.3. *REUF online.* To provide continuing support for REUF participants, AIM maintains the private REUF Resources website and a listserv (the REUF Forum) for all previous REUF participants to alert REUF alumni of relevant opportunities and facilitate exchanges of advice and information among faculty. To support the broader community of faculty interested in doing research with undergraduate students, AIM maintains the public REUF Undergraduate Research Resources website: reuf.aimath.org/resources/undergraduate-research. This page includes information contributed by REUF participants.

5. Outcomes

In this section we provide data showing that REUF is achieving its goals.

Research with undergraduate students. With the exception of the first REUF workshop, more than half of participants mentor undergraduate research, although it often takes some time for this to happen (see Tables 2 and 3). Continuation activities that have taken place since the second REUF workshop in 2009 appear to play an important role in supporting this.

Faculty research outcomes of REUF. As the first REUF workshop was envisioned, the expected outcomes were research with undergraduates and fostering faculty engagement with mathematics, but it was not expected that faculty research would be an outcome. However, a few successful long-term faculty collaborations were established (see Table 4). Some of the continuation activities, including the small

REUF workshop	# participants	# participants mentoring undergraduate research	% participants mentoring undergraduate research
2008	20	6	30
2009	20	12	60
2011	24	11	46†

Table 2. REUF participants mentoring undergraduate research, as of June 2012. †Two more participants in REUF 2011 reported plans to work with specific undergraduates, bringing the total for REUF 2011 to 54%.

		2008	2009	2011
Students mentored		23	53	23
Senior theses	completed	1	6	1
	planned	0	0	1
Presentations (national)	completed	some	10	1
	planned	0	0	4
Presentations (regional/local)	completed	some	some	3
	planned	0	0	4
Publications	accepted/appeared	1	9	1
	under review	0	2	0
	in preparation	0	5	1
Other	grants	2	1†	0
	national student prize	1	0	0
	started math research seminar	0	0	1

Table 3. Undergraduate student research outcomes of REUF, as of June 2012 ("some" indicates that exact data are not available and † indicates a NSF REU site grant).

		2008	2009	2011	2012
Publications	accepted/appeared	2	1	0	0
	under review	1	0	0	0
	in preparation	0	0	2	2
Continuing research groups		0	3	4, 2†	3, 2††
Long-term individual collaborations		2	0	0	0
Participated in other AIM workshop or CBMS conference		3	5	0	0

Table 4. Faculty research outcomes of REUF. The last formal data collection was June 2012, but data known to the authors through December 2012 is included.

† Each of these two will meet for a week at AIM in summer 2013.

†† One of these two will meet for a week at ICERM in summer 2013 (with REUF funding), and the other will meet at a university in the summer of 2013 (with funding from the university of the group leader and colleges of some participants).

research group meeting, have been established to foster continuing faculty research collaborations. The first of the small research groups to be funded through REUF will meet at AIM in summer 2013.

Research experience for faculty. Responses to questions about the experience in participant surveys immediately after the REUF workshops has been overwhelmingly positive (85–100%). Here is a sample of comments:

- *Being here makes me miss doing math so much that I have experienced a bit of heartache, but I have had to make the choices I made* [*due*] *to the demands of my time. It was fun being here*!
- *Fantastic*! *I had a great time working in groups.*
- *This was a great workshop all around* (*great people, great format, flexible schedule, intense and exciting collaboration opportunities, wonderful ideas for future work*).
- *The format of AIM workshops is the best model — organizers and staff greatly succeeded in creating the best environment for discussion and research work*!
- *It was the most useful workshop I've ever attended. It proved to me that faculty with high teaching loads that haven't done research in a long time still have it in them. They just need some open problems, some collaborators, and some free time*!

Network of faculty to support undergraduate research. The establishment of the REUF Network is still a work in progress. The regular gatherings of all prior REUF participants who are at attending national meeting such as MathFest and Joint Mathematics Meetings is described in Section 4.2. The REUF Forum and REUF Resources website for REUF participants is described in Section 4.3.

6. Growth and development

The REUF program continues to grow and evolve. While the workshop itself is an established and successful model, we continue to explore options for faculty research continuation and supporting research with undergraduates after the workshop, including building a community to support undergraduate research (for information about the latter, see Sections 4.2 and 4.3).

This summer the first four REUF small research groups will meet, two from the 2011 workshop and two from the 2012 workshop (see Section 4.1). After we receive feedback from participants we will analyze the outcomes and perhaps revise this part of the program. We are also gathering information about three smaller collaborations that grew out of REUF, and how these might be replicated. We plan to explore these ideas with both alumni at the REUF gatherings at meetings and participants in each future REUF workshop.

One small collaboration involved three members of a 2009 REUF group at two colleges thousands of miles apart; they continued work on the problem presented at the workshop, and published a paper in *Journal of Number Theory*. Two of the collaborators (at the same college) later produced several related papers with undergraduates. Much of the work on the faculty paper was done during the REUF workshop at AIM and most of the rest was done electronically, although all three did meet briefly in person at a national meeting.

Two pairs of researchers from the 2008 REUF workshop began collaborating on separate topics different from the workshop problems, and produced papers that appeared in *Linear Algebra and its Applications* and *Electronic Journal of Linear Algebra*; a second paper from one of the teams is also under review. For both of these, multiple week-long research visits (totaling at least a month) were necessary for the development of shared background between the collaborators as well as for the advancement of the research itself (much of the communication about writing was done via e-mail); in addition there were numerous in-person consultations when both participants were at the same meeting.

We are exploring ways to expand the REUF program, since demand substantially exceeds capacity (by a factor of 2 in 2012). This has led to a partnership between AIM and ICERM. The 2012 and 2013 REUF workshops were held at ICERM; in 2014 we will be back at AIM, and are exploring the possibility of holding two REUF workshops per year, one each at AIM and ICERM.

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Institutional support for undergraduate research

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(Communicated by Darren A. Narayan)

Institutional support is critical for establishing and maintaining an undergraduate research program. This paper discusses some of the challenges that one may encounter when seeking to institutionalize undergraduate research, including budget and personnel issues. It provides various views and ideas from schools that have been successful in securing institutional support for undergraduate research, and makes some suggestions of rationales for effectively arguing on behalf of undergraduate research.

This paper is based on ideas generated at a breakout session at the 2012 national conference on Trends in Undergraduate Research in the Mathematical Sciences. Additional resources for a more in-depth discussion of the ideas presented in this paper are also provided.

1. Introduction

Institutional support is critical for establishing and maintaining undergraduate research (UR) programs in all disciplines. In recent years, we have seen many short- and long-term UR programs flourish across the nation. In addition to approximately 75 federally funded NSF–REU sites and the NSA Director's Summer Program, there are many programs supported by the National Science Foundation in which UR plays an important role: the Research Experience for Undergraduates Supplement Program, the Science and Talent Expansion Program (STEP), the Mentoring at Critical Transition Points program (MCTP), the Undergraduate Biology and Mathematics program (UBM), the Enhancing the Mathematical Sciences Workforce in the 21st Century (EMSW21), and the Louis Stokes Alliances for Minority Participation program (LSAMP).

Even with these federally funded programs, there are not nearly enough UR opportunities. We know that many of the NSF-funded summer REU sites accept nine to twelve students but receive hundreds of applications. Further, given the

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current economic conditions, how long these programs can be sustained is not certain.

As a result, the 2012 national conference on Trends in Undergraduate Research in the Mathematical Sciences (TURMS) challenged participants to tackle two major themes:

(a) How to sustain REU programs after the federal money runs out.

(b) How to create institutional UR with no federal money at all.

Here we will discuss three aspects of securing institutional support for UR: challenges to address, examples of ways that schools have been successful in securing support, and suggestions/rationales for arguing effectively on behalf of UR. The ideas presented here were generated at a breakout session designated for this purpose at the 2012 TURMS conference.

2. Challenges for institutional support for UR

Arguments one might encounter when seeking to institutionalize UR can be divided into two categories: budgetary issues and personnel issues. In [SUR 2006; Davis et al. 2008], the authors give an excellent discussion of both categories, particularly about issues related to faculty time. Here, we make a few additional brief comments.

Budgetary issues. Convincing a dean/provost/president to allocate continuing funds to a new program can be difficult, but this is the ultimate goal in establishing UR as a program that students and faculty can count on and build upon. One of the often-mentioned challenges to securing institutional support for UR, particularly at larger institutions, is the fact that federal grants for UR bring little or no overhead. Administrators are therefore reluctant to allocate resources to programs that support UR. This challenge, however, is not as prevalent at small liberal arts colleges, which gain visibility by establishing national REU programs. In addition, many federally funded summer REU programs bring students from outside of the institution where the REU program is conducted, and institutions are reluctant to invest in programs that include students from other schools.

Personnel issues. UR brings several direct and indirect benefits, and faculty should advocate its significance to their department and university, particularly its benefits to faculty. First and foremost, faculty enjoy an intellectually stimulating environment where diverse perspectives and exchange of ideas happen. Getting undergraduates involved in research, both disciplinary and multidisciplinary, is one way to cultivate and promote this type of environment across departments, colleges and the university. UR may create the opportunity to renew and/or reinvigorate a program, and can be a path for internal and external recognition. The pleasures of passing knowledge and witnessing a student's intellectual growth are lifelong benefits faculty can reap from getting involved in UR.

Faculty can benefit from UR in specific ways as well. A faculty member may get credit for tenure and/or promotion if research conducted in collaboration with undergraduates leads to peer reviewed publications and conference presentations. Mentoring undergraduates in research may count as independent or directed studies and can be part of the faculty's teaching load. Getting involved in UR can help faculty in securing grants as many of these encourage involving undergraduates in the proposed program. Getting involved in multidisciplinary UR can create a venue for faculty-faculty as well as faculty-student collaboration across a variety of disciplines. Faculty may also use the UR experience to improve their teaching and initiate a new area of research in the scholarship of teaching and learning.

Even when the budget is not an issue and there is clear evidence of the benefits of UR, it has been difficult to implement UR across all institutions. One major challenge is enlisting faculty to establish and sustain a UR program. Anyone who has mentored UR in mathematics knows that it is time consuming and requires careful thought on student selection as well as finding problems that are appropriate for undergraduates. As a result some faculty may choose not to get involved. At some institutions, junior faculty invest their time in their own research and writing proposals to secure external grants as these are the major accomplishments that count towards faculty rewards. At other schools, some administrators view UR as a distraction from classroom teaching. Finding a balanced way to compensate faculty for doing UR will help alleviate this challenge.

In the next section we will show that some schools have found ways to address these challenges.

3. Ideas from schools that have been successful in securing institutional funding

We remain convinced that UR programs in mathematics can work at all types of schools — large R1 institutions, comprehensive masters, small liberal arts — but what works at one institution may not work at another. That being said, adapting ideas from schools that have been successful in getting institutional support for UR is a good way for schools to begin a program or increase opportunities. Below is a summary of ideas that have worked in at least one school. For each item, we give at least one example of a place where the idea has been implemented successfully, without attempting to give a comprehensive list.

(1) Look for university-wide programs with wider purpose for which mathematics UR might be a part. At Michigan State University (MSU), some faculty take advantage of the MSU Professorial Assistantship Program to get students involved

in UR. (See MSU's Honors College brochure *HC Connections*, available online at http://honorscollege.msu.edu/_documents/HConnections_2011.pdf.) Each year, approximately 200 freshmen are appointed as professorial assistants (PAs). The PAs work with regular members of the faculty on tasks directly related either to scholarly research or to innovative teaching. Students whose academic records place them in the top one percent of entering college freshmen nationwide are offered the opportunity to participate in the Professorial Assistantship Program. The PAs work an average of eight to ten hours per week and are paid a stipend of approximately $2500 for the academic year. Those who perform satisfactorily are reappointed for a second year at a higher stipend.

(2) Look for other programs on campus with a budget for which mathematics UR might fit the goals. MSU has a residential college system where students in a particular field of study (like sciences or social sciences) live together and take classes together in the first two years. Each college has its own dean and its own budget that can sometimes be used for UR funding.

(3) Build internally to support external funding. In other words, use obtaining external funding as an incentive for president/provost/dean to provide new funds from internal sources for UR. At a large R1, this might be dependent at first on finding external funding that provides some overhead to the school. For a small liberal arts college where the amount of indirect cost money received is often less important than the total amount of external funding, this incentive may have more impact. Middle Tennessee State University piggybacked on significant funding for UR from an NSF STEP grant to convince its president to fund a center with additional recurring funds from the university for UR.

(4) Work with the development office and alumni. Illinois State University tied a large donation from an alumnus designated for scholarships in the sciences to a scholarship for UR students. Some schools prepare a nice brochure describing such projects to share widely.

(5) Pay attention to smaller grant opportunities. The Rochester Institute of Technology got travel vouchers from JetBlue for students to be able to make conference presentations. The University of Richmond has funding from the Virginia Foundation of Independent Colleges for one student researcher in the summer that can sometimes be used to fund a student in mathematics.

(6) Make UR part of the curriculum or at least part of a course. At Middle Tennessee State University, if students engaged in UR in the summer are receiving course credit, the faculty mentor can receive a small stipend from summer school funds. When faculty members supervise individual projects as independent studies or some other way that is not a regular course, they are more often than not expected to

do this in addition to their regular course load. Some schools have successfully addressed this issue by including UR in the design of their curriculum.

(7) Develop relationships with industry. The Center for Industrial Mathematics and Statistics (CIMS) at the Worcester Polytechnic Institute, created as a mathematical resource to industry that faces highly technical problems involving sophisticated mathematics, has opportunities for UR. The Rochester Institute of Technology is another example of a school that has found opportunities for UR by working with industry.

The session participants also noted that schools, especially those that have active UR programs, need to form partnerships with other institutions that do not. A place where this is especially needed is community colleges, where we have a large population of mathematics students and very little current institutional support or a critical mass of faculty members involved in research.

4. Arguing effectively on behalf of UR: suggestions and resources

Beyond being creative in finding sources of funding, most faculty who are working to establish, sustain, or expand a UR program find themselves in the position at some point in time of asking for funding. Below are some ideas generated by others who have been in this position.

(1) Convince the dean/provost/president that UR will benefit the university by dramatically affecting the education of the students involved. With research experience, students will:

- gain self confidence;
- improve writing and speaking skills;
- learn teamwork;
- develop an understanding of what professional mathematicians do;
- be more likely to enter graduate school [Petrella and Jung 2008; Lopatto 2007];
- be more likely to receive admission into a better graduate school;
- be more likely to perform better research in graduate school;
- be more likely to earn a PhD degree;
- be more likely to receive a better job offer;
- be better prepared to manage projects in their future profession.

Numerous studies that quantitatively assess the great impact of UR experiences on the professional formation of students can be cited. For example, see [Petrella and Jung 2008; Thiry et al. 2011; Gregerman 2008; Hunter et al. 2007; Lopatto 2007;

Carter 2011; Hensel 2012; Kinkead 2010; Boyd and Wesemann 2009; Karukstis and Elgren 2007; Kinkead and Blockus 2012; Karukstis and Hensel 2010].

(2) Whenever possible, invite top administration officials (dean/provost/president) as well as alumni and potential donors to UR presentations, to build institutional support and ensure top officials are aware of students' accomplishments.

(3) Advocate the importance of UR as a tool to close the achievement gap among students of different ethnicities. (See, for example, [Boyd and Wesemann 2009] or [CUGESEWP 2011].)

(4) Convince the administration that UR inevitably competes with part-time external jobs: monetary resources are required to recruit the best students and allow them to engage in research without the burden of a part-time job on the side. In particular, a research opportunity without funding for the student significantly limits the participation of students who rely heavily on financial aid. One often low-cost way to help students get involved in UR is by offering them free or reduced-cost summer housing.

(5) If possible, bring up the issue of internal funding for UR in the context of accreditation discussions. With accreditation issues fresh in their minds, administrators might be more willing to invest funds in extracurricular activities that improve the delivery of undergraduate education (such as UR). (See [Hensel 2012].)

(6) Establish a mentoring system whereby faculty members with experience in leading UR (and obtaining intra- and extramural funding to support it) coach their junior colleagues on effective bargaining strategies with administrators.

(7) Convince the development office (or any other office in charge of relationships with prospective and current donors) of the strong relationship between participation in UR during college and future success in the market place. Donors like to invest in initiatives that directly affect students' success rate in finding top jobs.

(8) Meet with a dean/provost/president or approach the sponsored projects office directly and discuss possibilities for cost-sharing and/or partial overhead return for extramural funding dedicated to UR. Useful forms of support include both cash (e.g., funding to support additional students, materials and supplies, pedagogical equipment) and in-kind (e.g., time release for faculty). The latter might be easier to obtain and is of critical importance in finding motivated faculty willing to supervise UR. This strategy goes beyond support of UR-only grants. For example, Cal State Fullerton offers time release (on a competitive basis) for faculty submitting extramural grants; since the dean is particularly appreciative of UR, there is incentive for faculty to include funding for UR in any research grant in order to become more competitive on their released-time application.

(9) Convince the administration that support of the faculty involved in UR is absolutely essential for the success of the program. This support may come in different forms, but will have to include recognition of efforts spent on UR in personnel cases (tenure, promotion, etc.). (See [Hensel and Paul 2012].)

(10) Convince the administration to establish a campus-wide centralized office for UR to maximize the visibility of the initiative and hence the impact on the reputation of the institution. Explain that most highly successful undergraduate programs are associated with a central office of UR, which oversees campus-wide UR activities, including but not limited to on-campus research symposia, summer research, student workshops, mentorship training, and disbursement of funds for student travel. Some UR offices award internally or externally funded summer research assistantships to students and/or to faculty. Such a center would retain all information on participating students, and allow effective statistics on impact, success in the market place, etc. Even more importantly, a centralized office would identify and coordinate potential multidisciplinary projects, and act as a unified "lobbying center" for intramural and extramural funding support. The establishment of a designated position for a UR program director provides a clear statement of the importance and expected potential of the UR enterprise on a campus.

5. Additional resources

Several resources exist that have a more in-depth discussion on ideas presented here. Three excellent places to start are the Council of Undergraduate Research (CUR) website (http//www.cur.org/publications/publication_listings), the *Report of the MAA committee on the undergraduate program in mathematics*, available at http//www.maa.org/cupm/CUPM-UG-research.pdf, and the MAA column *Resources for undergraduate research in mathematics*, which can be found online at http//www.maa.org/columns/Resources/resources.html.

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Experiences of working with undergraduate students on research during an academic year

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Over the last three years, I have worked with four undergraduate students on research during the academic year in addition to mentoring undergraduate students at the REU at Rochester Institute of Technology. All four of these students had taken proof-based classes with me. These students had a high level of mathematical maturity with excellent motivation and work ethic. In this paper, I share my experiences working with them during the academic year and share my principles in mentoring undergraduate students.

1. Introduction

I joined the School of Mathematical Sciences at the Rochester Institute of Technology as an Assistant Professor in Fall 2009, after finishing my PhD from Clemson University. At Clemson, I was fortunate to serve as a graduate student mentor for three undergraduate students participating in the Clemson REU. As an undergraduate, I never had an opportunity to experience undergraduate research. I enjoyed the experience of mentoring undergraduate students at the Clemson REU, and decided that as a faculty member I wanted to work with undergraduate students in research as well.

My area of research is graph theory, and in particular, graph labeling. I was assigned to teach proof-based courses (discrete mathematics and graph theory) at the Rochester Institute of Technology (RIT) during my first three years. All four of my students — Ryan Held, Samuel Kennedy, Christopher Wood and Shamalie Peiris — were in these classes. These students had a high level of mathematical maturity with excellent motivation and work ethic. From our work, we were able to submit and publish multiple manuscripts in refereed journals. Currently, two of these students are applying to graduate school, one of them is working in industry, and the other student is looking for a job in industry.

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Keywords: undergraduate research, student mentoring, $L(2,1)$ coloring, ranking.

In this paper, I share my experiences working with these students during the academic year. I describe how I met these students, the problems I selected for them, and the principles I used while mentoring them in research.

2. Finding students and selecting problems

I mentored four undergraduate students in research over the last three years during the school year. I met all these students in my classes. Some of them approached me looking for a faculty to work with them in research, while others I approached to work with me. All four of these students took at least one proof-based course, like discrete mathematics or graph theory, from me. One of the main advantages of finding students from classes you have taught is that you get a first hand sense of their mathematical maturity and work ethic.

My first student was Ryan Held who was in my discrete mathematics course. He approached me to ask if I can help him to find a research opportunity for that summer. While I was not able to find him an opportunity for that summer, I asked him if he would like to work with me during the next academic year. Ryan worked with me for two years, and we were able to publish two papers in peer-reviewed journals as coauthors [Held and Jacob 2011; 2012].

I had Sam Kennedy in my graph theory class during Spring 2011, and he applied to participate in the RIT REU in the summer of 2011. We accepted Sam into our REU program without hesitation knowing that this could lead to successful collaboration beyond that summer. I mentored Sam and another student during the REU. After the REU, I asked Sam if he would like to continue working on the problem with me during the academic year, and he is still working with me on research. We submitted our results [Dobosh et al. 2013] to a refereed journal.

Chris Wood was in my graph theory class in the fall of 2011. He approached me toward the end of the quarter to ask me if I could offer advanced graph theory as an independent course. He was not a math major; however, he had a good appetite for mathematics. I agreed, but, suggested that he register for undergraduate research instead of independent study, because I knew he was capable of doing research. He was thrilled that I made the suggestion, and worked with me for a year on research. We submitted two papers for publication in peer-reviewed journals [Wood and Jacob 2013a; 2013b].

The fourth student, Shamalie Peiris, was in my graph theory class in Spring 2012. She did extremely well in class, and her attention to detail was extraordinary. I asked her if she was working on research with anyone, and when she said that she was not, I offered to mentor her in research. She was very interested, and we started working on a problem in the fall of 2012 and made significant progress.

To each of the students, I gave different problems, even though there was some overlap. I was not convinced that working on the same problems with them in a group would be a good idea, and I am glad that I did not give them the same problem. This way, they made significant progress on the problem I gave them. In general, I did not give them a set of problems to work on, but rather had them each focus on a single problem. To each one, I presented the problem and gave a simple question or two to look at, and asked them to come back the following week with their results. I also reminded them that if they found the problem uninteresting or too hard, then we would change the problem. However, I stayed away from giving them a choice of problems in the beginning.

The problems I gave them were in graph labelings, my area of research. I asked Ryan to look at the irreducible no-hole $L(2, 1)$ coloring of some graphs. He was thrilled about the problem, and he generalized his ideas to $L(h, k)$ labeling. Sam continued his problem from the REU — to find the rank number of some classes of graphs. He was able to find rank numbers of many classes of graphs that are Cartesian products of some graph with a complete graph.

Chris was not a math major; he was a computer science and software engineering major. While he was very good at proofs, I figured problems that involve computational components might suit him better. I asked him to study a 20-year-old conjecture on the $L(2, 1)$ span of trees, suggesting that he consider the conjecture from a computational angle. He "ran with it" using a program called Nauty to generate large numbers of trees, and combed through those trees using the subroutines he developed. While I was confident that he would have been successful with a problem that did not involve a lot of programming, this problem suited him well as we were able to use his programming experience as an asset.

Shamalie started working on a ranking problem. She was able to come up with solutions to the questions I asked her. However, the next level of problems I gave her was a little too hard. She tried hard, and looked at solving the problem from different angles, but we were not able to make progress and were running out of ideas. So I decided to change the problem to finding a parameter called the *azero-forcing number* of some graphs. She is making good progress on the problem. I was able to change the problem at the right time and keep her interest in research.

3. Working with the students during the school year

Typically, I meet with the students once a week to discuss their progress and suggest some ideas. As a graduate student, I served as a mentor for three undergraduate students during the Clemson University REU. One lesson I learned there was that it is better, if possible, to let the students identify issues through their own efforts rather than tell them how to proceed. That approach has worked well for me so far.

When I meet with my students, I ask them about their progress. Usually each of the students has made progress on the problem, and has ideas on how to proceed. There are times when I feel that their ideas are not going to be successful; however, I almost never tell them that. For me, this approach has many advantages. One is that the students come to see the issues themselves, and understand the shortcomings of a given approach much better than if I point them out. I feel that it is part of the research process to get stuck on ideas, and to go back to the drawing board, and I constantly remind them that this is part of the process. The added advantage of this approach is that when I am wrong about their ideas not working out, I don't have to eat my words. However, if it turns out that I am wrong in the beginning about their ideas not working out, then once they succeed, I tell them that I was not convinced in the beginning about their ideas. This approach allows me to convey to the students that just because I am a faculty member, I am not always right about which ideas will work out. The best way to find that out is to run with the ideas.

One of the challenges I face, at least in the beginning, is making sure that the students understand the difference between doing research and doing coursework. I convince them that the main difference is that the problems I give them for research might not be solvable, and that I certainly do not have solutions to them. I tell them that just like them, I am working on this and don't know how to solve the problems. So they know that when I make a suggestion, that does not mean that it would work out. One piece of great advice I received as a graduate student is that research is about making baby steps. Very rarely does someone take a giant step in research. I make sure to tell my students this, especially when they think that they are not making progress. I remind them about the progress they made since they started working on the problem, and usually that makes them feel better, and rightly so.

I meet with these four students separately, usually in my office. However, sometimes we meet in a different room with larger white board space. For example, Chris likes to talk about his research by explaining on the white board, so when I meet with him I usually meet in a room with plenty such space. The duration of meeting times varies: it can be anywhere from 15 minutes to 2 hours. Meetings with Chris, for example, tend to take an hour, but have occasionally gone to two hours or more. During the meetings, the students explain their progress on the problem. If they are looking for ideas on how to proceed then I may give them a couple of suggestions. I usually remind them that these ideas may not work. Also, to make them feel comfortable, I usually ask them a few questions about their interests and classes.

Apart from working on the problems, there are two rules I ask them to follow. If, for any reason, they have to cancel a meeting, then they should send me an email message so that I know about it. The other one is that they should write down or type up their research every week, even if the proofs or results are not correct. I encourage them to keep a research notebook, and encourage them to carry it with them as much as possible. I strongly suggest to them that they work on research in the notebook, so that they have a record of all the work. I am happy to say that these four students have exceeded my expectations in every way.

4. Conclusion

I enjoy working with undergraduate students in research. The students I worked with were motivated and had a good work ethic. Even though most of these students approached me to do research, in some sense I was able to select them from my classes and knew about their mathematical maturity and work ethic before I started working with them. We were able to submit and publish multiple manuscripts as coauthors. These students told me that these experiences were helpful to them, and, more importantly, that they enjoyed them too.

As an undergraduate student, I did not have any opportunity to do research, and in fact, I did not even know about research until I was a graduate student. As a faculty member, I enjoy mentoring undergraduate students in research. These experiences undoubtedly are some of the highlights of my career so far. And as a byproduct, mentoring and publishing papers with undergraduate students are excellent line items on my resume. I am looking forward to my future years working with undergraduate students on research.

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The role of graduate students in research experience for undergraduates programs

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In this article, we consider the role of graduate students as mentors in research experience for undergraduates (REU) programs, as reflected by a breakout session at the Trends for Undergraduate Research in Mathematical Sciences (TURMS) conference. We consider the benefits of using graduate students to the institution running the program and to the participating undergraduates. We also consider the benefits that the graduate students themselves gain from working in an REU, and we warn of potential problems that can arise when employing graduate students in this context. We discuss the role of postdoctoral fellows and other undergraduates in REU programs and conclude with questions about graduate student mentors that merit further discussion.

1. Introduction

At the Trends for Undergraduate Research in Mathematical Sciences (TURMS) conference, there was a breakout session that discussed the role of undergraduate students, graduate students, and postdocs as mentors in undergraduate research. The discussion focused primarily on the role of graduate student mentors, so that will be the primary focus of this article as well. We begin by considering the role of graduate mentors in undergraduate research and the benefits that they provide to both the research program and the undergraduate participants. Next, we consider the benefits to the graduate students in being mentors. After this, we discuss some concerns and possible problems with using graduate students in a research program. We then turn to the roles of other undergraduates and postdoctoral fellows in the undergraduate research experience. Finally, we conclude with some additional questions regarding the use of graduate students in research experience for undergraduates (REU) programs.

MSC2010: 00B20, 00B25.
Keywords: research experience for undergraduates, graduate students.

2. The role of graduate students

Graduate students have two broad roles in an REU setting: the first is administrative, and the second is mathematical and technical support. In their administrative role, graduate students plan and attend social events, spot potential compatibility problems, and are often available when faculty members are not accessible. During our breakout session, many REU organizers cited examples of graduate students planning lunches and weekend outings when the faculty do not have time. Since the graduate students are closer in age to the undergraduates and, in many cases, live with the undergraduates during the program, graduate students are more likely to spot potential personal problems that a student may have. Graduate students may also be able to detect compatibility problems within the group work environment. A distinct advantage of having the graduate students live with the undergraduates is that the graduate students are more likely to be accessible at times when a faculty member is not available. Occasionally, some REU directors in our session have to leave their program for a couple of days, during which time graduate students are able to keep the program running smoothly.

In their mathematical and technical support role, graduate students perform many functions that are vital to the success of an REU. Graduate students help provide the undergraduates with mathematical background and guide them in their research projects. In most programs graduate students provide computer training for the undergraduates. For example, many REU directors have their graduate students teach the participants LaTeX, Beamer, or any other computer programming skills that are needed to complete a project.

Some REU mentors in our session stated that the research area of the graduate students is less important than the ability of the graduate mentors to teach the technical aspects of a project, such as LaTeX. At the conclusion of an REU, graduate students can help train the undergraduates in presenting their work, listen to practice talks, and give feedback. This is particularly valuable at large REUs, where faculty members might not be able to listen to every practice talk. Graduates can also assist undergraduates with the writing and editing of REU papers. Since this process may take a significant amount of time (up to a year, or longer, in some outstanding cases), having graduate assistance for an extended period of time is invaluable.

We conclude this section by noting that graduate students do not need to stay at the REU for the full length of the program in order to have a positive effect. In particular, at the University of Minnesota Duluth, Joe Gallian has some graduate students come to his program as one-week visitors. The task of the visiting graduate student is to make a personal connection with one or two of the undergraduate participants.

3. Benefits for the graduate students

In a session populated with REU directors, it is not surprising that the first potential benefit mentioned for graduate students is that they will be more prepared to direct REUs themselves in the future. While this is certainly true, there are many other benefits. In a job market that seems only to get more competitive, participation as an REU mentor provides graduate students with practical experience that enhances their standing as a job candidate. As more colleges and universities stress undergraduate research, any experience with undergraduate research (such as participating in an REU as a mentor) will help a candidate immensely. As a short term benefit to a graduate student, participating in a research project can help jumpstart a Master's thesis or a dissertation. In short, the experience that graduate students gain in advising undergraduates can only help them in their future careers.

4. Concerns and potential problems

While the use of graduate students has many benefits for everyone involved, our session did address some concerns and potential problems with using graduate students as mentors in an REU. In order to begin their work, graduate students must receive proper training beforehand. In particular, graduate students should have their roles clearly defined by the faculty members who are directing the REU. Faculty directors need to describe explicitly what is expected of the graduate students during the program. Here are some examples of expectations for graduate students that were mentioned: do not dominate a project; assist undergraduates with work on a project but don't solve it completely, and make sure this work is treated as a priority. The REU director should clearly specify the role of the graduate mentors, whether they are meant to be a coauthor or just part of a student support system. Finally, if possible, graduate assistants should be chosen who are not under pressure to study for qualifying exams or finish a dissertation, as these students may be too distracted to fulfill their role as a graduate mentor.

5. Postdoctoral fellows and other undergraduate students

In addition to graduate students, our session also briefly discussed the roles of postdoctoral fellows and other undergraduates as mentors in REUs. The REU directors in our session said that postdocs generally act in a role closer to faculty than graduate student. Both the Iowa State and DIMACS REUs have used postdocs in this way as project mentors. However, there was a range of other experiences with postdoc involvement in REUs. For example, one former postdoc in our session participated in the MSRI-UP REU where his role was more that of a graduate student mentor than a faculty member. It seems that the greatest benefit of involving postdocs in REUs is that they are able to give graduate students the necessary training that was mentioned previously.

Participants in our session had varying opinions on the use of undergraduates as mentors at REUs. Most said that they would use undergraduates only in an administrative role and not in a mathematical mentoring role. While some REU directors do involve undergraduates in administrative work, other directors think that it is better for undergraduates to participate in another, different REU. One director said that he invites undergraduates who have already participated in his REU back to serve in an unofficial mentoring role. He does this for two reasons: first, if the student did not finish a paper for the previous REU, then bringing the student back can be an effective way to motivate them to finish their work. Second, this director sees value in bringing exceptional participants back, as this can be part of a grooming process to prepare these students to direct future REUs.

6. Concluding questions

(1) If an REU site does not have graduate students, what is the best way to choose graduate students from other institutions? It seemed that most directors in our session use alumni from their own programs (as Joe Gallian from the University of Minnesota Duluth put it, his alumni have an "institutional memory" that he does not have to teach). However, the Denison and Valparaiso REUs have experimented with recruiting graduate students from larger universities nearby.

(2) What is the best way to train graduate students for their participation in an REU? We discussed the goals that should arise from the training, but we did not discuss logistical issues, such as how early the graduate students should arrive before the program.

(3) In our session, our discussion focused on REUs. What roles can graduate students have in other methods of undergraduate research? In particular, do graduate students ever formally mentor undergraduate research during the academic year?

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An unexpected discovery

Erika L. C. King

(Communicated by Darren A. Narayan)

One summer, I chose two undergraduate students to work with me on a research project. Our goal was specifically to find a new approach to proving a theorem I had already proved several years before. We were looking for a new approach because the proof I had written is too long for publication, but the result itself is interesting. As is common in mathematics, our work led us to an unexpected discovery. This article leads the reader through our journey.

In 2010, I enlisted two students, Trevor J. Gionet Jr. and Yixiao Sha, to work with me for eight weeks in the summer on a research project funded through our provost's office. What follows is the story of our research journey. Our project was in the field of graph theory. I will use terminology from graph theory, though it is not necessary for you to know it to follow our adventure.

At the start of the project, Trevor had just completed a first graph theory course with me. Yixiao had never had graph theory, but had read through portions of the textbook [Chartrand and Zhang 2005] that had been used for Trevor's graph theory course. Our goal was to take a long proof I had written about ten years previously and make it shorter. The proof I had was over 350 pages long and therefore virtually unpublishable. However, some graph theorists thought the result it proved was interesting enough that they encouraged me to rework the proof and get it published. I hoped that if some students worked with me, they could offer some fresh ideas to help create a lemma or two (or five!) that would shorten the proof.

The result in question classifies a set of graphs with certain properties. I gave Trevor and Yixiao the statement of the theorem.

Theorem 1. *There are precisely seven connected*, 4*-regular*, *claw-free*, *well-dominated graphs.*

However, I did not give them the list of those seven graphs. In order to gain an understanding of what graphs with these properties look like, I wanted them to experiment and try to find the seven graphs on their own. So their first assignment was to find as many graphs as they could with these four properties.

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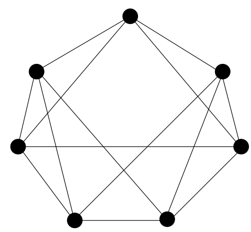
Keywords: undergraduate research, well-dominated graphs, well-covered graphs, claw-free, regular.

At our next meeting I asked them what they had found and whether they could justify that their graphs had the properties we were looking for. Yixiao and Trevor both had graphs to share. For one of his examples, Trevor drew this graph on the board:

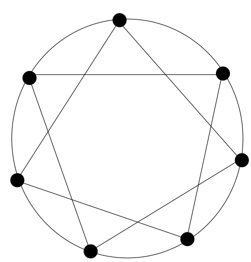


I asked him to be more and more specific about why this graph fit into our class of graphs. We went on to discuss other graphs and ideas. After a while, we finished our meeting. They left to do more exploring. I stared at my board. I had not told them, but this graph was not on my list of seven. As you can see, it is a small graph, so checking the properties was not difficult, but I did it several times before I sat down to write Michael Plummer and Bert Hartnell.

Trevor's graph is not just connected, it is 4-connected. My long proof had attacked this classification by breaking the possibilities into cases by connectivity. Since the graphs are 4-regular (meaning each vertex has a degree of four) and connected (meaning there exists a path between every pair of vertices in the graph), we know they are at least 1-connected and no more than 4-connected (we need to delete at least one vertex, but no more than four vertices, to disconnect the graph). This approach had given me four cases. More importantly, there was already a result published in *Discrete Applied Mathematics* by Hartnell and Plummer [1996] classifying 4-connected, 4-regular, claw-free, well-covered graphs. This was helpful since Finbow, Hartnell and Nowakowski [Finbow et al. 1988] showed that well-dominated graphs are a subclass of well-covered graphs. This meant that for one of my cases (the 4-connected one) I needed only to determine which well-covered graphs in Hartnell and Plummer's classification were also well-dominated. But since Trevor's graph was 4-connected, it not only should have appeared in my result, it should have appeared in Hartnell and Plummer's well-covered result as well. It did not. Hence, my impulse to write graph theorists Michael Plummer and Bert Hartnell.

Both Hartnell and Plummer responded quickly that we were correct and the graph should have been in their characterization. Furthermore, Plummer realized that the error was actually in an even earlier paper he had written on classes of claw-free graphs [Plummer 1995]. In the earlier paper, Plummer characterized 4-regular, 4-connected, claw-free graphs. He and Hartnell then used that characterization to determine the well-covered ones. However, Plummer had neglected to consider a case which included some graphs with an odd number of vertices. This was somewhat understandable given the topic of his paper primarily concerned graphs with an even number of vertices. However there were now two results based on his incomplete characterization and those results together with his characterization needed revising.

I shared the news with the students and their surprise and excitement were palpable. The goal of our summer project shifted to working on trying to complete Plummer's original characterization of the 4-regular, 4-connected, claw-free graphs, and to revising the Hartnell and Plummer result on well-covered graphs that followed. The students worked hard and we discovered that Plummer's characterization actually omitted an *infinite number* of graphs of odd order! However, Plummer's proof was still quite good and we needed only to rework a small portion of it to incorporate the missing graphs. In the process, Trevor realized that all the graphs in the characterization, including the original graphs and the new ones, were in a class of graphs we had read about in our textbook [Chartrand and Zhang 2005]. The textbook authors, Chartrand and Zhang, called this class Harary graphs. These graphs are very symmetric. Here is a redrawing of Trevor's graph, which is called $H_{4,7}$ in the class of Harary graphs, that makes the connection to Chartrand and Zhang's construction clearer.



$H_{4,7}$

By the end of the summer, we were able to complete Plummer's characterization of 4-regular, 4-connected, claw-free graphs, and revise Hartnell and Plummer's result about which of those are well-covered. Interestingly, even though there were an infinite number of odd graphs omitted from Plummer's characterization, *only two* of those graphs are well-covered ($H_{4,7}$ and $H_{4,11}$)! Both of the new well-covered graphs are also well-dominated, so now the theorem I first handed the students has been revised as follows:

Theorem 2. *There are precisely nine connected*, 4-*regular*, *claw-free*, *well-dominated graphs.*

We published a paper with our results, including the two revised results and a theorem classifying 4-connected, 4-regular, claw-free, well-dominated graphs, in *Discrete Applied Mathematics* [Gionet et al. 2011]. It was an exciting adventure and taught the students that although we start working on one problem, our research may lead us down a different path. Now back to figuring out how to shorten this 350 page proof!

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Alternative resources for funding and supporting undergraduate research

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At a time when funding for programs on academic campuses around the country is tight, financial support for undergraduate research has also become increasingly difficult to find. We discuss some suggestions for funding and supporting undergraduate research programs in mathematics from the 2012 "Trends in Undergraduate Research in Mathematical Sciences" conference held in Chicago, October 26–28, 2012.

1. Introduction

The "Trends in Undergraduate Research in Mathematical Sciences" (TURMS) conference was held in Chicago, October 26–28, 2012. Members of the mathematical community came together with the common goal of fostering research programs in mathematics for undergraduate students.

At a time when funding for programs on academic campuses around the country is tight, financial support for undergraduate research has also become increasingly difficult to find. Funding for Research Experiences for Undergraduates (REUs) has traditionally come from the National Science Foundation (NSF). However, with the NSF facing budgetary constraints in the coming years, funding for REUs is uncertain and the number of REU sites has remained relatively flat over the last few years despite increasing interest from institutions wanting to host such programs. Jennifer Slimowitz Pearl, program director in the Division of Mathematical Sciences at the NSF, stated at the conference that the demand for REU funding from the NSF currently exceeds the ability to fund programs. She went on to say that currently there are great proposals that go unfunded simply for the fact that there is not enough room in the budget to support them. Furthermore, the budget for 2013 is not expected to increase. It is therefore necessary for institutions interested in funding undergraduate research to explore doing so by obtaining funding from sources outside of the NSF.

MSC2010: 97-06.
Keywords: undergraduate research.

Some government organizations, such as the Department of Defense, contribute funding for undergraduate research. These organizations give funding to the NSF instead of awarding grants directly, and it is the NSF that decides on and awards this funding to institutions. Representatives of the NSF have stated that they would be willing to work with additional organizations in this manner.

Some participants of the TURMS conference asked NSF representatives if it would be possible for host institutions to cost-share on REU expenses with the NSF. The representatives answered this question in the negative. One reason behind this decision is concern that the success of a proposal would be based on the amount of institutional cost-sharing in the budget and not necessarily the merit of the proposal.

In this article we describe possible answers and potential challenges to two questions of Pearl during her presentation in the opening banquet of the 2012 TURMS conference:

- How can undergraduate research be assimilated into the curriculum without outside funding?
- How can funding agencies structure an initial investment in an undergraduate research program (REU or otherwise) so that some parts of it are sustainable after the grant money ends?

2. Research in the curriculum

Many participants of the TURMS conference shared their experiences incorporating undergraduate research as part of the mathematics curriculum, both as explicit research courses and senior theses, or as courses centered around a model of inquiry-based learning. We present some discussion on both of these models in this section.

Research as a graduation requirement. One example where undergraduate research has been included as part of the curriculum is at East Tennessee State University (ETSU), where all math majors must complete a one-semester undergraduate research course. Anant Godbole, former math department chair at ETSU, explained that this course is possible both by the relatively small number of math majors and the willingness of faculty to advise projects in addition to their usual teaching responsibilities. At ETSU, Godbole reports that about half the faculty have volunteered to advise at least one student. The research experiences range from projects resulting in publications for the most able students to novel presentations of fundamental mathematical ideas for weaker students. Godbole reminded the group that even “C” students have to complete the course and these students also deserve a suitably challenging project. Another participant from Wartburg College mentioned that the Physics department there tried a similar model but found that the effort needed to run such a program was unsustainable. At some liberal arts colleges, a research-type experience is also a graduation requirement. For instance, at Harvey Mudd College, mathematics majors must either complete a senior thesis or participate in a clinic project, a team project where students consult for an outside company to solve a real-world problem over the course of a year.

Inquiry-based learning. Inquiry-based learning (IBL) can be used as an educational tool to help students gain research experience in the classroom without the manpower needed to implement the ETSU model. Michael Starbird, professor at the University of Texas at Austin, proposed that IBL courses share many of the same learning outcomes with undergraduate research. In an IBL course, students are asked to work on problems to develop concept knowledge and problem-solving skills. At the end of a well organized IBL course, students should develop an ability to understand a research question, find strategies to solve a problem, work in teams, and learn how to raise questions in the process. Such IBL courses could serve as mini-REUs and translate research experience into the classroom. Incorporating IBL techniques into an undergraduate curriculum can provide a cost-efficient way of introducing research skills to a wide audience who might not otherwise be involved in undergraduate research in mathematics.

IBL courses bring together students from different backgrounds and with a wide range of skill sets to work and communicate as a team to solve a problem. Such classes might not only attract mathematics majors but also students who are just interested in mathematics. Consequently, instructors have more flexibility to introduce interdisciplinary research questions in the course. IBL techniques might be implemented in many different courses. However, incorporating IBL teaching methods for some courses might be challenging. Developing an IBL course requires careful planning and preparation by the faculty member. Some research questions might not be suitable to capture the essence of an IBL experience and might not attract students' interest. Since IBL course registration is open to all eligible students, instructors might spend some time motivating and encouraging students during the term. In contrast, the REU selection process filters for students who are already motivated and interested in mathematical research. Starbird states that benefits of this program include that in an IBL class, each day students are "overcoming the unknown" and sometimes experiencing the "joy of success" of solving a challenging problem new to them.

3. Creating sustainable research programs

If funding for undergraduate research continues to be scarce, then mathematics departments wishing to foster undergraduate research will be forced to find sustainable sources of funding. Departments with graduate programs could use graduate students as mentors for undergraduate research without placing additional financial burden on the institution. Research mentors could also look to the local community to find support for undergraduate research. Participants of the TURMS conference also brought up the concern for compensating faculty and students who participate in undergraduate research. We discuss these issues in the following section.

Graduate students as research mentors. While graduate students are not the sole mentors in many REUs, their participation is essential in organizing larger REUs with a variety of different research projects. The workload in maintaining a REU program for a large group of undergraduate students can be overwhelming. As Leslie Hogben, professor at Iowa State University, explained in her presentation, a graduate assistant serves as a bridge between the faculty mentor and the student research groups in the REU process. They make sure that students are meeting the expectations and working effectively in a collaborative environment. If there is a problem in the team dynamics, it has been Hogben's experience that students are more likely to share it with a graduate assistant than a faculty mentor. Students might also feel more comfortable talking to graduate assistants about graduate schools and mathematics as a career option. Serving as mentors in REUs will help graduate students to become better teachers and researchers in the future. During the term of the REU, they will discuss a variety of research topics with a diverse and wide audience. Such exposure early in their careers motivates them to improve their teaching methods, benefiting both the graduate students and undergraduate students at once. For a more detailed account of the benefits of graduate student mentors for undergraduate research, see [Bliss and Isaksen 2000; Hartke et al. 2007].

Community-based learning. If the federal government will not fully support undergraduate research, then the mathematical community must look to other sources of funding. One source of sustainable research problems and funding is community-based learning. In community-based learning local individuals or organizations consult with an institution and engage students to solve a problem related to their business or nonprofit organization. One benefit of this is that students get to work on real-world problems that directly impact their community. At some institutions, such as Davidson College, there is a Center of Civic Engagement that promotes courses that incorporate community-based learning assignments. They provide a small stipend (typically around $1500) to encourage faculty members to add community-based aspects to their classes and compensate them for the additional preparation time these projects entail. Organizations that come to an institution for advice may also be asked to provide support for students to present their work at an undergraduate research conference.

Another way students can receive an experience akin to mathematical research is through internships. At Worcester Polytechnic Institute there is a graduation requirement where students of every major must participate in an internship or consulting experience with business partners either locally or through established global partnerships.

Compensation for student and faculty research. One major concern of many conference participants is that their current institutions do not have the funds or are unwilling to compensate faculty directly for work with students during the academic year. Even small amounts of monetary compensation can indicate to a faculty member that their efforts are appreciated by their institutions. One way that faculty can be rewarded is via the tenure and promotion process. Another suggestion was that students could be paid for conducting research as a form of work-study.

Departments may be able to receive funding from alumni and corporate sponsors. Darren Narayan, professor at the Rochester Institute of Technology, gave the example of getting JetBlue to sponsor student travel and asking alumni to sponsor a student. This approach could eventually turn into an endowed fund providing a perpetual and sustainable source for student research funding. It may also be the case that alumni are more willing to give when they see the direct benefit of their contribution. Narayan also suggested that it is more effective to contact alumni with opportunities to support student travel to conferences, with a tangible purpose of their donation. Faculty interested in these kinds of initiatives should coordinate with their development office.

4. Conclusion

The level of participation at the 2012 TURMS conference shows anecdotally how far undergraduate research in mathematics has come in the past two decades. Nearly 500 undergraduates presented posters at the 2013 Joint Mathematics Meetings in San Diego, California. Joseph Gallian [2012] states unequivocally that "more REU-like summer programs, more academic year opportunities for undergraduates to engage in research, and more undergraduates attending conferences and presenting" are the future. However, it seems that it will take greater effort to find funding for this research. Whether the money is to fund an REU site, to pay students and faculty for institutional summer research programs, or to help undergraduates and their mentors travel to conferences to share their findings, the mathematical community must be creative in seeking out this financial support. Inquiry-based learning-type classes could be used to help foster the same kind of problem-solving skills and senses of mathematical discovery that are the intended goals of undergraduate research. Reaching out to the community in the form of service-learning and internships could lower the burden on the institutions and the government.

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Academic year undergraduate research: the CURM model

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The Center for Undergraduate Research in Mathematics (CURM) provides funding and training for mathematics faculty to engage groups of students in academic year research. This paper provides an overview of the CURM model and its impact on mathematics students, faculty, and institutions across the country. We also present three case studies describing the transformational effects of CURM mini-grants at three markedly different institutions.

1. Overview of CURM

The NSF-funded Center for Undergraduate Research in Mathematics (CURM), founded in 2006 by Michael Dorff of Brigham Young University (BYU), awards mini-grants to fund academic year undergraduate research groups led by faculty at institutions nationwide. The CURM model draws on successful practices at BYU where student researchers across disciplines produce an impressive array of new results each year. The core of the CURM model is straightforward:

- Train faculty mentors to lead students in research projects.
- Provide funds for one course reassignment so faculty have time to effectively mentor students. Provide funds for supplies and travel.
- Pay students so that research becomes an alternative to other employment. Demand 10 hours per week per student on the job.
- Encourage students to work in groups.
- Celebrate student research success during a Spring Research Conference.

Faculty training. When awarding faculty mini-grants, CURM strives to balance experience, type of institution, gender, and ethnicity within each faculty cohort. A substantial number of mentors each year come from institutions where students might not otherwise have access to research.

MSC2010: 97-06.
Keywords: undergraduate research.

Each cohort begins the minigrant program at a Faculty Training Workshop held during summer at a venue where awardees share rooms, cook and eat together, and hold formal and informal discussions. Formal discussion topics cover a wide field, including how to select a good research problem, what to do when you realize your research problem is too challenging, how to handle interpersonal disputes between group members, where to publish student work, where to find additional money to fund research groups, logistics of running a research group, how to support struggling students, and how to find/select students [Leonard 2008] (see also maa.org/external_archive/columns/Resources/resources.html).

The rich content of the discussions comes in large part from the diversity of perspectives represented in each faculty cohort. Relationships built during the workshop often evolve into lasting friendships or collaborations.

Research group. Faculty must identify potential research group participants when applying for a CURM minigrant. As a result, they often reach out to students who might not otherwise know about student research opportunities. As with selecting faculty mentors, CURM strives to balance students with regard to gender, ethnicity, and economic opportunity. Additionally, CURM encourages faculty to recruit students as early in their academic progress as possible.

Students are required to work 10 hours per week for the academic year, approximately thirty weeks, and are paid \$3000. At \$10/hour, the CURM stipend is equivalent in pay to a typical campus job. Paying students for research affords the time for students who must work during college to participate in research. Students almost always work in small groups, enabling projects to move forward even when one group member must attend to other academic responsibilities. Often, CURM research groups evolve into self-organized learning communities where students enroll in the same courses, study together, and socialize together long after the project ends.

The precise structure of a research group varies from mentor to mentor, but a typical structure involves at least one weekly group meeting with the faculty mentor, one group meeting without the faculty mentor, and individual student meetings with the mentor. Faculty oversight often decreases as the year progresses and the students learn intellectual independence. CURM advises faculty mentors to require weekly presentations and weekly papers so that students can build their presentation and writing skills gradually over the funded period.

Students must submit a final research paper to CURM in May in order to receive their last paycheck. Ideally, these final research papers serve as a draft for a future publication.

Spring research conference. Mid-March, CURM students and faculty travel to BYU for a two-day research conference. Students present their results in 20-minute talks and receive extensive feedback from faculty reviewers in the audience. Keynote speakers known for their ability to engage students give one-hour presentations on current mathematical topics. Half-hour "What is…" presentations introduce students to concepts like cryptography, knot theory, or operations research. Panelists describe graduate school applications and industry employment opportunities. Students interact with each other through ice-breaker activities, hikes, and a pizza party.

For CURM students at smaller, more isolated schools, the Spring Research Conference offers a series of firsts: the first visit to a research university, the first mathematical conference, the first research presentation, and the first conversations with other student researchers. Even for more experienced students, the CURM conference is often their first conference aimed entirely at students.

Outcomes, 2006–2011. During the first grant period (2006-2011), CURM funded 64 faculty mentors and 195 research students from 54 distinct institutions. Among faculty, 41% were female and 19% were from ethnic/racial groups traditionally underrepresented in mathematics. Among students, 54% were female and 29% were from traditionally underrepresented groups. CURM students generated 60 research papers, 158 conference presentations, and 29 awards. Graduate school attendance rate among CURM graduates was 63% as compared to the national average among math majors of 18% [AMS 2009]. Several CURM faculty alumni have successfully applied for larger NSF grants to support student research through programs such as REU, UBM, MCTP and CAREER. Others have succeeded with private foundations such as the Keck Foundation.

Subtler impacts also surfaced. Based on a survey administered to students before and after their CURM experience, CURM students' perception of interest in studying mathematics rose from just under 60% to just over 80%, and perception of interest in graduate school rose from just under 50% to 80%.

CURM's transformational power is perhaps best captured through anecdotes from participants:

- *CURM has opened many doors for my future. It encouraged me to apply for [be accepted to, and attend] a summer* 2008 *REU … If it weren't for CURM, I wouldn't be where I am today. I wouldn't know what it meant to do research, and I wouldn't be applying for graduate school.*

- *Two of my students were funded by the CURM grant for doing undergraduate research. Being from a minority institution these students were not exposed to this kind of work/project before. This work made them more disciplined, organized, and independent researchers. Also, participating in the CURM research conference in March of* 2009 *enhanced their communication skills significantly (note that English is not native language for either of these students) and thus, helped to improve their confidence levels.*

• *For several years the math department... has wanted to start encouraging interested students to do undergraduate research. However, they never got past wanting the research to happen, since faculty didn't have the time, energy or ideas to start undergrad research projects, and students didn't know to ask about the opportunity of doing research in mathematics. Because of the CURM grant, I was able to work with a large number of students* (7 *total, while only* 2 *were supported by CURM*). *All it took was this one year of the CURM grant to fan the fire, and our department has begun to foster an environment that encourages undergraduate research. This coming year there will be* 4 *professors working with students or groups of students on research projects.*

• *Another important impact that I believe the CURM experience had... is at the institutional level. We are in the process of revising our general education requirements and there is interest of somehow including a component or option related to research and scholarly activity. Recently, more external funding such as the CURM funding has been coming in... This has greatly helped us gain the support of the administration for the recognition of the educational value of undergraduate research.*

• *Before joining the CURM team in* 2007, *I had directed undergraduate research projects, so considered myself an experienced faculty in this matter. But through your CURM program, I realized that there is so much more to learn!... A very beneficial part I consider from the CURM program is the opportunity to collaborate/communicate with other faculty mentors in the program. Most of us are from undergraduate universities, at the similar stages of career, and facing the similar challenges... The network formed for this group kept active throughout the year and it was a very useful resource.*

In the next sections, we present case studies of the CURM model at three institutions: a large public university (close to 40,000 students), a small public university (around 3500 students), and a historically black university.

2. Case study: California State University, Fullerton, a large public university

Context. We begin by providing a quick snapshot of California State University, Fullerton (CSUF) as a whole (see fullerton.edu/analyticalstudies):

- Serves Orange County, located 25 miles from downtown Los Angeles
- Second largest campus in the California State University (CSU) system
- Over 37,000 students in its bachelor's and master's programs
- Among students who are US citizens or permanent residents: 57.0% women, 43.0% men

- In the same group, 37.2% self-identify as Hispanic, 31.9% white, 23.7% Asian and Pacific Islander, 4.1% multi-race, 2.8% African American, and 0.3% Native American
- Hispanic Serving Institution that is first in California and fifth in the nation in awarding baccalaureate degrees to Hispanic students [Cooper 2011]

This larger picture is reflected in the mathematics department. In Fall 2011, out of a total of 280 declared math majors, 128 (79 female) were Hispanic, African American or Native American, with the overwhelming majority being Hispanic (121; 71 female). The diversity in the mathematics department is one of its strengths.

The teaching load in the mathematics department is typically between 11-12 contact hours (3-4 courses) per semester, which makes it difficult to maintain a student-centered research program without assigned time for research with students, especially in a department that also has research expectations. The socio-economic situation of the student population often requires them to work many hours outside of their coursework. The demographics of the students and the high teaching load for faculty create conditions where a program like CURM is able to have a significant impact.

Prior to funding through CURM, most of the student-faculty research projects were carried out by faculty in addition to their teaching duties and without any training on effective research mentoring. In very few cases, faculty had external grants from NSF and the National Institutes of Health (NIH) which included funding for undergraduate students, but those grants are rare even now. NSF and NIH grants also did not provide any training for faculty in best practices for mentoring undergraduates in research.

The CURM research stipend for students, as well as training and reassigned time for faculty, directly addressed the challenges for mentoring undergraduates at an institution like CSUF.

CURM grants at CSUF. Three separate student-faculty collaborations have been supported at CSUF through CURM mini-grants:

- Gulhan Alpargu (CURM 2009-10) mentored a project in statistics titled "Microarray Gene Expression Analysis" with two students (Kirsten Cunanan and Suzette Puente).
- Scott Annin (2009-10) mentored a project in pure mathematics titled "On kth Roots in the Symmetric Inverse Monoid" with two students (Troy Cannon and Carlos Hernandez).
- Angel Pineda (2010-11) mentored a project in applied mathematics titled "Statistical Modeling of the Fat Fraction in Magnetic Resonance Imaging

(MRI)" working with four students (Kevin Park, Anne Calder, Eden Ellis and Li-Hsuan Huang).

The CURM projects at CSUF from Fall 2009 to Spring 2011 were instrumental in raising the level of interest in student-faculty research collaborations in the department. Faculty leading the projects improved their mentoring skills. Students presented their work at national meetings, published their results, and most attended graduate school. In addition, the CURM grants increased interest throughout the department, from both students and faculty, in undergraduate research. Consequently, there is now an undergraduate research course offered in the department which provides assigned time for faculty to mentor students. During Spring 2012, there were 19 student-faculty research projects (involving 9 faculty members) which generated the equivalent of three 3-unit courses for faculty. There is also additional assigned time being provided by the administration. We do not have reliable records of student-faculty research activity before the CURM grants, but anecdotally the change is dramatic.

The awareness that there is external funding available for mentoring students has also increased the efforts from the faculty to find such funding for students. Currently, mathematics students are funded by LSAMP (lsamp.fullerton.edu) and the Minority Access to Research Careers (MARC) program at CSUF (marc.fullerton.edu). There has also been increased submission of external grant proposals for additional funds to mentor students.

The CURM mini-grants provided not only the support for three research projects, they also created a cultural shift within the department by increasing awareness of the possibilities available locally and nationally to support student-faculty research. CURM has served as a catalyst to materialize our potential for developing an excellent undergraduate research program.

In the next section, we describe in detail one of the projects.

CURM 2010-11: Statistical modeling of the fat fraction in MRI. The first component of the CURM project was the Faculty Training Workshop held from June 24 to June 27, 2010 in Draper, Utah. That workshop was critical in creating a support structure among the mentors of the CURM research projects for that year. During the workshop we developed materials for introducing our students to research and learned about sources of information regarding mentoring of students [Leonard 2008] (see also maa.org/external_archive/columns/Resources/resources.html). The diversity in terms of experience, type of institution and type of mathematics studied made for an extremely informative exchange of ideas. We also developed friendships which helped us create a mentor network to support each other. We describe one such network at the end of the section.

In my case (Pineda), I originally planned to have three CURM students, but when a fourth student wanted to join the group, we combined funds from the Louis Stokes Alliance for Minority Participation program (LSAMP, CSUF) and the CURM mini-grant to provide support for all four students. The flexibility provided by having two funding sources played an important role in creating opportunities for the fourth student.

The four students divided themselves to work into two teams of two working on complementary projects involving statistical modeling in MRI, with one group focusing on the numerical simulations and the other on analytical derivations. The students met without me once a week and we all met once a week to discuss progress. Because the students were paid, they maintained a time log of the work they had done each week. A critical component for the success of the project was having reasonable expectations. The summer workshop and discussions with other mentors were extremely helpful for a realistic understanding what a team of undergraduates can do. The students worked hard and discovered new and interesting results, eventually identifying a situation where the current method for estimating the fat content of tissue fails and deriving the probability density function for a new random variable to quantify fat content.

The CURM team gave several presentations, including a poster at the Undergraduate Poster Session at the Joint Mathematics Meetings (JMM) in New Orleans, LA, a talk at the Pacific Coast Undergraduate Math Conference (PCUMC) in Los Angeles, CA, and talks at the CURM Conference, Provo, UT. They also published a paper in the undergraduate research journal of our college [Calder et al. 2011] and a paper in a peer-reviewed journal [Calder et al. 2012].

All four students who participated in this project decided to continue their studies. Li-Hsuan Huang and Kevin Park both attended California State University, Northridge as part of the LSAMP Bridge to the Doctorate. They are currently applying to PhD programs in applied mathematics and statistics. Anne Calder is now a master's student in applied mathematics at CSUF. Eden Ellis is currently applying to PhD programs in statistics. The experience and success provided by the CURM project played a role in these students deciding they would like to continue their studies.

The summer workshop preceding the CURM project facilitated the collaboration of three CURM groups working in southern California. Kathryn Leonard at CI, Herbert Medina at Loyola Marymount University (LMU) and Angel Pineda at CSUF created SoCal CURM, a regional version of CURM, where faculty and students travel to each other's universities to share their progress. These external but local presentations of intermediate results were critical for students to develop their communication skills and to create a local network of students with similar interests. The meetings also gave faculty an opportunity to work through various challenges together as they arose. SoCal CURM continued for a second year, but is currently on sabbatical-related hiatus.

3. Case study: California State University Channel Islands, a small public university

We begin with a quick snapshot of CSU Channel Islands (CI):

- Only four-year public university in Ventura County, a highly rural area near Los Angeles
- Accepted first students in 2001
- Demographics similar to CSUF, also a Hispanic Serving Institution
- Enrolls ∼3,500 full-time-equivalent students
- Employs 85 tenure-track faculty

Most of the math majors at CI transfer from local community colleges. Understandably, we have some difficulty acculturating students to the mathematics profession. Faculty members are stretched by the standard campus service responsibilities spread over such a small number of individuals. Upon my (Leonard's) arrival at CI, no mathematics faculty were engaged in student research during the academic year, in large part because of the time demands of mentoring. I wanted to involve students in my research, knowing it might be the only way to keep my scholarship alive, but had no experience or exposure to undergraduate research and no departmental models to draw from. CURM's funding and training program was precisely what I needed. I applied for and was awarded a CURM mini-grant during the first two years of CURM's existence.

The first year, I soaked in wisdom from the Faculty Training Workshop. I carefully analyzed my group structure and my research problem and planned the first few weeks' activities. Our research project involved modeling textures in digital images that can be viewed as continuous deformations of periodic patterns. The semester began, my three students started work, and immediately disaster loomed. My problem was far too challenging and the group dynamics spiralled downward. Drawing largely on resources and relationships from the Faculty Training Workshop, I salvaged the problem and patched relationships between group members. Without CURM, I might not have overcome the challenges of that first year. I detailed my experiences and lessons learned that first year in [Leonard 2008] to help others avoid my rookie mistakes.

The second year, CURM funded two new students for my research group while two students from the previous year continued without funding. The year progressed smoothly: students successfully identified relationships between wavelet coefficients of periodic functions and the coefficients after a deformation of the function, presented their work in multiple venues including the Undergraduate Poster Session at JMM 2009, and submitted a paper for publication.

Of the five students funded through CURM, two women (one Caucasian, one Filipina) have master's degrees in mathematics, one (Latino) is in a master's program in astronomy at CSU Northridge through the Bridges to the Doctorate program, and one (male Caucasian, first-generation college student) is in a PhD program in mathematics at Univ. of Nebraska. The fifth student (Latina) married and drifted away before graduating. My research group has grown each year since the first CURM award so that I have now mentored over 20 students. Among those who have graduated, all but two have continued on to graduate study in mathematics, statistics, computer science, or physics. The other two now work as mathematicians in industry.

CURM contributed to my professional development as well. The two CURM grants awarded early in my tenure process provided a foundation for two successful NSF grants involving student research. I continued attending the Faculty Training Workshops and Spring Research Conferences after my funded years, and am now a co-director of CURM.

Meanwhile, CURM ideas trickled into my institution. Inspired by the CURM academic year research model, CI's Dean of Faculty implemented a new course, UNIV 498, for faculty from any discipline to mentor senior-level research students for one semester per year. The UNIV 498 program led to a successful grant from the Keck Foundation to fund UNIV x98 courses, $x = 1, 2, 3, 4$, engaging students in interdisciplinary research at the freshman, sophomore, junior and senior level. CI now has a Student Research Steering Committee that offers travel funding for students presenting research at conferences, raises awareness about campus research opportunities, and strategizes about the future of student research at CI. Several campus programs offer research stipends that pay students to do research, including some funded through the Department of Education HSI-STEM grant. In addition, CI hosts an annual research conference funded by SAGE Publications where students showcase their research accomplishments. None of these campus efforts existed before my CURM mini-grant.

CURM has altered the department as well. Every tenure track mathematics faculty member has now mentored at least one academic year student research project. Increasing numbers of our students are attending the Joint Mathematics Meetings and local MAA meetings. Transfer students are learning about research from fellow students and seeking opportunities. This year, my research group of eight students includes two transfer students. Slowly but surely, we are becoming a department where students expect research to be part of their education. CURM provided the necessary push.

4. Case study: Jackson State University, a public, historically black university

We begin with a quick snapshot of Jackson State University (JSU) as a whole:

- Located in Jackson, Mississipi, a highly urban environment
- Founded in 1877
- Over 8,900 students in its bachelor's, master's, and doctoral programs
- Designated a research-intensive institution by the Carnegie Mellon Foundation

The Department of Mathematics at JSU offers a program of study in mathematics leading to two undergraduate degree tracks, the Bachelor of Science in mathematics and Bachelor of Science in secondary mathematics education. A majority of the students completing the Bachelor of Science degree in mathematics first seek employment with the federal government or in industry, then later pursue advanced studies in their areas of employment. Today, a degree in mathematics with additional course work in a related field such as computer science, engineering, statistics, or actuarial science is a more appropriate educational preparation for most industries. These industries expect employees to have interdisciplinary skills allowing them to team up with engineers, scientists, and other professionals [BLS 2013]. In addition, they are expected to explore quantitative data, to explain mathematical theories and solutions to people who do not have extensive knowledge of mathematics, and to devise new solutions to problems encountered by scientists or engineers. Because course work for the typical undergraduate degree in mathematics is not sufficient to meet these minimum skill requirements for employment, the JSU Department of Mathematics sought to address the deficiency with undergraduate research experience in mathematics.

We first chose to collaborate with the Department of Biology to train students in exploring data, explaining abstract mathematical theories, and deriving new theories [Robeva et al. 2010]. The vision was that JSU students would take part in collaborative research work during the academic year for a minimum of two years under the supervision of faculty teams from the biology and mathematics departments. As implemented, a mathematics faculty member usually initiates a research project in consultation with a biology faculty member whose research interests align with the objectives of the project. The two faculty mentors design a project and choose a team of undergraduate mathematics and biology majors to participate in the research. Implementation began with a successful National Science Foundation grant (0531927) for an Interdisciplinary Training of Undergraduates for the Biological and Mathematical Sciences (UBM) program.

Following the success of UBM, PI Tor A. Kwembe decided to expand undergraduate research in the mathematics department beyond mathematical biology and to make it permanent. He applied for and was awarded a CURM grant for the 2010-2011 academic year to support five undergraduate mathematics majors to conduct research in applied mathematics. Currently, all five students are in good academic standing. Two will graduate in December 2012, one in May 2013 and the other two in May 2014.

Unlike in the previous two case studies, the UBM program at JSU had already established a firm foundation for undergraduate research. Nonetheless, receiving a CURM award shifted the paradigm at JSU. The CURM model of awarding mini-grants to faculty from different universities led JSU to establish a Center for Undergraduate Research (CUR) that awards mini-grants of $7,500 to twenty faculty members drawn from the five academic colleges. Each awardee conducts a yearlong research project with five undergraduate students. Participating researchers are encouraged to design research projects that are interdisciplinary in nature. Thus, interdisciplinary undergraduate research at JSU is institutionalized with the creation of CURM-inspired CUR. The Department of Mathematics has already benefited: two of the recipients of the first cohort and one of the second cohort of CUR mini-grants were awarded to mathematics faculty.

Samples of completed projects to date are:

- Effects of water depth and turbidity on spectral signature of submerged aquatic vegetation
- Numerical analysis of distal vasculature pressured fluid velocity and stresses on the walls of cylindrical shaped aneurysms
- Vegetation indices for remote sensing of canopy-forming submerged vegetation
- Detection of submerged plants using closed range hyperspectral remote sensing
- A mathematical model of the effects of aquatic contaminants on freshwater mollusks
- Modeling probability density functions for detecting quantum dot crystalline nano-particles by transmission electron microscopy (TEM).

5. Conclusion

The simplicity and effectiveness of the CURM model to inspire and sustain active undergraduate research programs is unmistakable. Repeatedly, a single CURM award has led to departmental or institutional transformation. By providing faculty and students with the support they need to succeed in undergraduate research, CURM funding demonstrates how undergraduate research bolsters student and faculty achievement. The manageable expense and high returns of academic year research groups appeal to department chairs and administrators as a way to institutionalize a high-impact practice. For more information about CURM activities and opportunities, please see http://curm.byu.edu.

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Information for faculty new to undergraduate research

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(Communicated by Darren A. Narayan)

In this article we provide information for faculty interested in engaging in undergraduate research with their students. Most of the information discussed was gathered from the 2012 Trends for Undergraduate Research in Mathematical Sciences conference. The article includes information on finding appropriate research projects and students to work with. It also discusses various opportunities for faculty as well as some general advice for those new to mentoring undergraduate research.

1. Introduction

Student demand for undergraduate research is growing and the number of universities who value and support undergraduate research is expanding. Consequently many faculty members have become interested in getting involved in undergraduate research projects. Undergraduate research can take many forms: from research projects conducted during a class, to academic year projects or summer REU (research experiences for undergraduates) programs.

The Trends for Undergraduate Research in Mathematical Sciences (TURMS) conference was held October 26–28, 2012 in Chicago, Illinois. During the conference many ideas, suggestions and pieces of advice were given for faculty who are new to undergraduate research. In this article we will present some of that information.

2. Starting undergraduate research

While REU programs provide a beneficial undergraduate research experience, they are restricted to a small percentage of students. In order to reach a larger number of students many faculty are interested in undergraduate research activities they

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can implement at their home institutions. Fortunately there are a number of ways interested faculty can provide research experiences for their students.

One of the ideas suggested at TURMS is to engage students in academic year research projects. This can be done through independent study classes, undergraduate research courses or capstone courses. While independent study and research courses are usually done with a small group of select students, they can provide genuine research experiences. On the other hand, a required capstone course can be a great way for all math and math education majors to get involved in a research experience.

Another way to provide students exposure to mathematical research is to incorporate research projects into mathematics classes. Projects can be a rewarding experience for students and depending on the class and type of students involved, these research projects can lead to future undergraduate research experiences. Carefully chosen and structured in-class projects can be extended to larger projects later on. As mentioned in a later section of this paper, this can also be a good recruiting tool for finding students to work on undergraduate research projects.

An additional idea mentioned at the conference was that mathematics courses can be structured in such a way that students discover and prove course material with guidance from their instructors. This student-centered method of teaching is known as *inquiry based learning* (IBL) and has become more popular in recent years. IBL is regarded as a modified Moore method (legacyrlmoore.org/reference/FOCUS.html). IBL provides an opportunity for all students to get involved in discovery based learning and some instructors regard this as a valuable type of research experience. The Academy of Inquiry Based Learning (AIBL) website (inquirybasedlearning.org) provides useful information about this method and its variations. The website also gives information regarding mentoring programs, workshops, and grants offered to support new and experienced IBL instructors in their efforts to use or develop IBL course materials.

Finding project ideas. There are a number of places to find project ideas suitable for undergraduate research. Places to consider when looking for project ideas include your own research, articles accessible to undergraduates, problems from math competitions and problems from industry.

Considering problems from your own research can be a good place to start with undergraduates. In addition to being familiar with the material you can ensure you are making progress with your own research program while continuing to work with students. If your research involves mathematics that is not appropriate for undergraduate level research you may want to look for articles accessible to undergraduate students. By changing the hypothesis of the main result in an article students may be able to discover a new result. It is also interesting to determine whether the main result of an article can be generalized or expanded in a new way.

Problems from math competitions such as the Putnam Competition or the COMAP Mathematics Modeling Contest are another good source of project ideas. Often problems from these competitions can be generalized or expanded to become suitable for an undergraduate research project. Information on the math competitions mentioned above can be found at their websites; see math.scu.edu/putnam for information on the Putnam Competition and comap.com/undergraduate/contests/ for information on the COMAP Mathematics Modeling Contest. Additionally, the Open Problem Garden is a collection of unsolved problems from a variety of areas in mathematics that can also be a good source of problems (openproblemgarden.org).

If students are interested in working on an applied problem from industry contact local companies to see about establishing collaborations or partnerships. In some situations companies have problems suitable for undergraduate research and they are happy to share these problems.

Finding students to work on undergraduate research. To become involved in undergraduate research one has to recruit students. One idea suggested during TURMS is to offer financial support to students participating in undergraduate research. Many universities have special funds for undergraduate research; however, if your university does not have such a fund available, two possible sources of funding are donations from alumni and donations from faculty. At Coastal Carolina University an alumnus made a donation that was to be used to support an undergraduate research project for one faculty member and three to four students. In addition to the alumni donation, some faculty members also donate to the department discretionary fund to support and encourage undergraduates in their projects.

Another recruiting technique is emailing students doing well in appropriate math classes to see if they would be interested in engaging in an undergraduate research project. Patrick Rault at SUNY College of Geneseo has had success in finding problems critical to his research that will not take much more time for students to complete than for himself. He then emails several promising students in early to mid summer, inviting them to work on the undergraduate research project. Typically one to two students will make significant headway on the problem before the end of the summer and those students can then be enrolled in an independent study course. He requires students to finish the project by the end of the spring semester, present at a regional conference, and possibly submit for publication. In instances where funding is unavailable for students, offering course credit can be a good motivator.

A third recruiting technique is to offer students the opportunity to continue a project they began in class. Although choosing a project that can be extended may be challenging, it can provide a great way to excite and motivate students.

Developing a culture of undergraduate research. In addition to recruiting students it is important to support a culture of undergraduate research at your institution. One way to do this is to invite speakers focused on undergraduate research to your campus to give talks targeting undergraduates. You may also want to organize gatherings after the talk and give the speaker the opportunity to speak informally with the students. Attending undergraduate poster and paper sessions can be a great way to find possible names and contact information for speakers. Make a note of the topics and advisors of the posters that stand out to you. In the future it may be useful to contact these advisors to invite them as speakers to your college or university.

In order to expose students to undergraduate research you may want to bring them to local conferences such as Regional Mathematics Undergraduate Conferences (maa.org/rumc/supported.html) and sectional MAA meetings. Additionally, any conferences that offer undergraduate paper or poster sessions are great venues for students to attend. For example the Joint Mathematics Meetings, MathFest, the Nebraska Conference for Undergraduate Research in Mathematics, and the UnKnot Conference can be a great experience for students.

Another idea is to partner with local colleges in organizing seminars for students. At these seminars students can present topics that interest them. The seminars can also be used to motivate students to continue with work on their projects.

3. Opportunities for faculty

There are a number of different programs undergraduate faculty can apply to if they are interested in learning more about engaging undergraduate students in research. We will discuss a few here, each of which has its own objectives and goals that will help interested faculty become better equipped to mentor and work with undergraduate students.

For individuals early in their career who are interested in learning more about undergraduate research, Project NExT and the various section NExT groups can be very helpful. Project NExT (New Experiences in Teaching) is a professional development program sponsored by the Mathematical Association of America for new or recent PhDs in the mathematical sciences. Although it addresses all aspects of an academic career, Project NExT fellows are able to attend special talks and workshops on undergraduate research and are made aware of opportunities for those interested in undergraduate research. Individuals who have completed their PhD and who will be in their first or second year of full-time teaching at the college level are eligible to apply to become a Project NExT Fellow. In addition to Project NExT, many MAA sections have section-level programs that resemble the national Project NExT. More information regarding Project NExT and the various section NExT groups can be found at the Project NExT website (archives.math.utk.edu/projnext/index.html).

A resource available to individuals at any point in their academic career is the mini-course "Getting students involved in undergraduate research", offered by Joseph Gallian and Aparna Higgins at the Joint Mathematics Meetings each year. This two-part MAA mini-course covers many aspects of facilitating research by undergraduates. It is designed for faculty who are new to directing undergraduate research and is a great resource for those beginning to work with undergraduates.

For those who would like to apply to a more comprehensive program for faculty interested in directing undergraduate research groups, The Center for Undergraduate Research in Mathematics (CURM) may be a good option. CURM is funded by the National Science Foundation (NSF) and Brigham Young University (BYU) and is directed by Michael Dorff. CURM trains professors as mentors, provides funding to establish undergraduate research groups, advises professors on how to organize undergraduate research groups, and prepares undergraduate students to succeed in graduate studies. CURM achieves these goals by offering mini-grants providing financial support to research groups. These research groups are composed of one faculty member and two to five undergraduate students. Professors accepted into the program receive funding to attend a faculty summer workshop at BYU and students and their faculty mentors receive funding to attend a spring research conference there. More information regarding CURM can be found at the CURM website (curm.byu.edu).

Another workshop and program individuals interested in undergraduate research may want to explore is the Research Experiences for Undergraduate Faculty (REUF) workshop. The American Institute of Mathematics (AIM) has sponsored one-week REUF workshops in 2008, 2009, 2011, 2012 and will be again sponsoring a workshop in 2013. These workshops bring together established research mathematicians with faculty at undergraduate institutions who are interested in involving undergraduate students in research. Research groups are formed during the workshop and the majority of the time is spent working in these groups on problems and formulating plans for future work. In addition to the workshop itself, there are also opportunities for continuing activities and collaborations. More information on upcoming REUF workshops can be found on the AIM website (aimath.org/research/upcoming.html).

An additional program undergraduate faculty members may be interested in exploring is the Undergraduate Faculty Program at the Park City Math Institute (PCMI). As stated on their website, the program's objectives are to help undergraduate faculty "renew excitement about mathematics, talk with peers about new teaching approaches, address some challenging research questions, and interact with the broader mathematical community". In the recent years some of the undergraduate faculty programs have been structured so that participants have the opportunity to both generate a list of research problems on a particular topic suitable for undergraduate exploration and form lasting collaborations that continue after the summer program. The 23rd Annual PCMI Summer Session will be held during the summer of 2013. Additional information can be found at the PCMI website (pcmi.ias.edu) by following links to the Undergraduate Faculty Program.

Finally, the Department of Mathematics at Fresno State is accepting applications for Faculty and Undergraduate Research Teams (FURST). This program provides support to research teams composed of one faculty member and two students at primarily undergraduate and minority-serving institutions. The program gives financial support to the teams and provides a one-month immersion program at Fresno State and support for travel both to Fresno State and the Joint Mathematics Meetings. The expectations for the teams are that they will recruit new members for the following year and that faculty will pursue internal funding to support the research group in the year following the FURST award. More information can be found at fresnostate.edu/csm/math/furst.

4. General advice

In this section we will discuss some of the advice given at the 2012 TURMS conference, as well as present some useful references for individuals beginning their work with undergraduate students.

The importance of networking with those involved in directing undergraduate research was mentioned several times during the conference. It was suggested that the best thing to do in starting an undergraduate research project is to attend conferences that focus on undergraduate research and talk to people to find out what has and has not worked for them. Examples of such conferences include TURMS and any of the workshops and programs mentioned earlier in this paper. Many faculty members have a lot of experience mentoring and working with students. These experienced individuals are often happy to share their knowledge with others.

Another piece of advice given is to document everything. By documenting successes and failures it will be easier to report on your progress and keep track of what worked well and what did not. Ideas that worked well can be used and adapted in future projects.

The final piece of advice, suggested for individuals interested in becoming involved with undergraduate research projects, is to look at and read the many websites and articles available that are relevant to undergraduate research. Although it is not possible to list all of these references here, we mention a selection of resources we hope the reader will find helpful:

- Two useful articles published in the Notices of the American Mathematical Society are [Leonard 2008] and [Gallian 2012].
- The website cur.org for the Council for Undergraduate Research (CUR) contains many useful links for those interested in undergraduate research including a link to the CUR Quarterly, a publication described by CUR as "the official public voice of CUR". The publication provides information about student-faculty collaborations from various types of institutions; see cur.org/publications/curquarterly.
- The Mathematical Association of America's website has useful resources for undergraduate research, including short articles on topics ranging from how to get started to finding students; see maa.org/columns/Resources/resources.html. The MAA main page also contains links to an index of programs for undergraduate students (see maa.org/funding/undergraduate.html).
- The National Science Foundation website contains information regarding REU site proposal funding (see nsf.gov/funding/pgm_summ.jsp?pims_id=5517).

5. Conclusion

Undergraduate research can be time consuming, but also a very rewarding experience for both faculty and students. We hope that the information provided in this article will be a good starting point for any faculty interested in establishing an undergraduate research group.

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Promoting REU participation from students in underrepresented groups

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Research experiences for undergraduates (REUs) are an important component of undergraduate education. However, at the 2012 Trends in Undergraduate Research in the Mathematical Sciences conference, questions were raised about why many REU programs see few applications from students that are members of underrepresented groups. We examine the benefits of REUs and factors preventing or promoting participation in REUs.

Research experiences for undergraduates (REUs) have become an important component of undergraduate education. An REU gives students the opportunity to work independently or in small groups on challenging problems, present to a mathematical audience, and communicate findings via technical writing that is often published. Considering the many aspects of professional and academic life addressed by REUs, it is no surprise that research experience is highly valued by both graduate schools and employers.

Perhaps more importantly, REUs play a key role in encouraging students to pursue careers in science, technology, engineering, and mathematics (STEM) fields in the first place. Indeed, surveys of former math REU students indicate that REUs "nurture the commitment of a student to pursue a career in mathematics" [Connolly and Gallian 2007]. Despite the demonstrated effectiveness of the many REU programs currently in operation, the number of students entering STEM areas — in particular, mathematics — remains very low. The February 2012 issue of *Notices of the AMS* showed that the number of undergraduate degrees in mathematics awarded annually decreased 5% from 2006 to 2010. A report by the President's council of advisors in science and technology states that in order to maintain its "historic preeminence" in STEM fields, "the United States will need to increase the number of students who receive undergraduate STEM degrees by about 34% annually over current rates" [PCAST 2012].

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Increased participation by underrepresented groups "is critical to ensuring a high-quality supply of scientists and engineers in the United States over the long term" [Hartline and Poston 2009]. "Women, underrepresented minorities and persons with disabilities comprise more than two-thirds of the U.S. workforce, but hold only about one-quarter of the science, engineering and technology jobs that underpin U.S. economic strength" [NSF 2003]. Moreover, the groups that are most underrepresented in STEM fields are within the fastest growing segments of the general population [COMURG 2011].

It has been suggested that one way to increase the rate of graduates in STEM fields is to diversify teaching methods [PCAST 2012]. REUs, which have been shown to improve retention and academic achievement, are a key demonstration of this principle [Osborn 2009]. It would stand to reason, therefore, that involvement of more underrepresented students in REUs could play a critical role in achieving the suggested 34% increase in STEM degrees. Accordingly, many REUs have included diversity aspects in their program designs.

An open discussion at the end of the 2012 Trends in Undergraduate Research in the Mathematical Sciences (TURMS) conference raised the question of why some REUs are receiving very few applications from students in underrepresented groups. As the conversation proceeded, it became clear that this is an issue that mentors at the REU students' home institutions as well as REU organizers find significant. This article examines how we might go about increasing participation in REUs from students in underrepresented groups. Here we expand the umbrella of underrepresented groups (URGs) to include "minority, low-income, first-generation, and disabled students" [Osborn 2009] each of which is indeed underrepresented within the STEM workforce. Appealing to input from colleagues, published research, and our own personal experience, we examine the issues that might prevent students from URGs from participating in REUs as well as what we can do to change this.

We thank Cindy Wyels and Tamas Forgacs for helpful input during the writing of this article. We also thank our many colleagues who openly shared their experiences sending students to REUs.

1. Benefits of a successful REU experience: stakeholder perspectives

Before considering how to better recruit students from URGs for summer research, it is beneficial to look at what is most important to the students we want to encourage, to their mentors at their home institutions, and to the REU organizers. For all three of these stakeholders, there are some additional expectations for a good REU experience that go beyond those that we most frequently consider.

Benefits that students gain from REUs can be (more or less) organized into the following four categories: gains in knowledge, academic achievement, professional advancement, and personal growth [Osborn 2009]. Students from URGs, like all students, expect an REU to provide them with knowledge, experience, and skills in mathematical research, but they also may need more emphasis on professional and personal growth. Particularly of interest to many students from URGs may be developing "stronger relationships with mentors and other professionals" and "deeper integration into the culture and profession of the discipline" [Osborn 2009]. Merging the sphere of academics with the social and family spheres is also often important [Webb 2009]. "Program components that encourage social interaction and link academic pursuits to community are often more important to historically underrepresented students and women than to other students" [Gregerman 2009].

Mentors from a student's home institution look to REUs to provide experiences for their student that cannot be obtained during the academic year. Students from URGs are more likely to have deficits in their academic background. First-generation college students may be particularly in need of information and resources related to graduate school or employment for mathematicians. Also, especially important for many students from URGs is the formation of an REU cohort. Through this cohort, students gain the experience of socializing, learning, and professionally interacting with others from URGs.

Because students from URGs "experience differential retention rates and inequities in academic achievement" [Bauman et al. 2005] mentors hope that having their students attend REUs will produce a halo-effect within the entire department. Ideally, these students will return to college with a newfound intellectual vibrancy and maturity, which greatly enhances the intellectual climate of a department and also improves retention. "A collaborative scholarly and creative atmosphere attracts motivated students, talented and committed faculty and staff members, and devoted trustees, all of whose involvement further advances the overall academic program of the institution" [Osborn 2009]. This is especially of interest as the amount of government-based funding for REUs has decreased, and we are looking for sustainable ways to increase the production of STEM graduates. The more effective the REU experience, the more likely other students in a student's academic sphere will be impacted.

REU organizers are interested in expanding opportunities to mentor, teach, and conduct research. They are also interested in effective ways to combine scholarship with teaching [Osborn 2009]. For REUs focused on engaging students from URGs, there are additional benefits. At present, funding agencies prefer to support programs that mentor students from URGs. Beyond possible increased funding benefits "mentoring underrepresented students allows faculty members to foster connections with a wide range of campus offices, better integrating undergraduate research into the institutional culture" [Osborn 2009]. These interactions are a benefit to all students participating in REUs. In addition, developing the talents of students from

URGs will increase the pool of prepared candidates that intend to pursue STEM careers.

2. Factors preventing involvement in REUs

The previous section highlighted some of the specific benefits stakeholders (students, their home institutions, and the REU programs) are seeking with regard to involving students from URGs in REUs. Here, we attempt to point out factors — both logistical and psychological — that could be contributing to a lack of applications from students in URGs. In addition, part of the perceived lack of applicants may be because applicants tend to cluster at particular REUs.

For many students from URGs, mentors at the home institution may have a strong influence on the programs their students apply to. Some colleges actively recruit minority students and involve them in programs geared towards helping them succeed. Generally, part of this involves encouraging students to pursue extracurricular intellectual activities like REUs. A recent article addresses the issue of faculty members placing their students in REUs noting that "faculty members at these institutions are highly protective of their students and highly selective in making recommendations to students regarding research opportunities" [Evansech 2009]. If an REU does not stand out to a mentor as a program where his or her student is most likely to have a successful and positive experience, it is unlikely the student will be encouraged to apply.

Many students from URGs do not have access to advanced classes in mathematics during high school. For this reason alone, they are less likely to stand out as good candidates for REUs. These students are less likely to declare a major in a STEM field; in fact, they are more likely to have no major upon entering college [Chen and Carroll 2005]. Even if these students are enrolled in mathematics classes at four-year institutions, they often come into college underprepared and may not complete calculus until their second year [Biermann 2009]. Students from URGs are also more likely to begin postsecondary education at a two-year institution, which may also limit their opportunities to find out about REUs.

Even if students are aware of REU opportunities, they may be intimidated by the process of selecting and applying for programs. A publication from the Council on Undergraduate Research (CUR) on mentoring undergraduate researchers notes that "students can become overwhelmed when they do not receive the support they need, and are often reluctant to ask for assistance" [Temple et al. 2010]. This is especially true for many students from URGs who may experience social and academic isolation and lack of confidence in their abilities. "Even if students are prepared and interested, they and their families may be intimidated by the higher education environment in which they have had little or no previous interaction" [COMURG 2011].

Students unfamiliar with the details of REUs may not understand that applications are competitive; this can result in weak applications that may not be given serious consideration. For the students who do seek help in crafting application materials, campus career services and writing centers may cover writing style and grammar, but sometimes miss the mark when it comes to the more technical writing required in STEM disciplines. Since mathematics courses do not always have a large writing component, the students may not realize that strong writing skills are expected by REUs. Talented mathematics students that struggle with writing skills are less likely to stand out as promising REU applicants.

Students from low-income households are more likely to be working while in school. This means that they will be concerned about leaving jobs to pursue summer research [Watkins 2009]. It is often the case that students are willing to participate in an REU as long as the stipend is comparable to their standard income. However, students unfamiliar with the structure of an REU often assume that REUs, like many summer internships, are unpaid. If a student has children or a spouse, the situation becomes more complicated, since attending an REU could mean losing health benefits and childcare.

3. Promoting more applications from students in URGs

One easy thing mathematics professors can do to promote more applications from students in URGs is early and often to encourage *all* students that demonstrate an interest or ability in math to apply to REUs. We can try to emphasize participation in REUs as an essential component of the math major much as summer internships are for students in other STEM disciplines.

Students who are unfamiliar with REUs sometimes do not know what they should be looking for, and this makes it difficult to apply to the right programs. In order to minimize confusion while also easing the strain on already busy professors, we suggest drafting a document outlining the steps for applying to REUs. The components of such a document may depend on the students it is geared towards but might include information about what an REU program entails, what to expect from an REU, advice on gauging one's own eligibility and needs, choosing programs, and putting together competitive applications. Resources such as contact information for students that have attended REUs in the past may also be helpful. Clearly communicating the expectations for applications will hopefully reduce intimidation and give students from various backgrounds a better chance of getting into an REU and succeeding in the program.

REU organizers can attract a more diverse applicant pool by making sure their program websites are accessible and welcoming to nonexpert undergraduates. In surveying colleagues from various educational institutions, we conclude that students are often the ones reading REU websites. Professors, who are extremely busy during the semester, will give interested students the NSF-REU website or other resources but leave it to the students to investigate individual programs. Overly technical project descriptions and large blocks of text that read like excerpts from grant proposals may be particularly intimidating. Especially where an REU is trying to attract students that have not yet experienced the culture of mathematics, websites and promotional materials should be carefully constructed with the undergraduate perspective in mind.

Another effective method for attracting students from URGs is to reach out to minority-serving institutions such as historically black colleges, Hispanic-serving institutions, and other similar institutions. Mathematicians who have been nationally recognized for their interests in building diversity are also great resources and advocates. It is helpful to promote REUs by connecting with organizations that serve URGs, such as the Society for the Advancement of Chicanos and Native Americans in Science (SACNAS), the American Indian Science and Engineering Society (AINSES), the National Association of Mathematicians (NAM), and the Louis Stokes Institute [Vélez 2011].

It is also essential for REUs to keep in mind that students' institutional mentors are interested in activities and programs that go beyond mathematics. Mentors of underrepresented students want to know that their students are likely to succeed in a particular REU; they are looking for language beyond the tagline "minorities and women are especially encouraged to apply". Key activities should include providing social support and assisting students in navigating a new environment. One way for REU organizers to provide more of these programs is to increase coordination within campuses and regions. Since diversity is such an important aspect of undergraduate education, there is usually a range of opportunities on campus. Mentors are also looking for signs of inclusive excellence, which the literature defines as programs "integrating their diversity and educational quality efforts" [O'Neill 2009]. All students will benefit from socializing with a more diverse group and participating in integrating activities, and these should be highlighted in REU program descriptions. In other words, mentors are looking for an REU to exhibit dedication to helping all students, regardless of background, do superior work.

As part of the ongoing discussion on this topic, Tamas Forgacs surveyed TURMS participants, asking what information would make them more likely to send their students to a particular REU. The following is a compilation of the responses.

(1) Strict and quantified criteria to be met for a given REU program. For example, if a given program scrutinizes GPA and courses taken, what is the minimum GPA and what courses are required?

(2) How research mentors are trained and selected.

(3) What the daily schedule is (how accessible is the mentor, how many meetings, etc.).

(4) What kinds of community-building activities there are.

(5) How the students are chosen — is it only the top tier or is there an effort to reach out to the promising students?

(6) Total number of applications/total number of offers made (in years past).

(7) The institutions, year in school (freshman, sophomore, junior) and mathematics GPAs of students admitted.

(8) The sex and race of students admitted.

(9) The sex and race of research mentors.

4. Conclusion

The United States is in need of more STEM graduates. While its population is becoming increasingly more diverse, the fastest growing segments are also the least represented within STEM fields. REUs are transformative experiences that provide students with motivation, experience, and mentorship. Reaching out to a larger group of REU candidates will provide immediate career goals and achievement benchmarks for all students. For this reason, an increased number of underrepresented students participating in our REUs could translate to an increase in the STEM workforce.

Well mentored minority students are often encouraged by their mentors to apply to specific REU programs. These programs, like all REUs, have a limited number of positions. This may mean that there is increased competition among minorities to attend some programs while other programs are getting very few applications. Students that are not mentored may not know about REUs or may be submitting subpar applications.

We conclude by summarizing our suggestions for increasing the number of applications from URGs to REUs.

For home institutions:

- Encourage all students, early and often, to consider REUs.
- Draft a document that informs students about the benefits of REUs and guides them through the process of selecting and applying for REU programs.

For REU organizers:

- Make program websites accessible and inviting to an audience that is not mathematically advanced.
- Highlight activities that go beyond academics to provide social support.
- When possible, partner with other campus programs that are geared towards diversity to improve the student experience.
- Make statistics about previous participants of the REU available to students and their mentors.

For everyone:

- Reach out to advocates for URGs to help connect students to the right programs.

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The Center for Industrial Mathematics and Statistics at Worcester Polytechnic Institute

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(Communicated by Darren A. Narayan)

"*I like math a lot, but what can I do with it other than teach*?"

In order to enhance the educational, research, and professional experiences for students and faculty at Worcester Polytechnic Institute (WPI) and to help make contacts with business and industry, the Center for Industrial Mathematics and Statistics (CIMS) was established at WPI in 1997. Faculty and students work on research problems that come directly from companies and are of industrial and mathematical significance. CIMS research activities have included projects for mathematical sciences majors during the regular academic year, and the WPI REU Program in Industrial Mathematics and Statistics, which is supported by the National Science Foundation, the Department of Defense, and our industrial partners. Here we give an overview of our experience with the industrial research program, highlighting the processes, benefits, and challenges.

1. Mathematical scientists in business, industry, and government

According to data collected by the American Mathematical Society (available online at http://www.ams.org/profession/data/annual-survey/survey-reports), in the last five years around 25% of new recipients of doctorates in the mathematical sciences from US universities who took their first job in the US went to work in business, industry or government (BIG); see Table 1. When we remove those who wrote their dissertations in statistics or biostatistics — that is, we consider only mathematics dissertations — the portion is still significant and is between 15% and 20%; see Table 2. Between a third and a half of statisticians take up business, industry, or government jobs.

It behooves us therefore to ask:

- What is the role of a mathematician in business and industry?

MSC2010: 00A69.
Keywords: project, industrial math, statistics, student research, business, industry, government.

Year PhD completed	No. of new PhDs employed in US (first job)	No. of new PhDs employed in US (BIG)	Percentage of new PhDs employed in US (BIG)
2006–2007	1012	256	25.3%
2007–2008	1026	270	26.3%
2008–2009	1166	305	26.2%
2009–2010	1263	292	23.1%
2010–2011	1191	316	26.5%

Table 1. US employment information for new recipients of doctorates in the mathematical sciences.

Year PhD completed	No. of new PhDs employed in US (first job)	No. of new PhDs employed in US (BIG)	Percentage of new PhDs employed in US (BIG)
2006–2007	689	116	16.8%
2007–2008	730	135	18.5%
2008–2009	695	144	20.7%
2009–2010	867	139	16.0%
2010–2011	825	158	19.1%

Table 2. US employment information for new recipients of doctorates in the mathematical sciences, omitting those who wrote dissertations in statistics or biostatistics.

- What is it like to work with technical experts on a problem that requires significant mathematics but also must satisfy real-world constraints?
- What kind of mathematical and statistical tools are used to solve problems in business and industry?
- What skills are important to be successful?
- What can university faculty do to better prepare students?

The 1996 and 2012 SIAM reports on mathematics in industry [SIAM 1996; 2012] investigate some of these questions. In addition, the more recent report presents 18 case studies of business applications of mathematics which include business analytics, communications and transportation, computer systems, information technology, mathematical finance, manufacturing, oil discovery and extraction, systems biology, modeling of complex systems, and software development. The Center for Industrial Mathematics at Worcester Polytechnic Institute (WPI) also seeks to get closer to answering these questions by providing opportunities for students and faculty to conduct research with business and industry.

2. WPI and the Center for Industrial Mathematics and Statistics

Some people are driven to do mathematics mainly because of the appeal of the intrinsic beauty of the subject, while others have a greater need to see its connection and relevance to societal needs. Recognizing this, the Center for Industrial Mathematics and Statistics was established at WPI in 1997 to make closer ties between the university's mathematical sciences community and the world outside the academic walls. In the last 15 years, over 125 projects have been completed with over 70 different companies. The projects begin with real-life problems that are generated by our industrial partners and are of direct importance to them. Individual students or small teams, under the direction of one or more faculty members and one or more industrial liaisons, work on a problem of both industrial and mathematical significance.

Companies that work with CIMS do so because it helps the companies address their needs for mathematical solutions and enhance their technological competitiveness. Industry benefits by having access to

- the expertise of faculty members who can help identify and solve critical problems,
- bright, energetic students,
- the latest scientific research developments,
- state-of-the-art computing facilities,
- active participation in the educational process, and
- the opportunity to identify and help train potential future employees.

The existence of CIMS at WPI is feasible because of the nature of WPI and its educational philosophy and academic structure. WPI is a private university of engineering, science, technology, and business. The institute has had hands-on student research projects at its core in keeping with the university's motto of "Lehr und Kunst", which we translate as "theory and practice". In order to obtain a bachelor's degree, *every* undergraduate must complete a Humanities and Arts Project, an Interactive Qualifying Project which makes the connection between science and technology and society, and a Major Qualifying Project (MQP); see http://www.wpi.edu/admissions/undergraduate/academics/projects.html. Also, WPI faculty are expected to advise student research projects as part of their regular teaching duties.

The MQP is a senior-year project completed in their major field of study. It is often the work of a team and spans three-quarters to a full academic year. The purpose of the MQP is to provide a capstone experience in the student's chosen major that will develop creativity, instill self-confidence and enhance the student's ability to communicate ideas and synthesize fundamental concepts. In completing a mathematical sciences MQP, students will have put the theory that they learn in their courses into practice. They will have the opportunity to gain mathematical depth and to develop skills in problem-solving, communication, teamwork, and self-directed learning. With industrial projects, they also get the opportunity to interact with the outside world before starting their careers.

Mathematical sciences graduate students may also complete industrial projects as part of their MS and PhD requirements. In particular, students in the Professional Science Masters Program in Financial Mathematics, and the Professional Science Masters Program in Industrial Mathematics choose this option.

3. WPI REU Program in Industrial Mathematics and Statistics

The opportunity to work on real-world mathematics and statistics projects is not just afforded to WPI's undergraduates and graduate students. Through the WPI REU Program in Industrial Mathematics and Statistics, undergraduates from other universities can come to WPI and work with CIMS [Heinricher and Weekes 2007]. The REU program has been supported for 15 years from 1998–2012 by the National Science Foundation (DMS 9732338, DMS0097469, DMS 0353816, DMS 0649127, DMS1004795) and the ASSURE program of the Department of Defense. We have hosted 167 students from 121 colleges in 37 states. Our REU program will continue for the summers of 2013–2015 under award DMS1263127 from the NSF.

The REU Program in Industrial Mathematics and Statistics at WPI introduces students to the ways that advanced mathematics and statistics are used in industrial and financial settings. It provides an excellent experience for advanced undergraduate students going on to graduate school, whether they choose to specialize in applied mathematics or not. The implementation of mathematics to a particular industrial question is certainly valuable for students interested in following nonacademic career paths, but it is just as valuable for students who enter "traditional" graduate programs and go into academic careers. Students are challenged to understand a given problem and the needs of the sponsor, get to work, and communicate effectively the approach, results, conclusions, and recommendations. Over the course of the project, the research team meets several times with the project's industrial advisor. The students' final deliverables to the company include an oral presentation, a written report, and any accompanying computer programs.

In this REU program, students will experience some of the differences between industrial and academic cultures. For example, the problem that a company presents to the students will be given in the context of the company's operation and the potential impact of the solution to the company's business. The problem needs to be formulated mathematically in order to be understood and solved. These mathematical solutions can have a direct, and potentially large, monetary value for the company's bottom line; this is an experience not found in the traditional academic research environment. In addition, the mathematical solutions found need to be reported in the original corporate context and in a form from which strategic decisions may be made. The mathematics needed to answer the company's questions may be applications of known mathematical concepts or a new mathematical formulation that merits its own study.

K–12 impact. The industrial mathematics projects completed by WPI and REU students have become the foundation for successful outreach programs based at WPI. CIMS has organized five summer institutes for high school and middle school teachers on industrial mathematics called the *Mathematics in Industry Institute for Teachers* thanks to financial support from the GE Foundation, the NSF, SIAM, and ExxonMobil. The ultimate goal of the institute was to increase awareness on the part of teachers, students, and parents of the demand for mathematical scientists in government, business, and industry. More than 200 teachers from more than 100 schools in 18 states came to WPI to spend a week working on industrial mathematics projects. The attendees heard directly from mathematical scientists working in industry, and heard also from faculty and REU students about their work.

Teachers worked in small groups with faculty and undergraduates on industrial research projects. For example, there was a team of REU students working on a project in which they were studying the fluid flow through an electropneumatic pulsed jet actuator, which consists of a miniature valve connected to a convergent nozzle by a small flow chamber. Three middle school teachers and the faculty advisor reduced this project to one that was accessible and interesting to middle schoolers and in concert with the middle school curriculum. Middle school students would work on experimentation, data collection and graphing skills as they respond to a (fake) call from the Hasbro company to "find the best combination of the amount of water and number of pumps to operate the Supersoaker 4000". They would, of course, need to be supplied with a large volume of water, waterguns and, possibly, a change of clothing!

In parallel, NSF-funded Research Experience for Teachers (RET) participants developed a library of industrial projects for the high school classroom: the WPI Industrial Mathematics Project for High School Students. For three summers, 2–3 high school teachers supported by the RET supplement, worked in parallel with the REU students. In addition to attending the regular REU activities, they developed materials needed for classroom work. These projects are available on the web (see http://www.wpi.edu/academics/math/CIMS/IMPHSS/). One can find over 20 projects for high school students drawn from a variety of real-world situations that lead investigations in subjects ranging from algebra to calculus and statistics.

4. The "reality" of industrial project work

Problems from industry do not necessarily arrive neatly packaged and presented like those found at the end of the chapter in a textbook. They are real problems and reality is not pristine. Once the industrial sponsor has presented us with the problem that we have agreed would benefit from mathematical or statistical tools and analysis, the first thing that students must do is understand the industrial context of the problem. Why is this problem important to the company? What has the company attempted, if anything, by way of analyzing the problem? Once we have a good enough understanding of the science, engineering, or business end of things, we look at formulating the problem mathematically. We have found that *formulating the problem* and figuring out what mathematical questions need to be tackled may be as hard or sometimes harder than problem solving.

We find that the *project goals may evolve* as the project moves along. This is because, as we learn more about the problem, we may ask and encounter questions that were not thought about beforehand or could not have been foreseen without the investment of time and thought that the students researchers bring to the problem. We bring fresh pairs of eyes to each project that we take on and having bright students with new ideas and new perspectives is valuable to the industrial advisors whose views may be occluded by their prior history with the project.

Mathematical scientists in industry are often part of a team of people working together to solve problems. The team members often have a variety of backgrounds. It is important that our students be able to work on their own, but equally important that they develop the *teamwork* skills so necessary for successful collaborations with a diverse range of colleagues.

It cannot be overstated how critical *communication skills* are. One has to learn the language of the core discipline in which the research problem lies so that confusion does not arise. It is important that students learn early to ask questions about the industrial problem and to keep asking questions until they understand. The problems that are being encountered are completely new to them and often completely new to the faculty advisor. We have found that when one is not quite convinced of something, it is often not correct, or, as sometimes happens, that we have been speaking different languages and need to take the time to define terms clearly. Questions also help refine the goal and direction of the research.

The most wonderful research is not of much use if it is not communicated effectively. It is not enough to convey how much work has been done; it must be conveyed what has been done and this explanation has to be at a level suited to the audience. Students in our REU program make weekly oral presentations showing their progress and, ultimately, the students are required to write up their research work in a formal document that is given to the sponsor.

Industrial mathematics projects have impact because mathematics and statistics are used to make real decisions. We may often have to aim not for the best solution, but for a very good solution since, in the real world, one works within time and resource constraints. There is rarely one right answer — we are dealing with models often, after all — but there are good solutions and bad solutions, and it is important to be able to examine, understand, and articulate their strengths and limitations.

Getting industrial research projects. To find industrial research projects that are suitable for students, one may start with the career center at one's university to see which employers are looking for mathematical sciences students — that is, students with quantitative and analytical skills. It also makes sense to see where graduates of one's department have gone on to work and to contact those alumni directly. Initial contacts with a company may be made via friends, alumni, trustees, and even strangers. It is then necessary to find out who would be the right person to communicate with at the company to advocate for the connection with you and your department, and then to put forward project ideas. Persistence and initiative are a must.

There are certainly many challenges when pursuing a company for projects. For instance, the company may be actually interested in building a research relationship but may be too busy to put the time and effort into such a new venture. The potential sponsor must feel that the time, hence, the money (since time = money) it commits to the collaboration will be worth the return. Earlier in this article the benefits of student research to industry were set forth, but new companies in particular may need to be convinced of these.

The appropriate faculty member with the right mathematical expertise and interest must be found who can discuss the project idea and scope with the project sponsor. The potential mathematical content of the project must be assessed to make sure that the level is neither too high nor too low for the student in question.

One must assemble the right team of students. Aside from looking at coursework and technical skills, one would want students who come with enthusiasm and an interest in doing mathematics that is truly applied and with a nontrivial multidisciplinary flavor. Ideally, the students would provide a mix of strengths; for example, some may be better programmers than others.

Finally, industry has a guarded culture of intellectual property compared to the relatively open culture of academic research. The degree of openness varies from company to company and from project to project. Both the needs for corporate discretion on specific technical details of a problem and the academic transmission of new general mathematical results need to be fulfilled. When we work with industrial sponsors, legal agreements must be signed so that the university and the company are clear with regards to the use of data, confidentiality and proprietary information, intellectual property, and payment. Such an agreement may require several rounds of revision between our lawyers and those of the sponsor.

5. Conclusion

Mathematics and statistics problems abound in business, industry, and government, and we have found that students and faculty can make real contributions and help solve important problems. More information about the Center for Industrial Mathematics and Statistics, abstracts of past projects, and current activities can be found at http://wpi.edu/+CIMS.

We conclude with feedback from two companies about two REU research projects.

From an industrial sponsor's letter: *I very much enjoy working with the students since they bring a level of passion to the projects that is not often seen in industry. They bring a "can do" attitude to every study and that is always refreshing.... The torque model that WPI has helped us develop over the last 3–4 years has helped us make progress in fastener joint design for the projects we design. The torque model has direct applicability in supporting and improving our designs for assembly....*

I believe that these types of efforts help the student and our engineers. The students get an idea of what happens in industry. They can see how their discipline can be applied to various manufacturing problems. It also allows them to see and understand the compromises that sometimes need to be made given the constraints of the problem.... I have enjoyed working with the WPI family over several years and I plan to keep WPI in mind for future student projects.

From an industrial sponsor's letter: *Because the REU program provides faculty support and enables students to access the university IT infrastructure we are able to specify much more complex projects to challenge the students.... We were extremely impressed with the skill that the students displayed in the areas of financial mathematics and computing. Not only did the students meet the high expectations required by the complexity of the project, they showed genuine insight and at times even stumped the experts....*

Overall, the REU program opens the door to a realistic and complex learning experience that is not easily attainable in either a pure classroom or pure internship setting.

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Nontraditional undergraduate research problems from sports analytics and related fields

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The purpose of this article is to encourage advisors to consider choosing a topic related to sports analytics for their next undergraduate research project. We discuss some of the advantages of working in problems related to sports analytics in an undergraduate research context. We also give a sense of the skills necessary to be successful in research, some ideas of what would make good problems, and avenues to present results. This article expands on the author's presentation at the 2012 Trends in Undergraduate Research in the Mathematical Sciences Conference.

1. Introduction

Over the last few years, analytic techniques to investigate problems in sports have grown in strength and popularity. In the 1990s and 2000s, the Oakland Athletics' "moneyball" approach was a novel way of examining the value of baseball players [Lewis 2003]. Analysts looked at more accurate baseball statistics than traditional stats such as batting average and earned run average. This provided an alternate way to find talented baseball players whose true skills were better than traditional baseball statistics indicated, and allowed the Athletics to find winning players with a limited payroll. One example of a less traditional statistic examined by analysts is on-base percentage. This statistic may give a more accurate representation of the run-scoring potential of a player than batting average. In the last few years, many professional sports teams have mimicked this approach by employing groups of researchers who analyze game and other team-related data searching for trends that will allow their teams to be more successful on the field. The improvements and suggestions are not necessarily complicated, nor do they require strong mathematics, and there are advantages for analysts who are novices to particular sports as this may allow fresh eyes to see an unnoticed trend.

Although we will focus on applications in sports, these types of analyses have become more prevalent in other fields. For instance, Nate Silver's election predictions

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(fivethirtyeight.blogs.nytimes.com) are a well known application of data analysis to political science. Many industrial companies use analytic techniques for supply chain optimization. Other retail-based companies look for ways to save money or to better price products such as airfares, grocery items or insurance policies. There are also companies such as Coremetrics (now IBM Enterprise Marketing Management; see ibm.com/marketing-solutions), Omniture (now part of the Adobe Marketing Cloud; see adobe.com/solutions/digital-marketing.html) and Webtrends (webtrends.com) that specialize in analytics related to websites. In the future, there are likely to be many employment opportunities in analytics with large corporations or with analytics-focused companies. In this article, we will describe types of sports analytics problems found in the literature, useful skills for a successful project, ideas to help advisors find interesting problems and ways for students to present research results.

2. Useful skills

There is no prerequisite to know anything about sports before analyzing sports-related data. Sometimes strong knowledge about a sport can make it more challenging to develop new ideas because new ideas may involve unconventional approaches or statistics. One reason to use sports as a starting point is that there are mountains of available public data that can be analyzed in a variety of ways. In addition to new knowledge gained, one benefit for students who work in sports analytics is that they can learn skills directly applicable to many career paths. In subsequent sections, we describe important problems in the field, but first we will discuss the types of skills students will learn while conducting research in sports analytics.

One substantial skill that can be obtained by working on sports projects is data analysis. Students may need to learn theoretical aspects of different types of regressions and other statistical techniques. For example, a logistic regression may be helpful in making predictions about binary variables such as predicting a win or a loss. In addition students may need to test theories and so must learn about hypothesis testing to make mathematically rigorous statements about their results. A sports analytics focus could fit nicely with class projects in an upper-level mathematical statistics class. One source of ideas for projects in such a course is to look at problems similar to those listed in the essays for the 2010 Math Awareness Month (see mathaware.org/mam/2010/essays). Essays here discuss problems such as analyzing what makes a successful golfer, predicting baseball outcomes with sabermetric tools and understanding how wind, altitude and track geometries affect times in track and field. Another related source of ideas comes from the 2012 Math Awareness Month topic of mathematics, statistics and the data deluge (see mathaware.org/mam/2012/essays); included here are essays describing the implementation of new analytic techniques. In addition, Amy Langville and Carl Meyer have recently published a new book [2012] on ranking appropriate for undergraduates.

Tools from learning theory for the classification of objects into different sets may also be required for certain problems. For instance, a project may involve determining what factors related to the dynamics of a NASCAR race are conducive to a large number of caution flags. Another example might be tracking baseball pitching patterns to detect subtle signs of fatigue. Such techniques, including clustering and using singular value decompositions may require deep forays into numerical linear algebra.

A more practical issue in many studies is obtaining and working with large sets of data. Students may have to know rudimentary or possibly more substantial programming in order to acquire and process data efficiently. This programming may take the form of simple if/then statements in Microsoft Excel or another spreadsheet program. It could also involve writing a script in Perl or another more sophisticated programming language so that large amounts of data can be pulled from a website storing the results of a particular set of sporting events. How to scrape data from a website will be discussed in more detail later. The diversity of skills required to be successful in this area makes it an excellent candidate for interdisciplinary projects linking students in mathematics, economics, computer science and related fields. It will also force students to be able to communicate ideas to students in related fields. Further, students can see how the strengths of their majors can work well with others to make more interesting projects. For instance, at the 2012 TURMS Conference, Michael Dorff, professor of mathematics at Brigham Young University and director of the Center for Undergraduate Research in Mathematics, remarked in his talk about careers in mathematics that many engineering firms are interested in hiring mathematicians because their thinking skills, when combined with the engineering know-how of others, is synergistic. Dorff mentioned that an engineering firm told him that by applying different schools of thought to a problem, groups with mixed technical backgrounds often find a solution faster and more effectively than a group with, say, only engineers. Another more objective measure, the 2013 jobs rating by careercast.com cited in the Wall Street Journal [Weber 2013], lists jobs related to analytics projects as six of the top 20 jobs in their ranking system: mathematician, university professor, actuary, software engineer, computer systems analyst and statistician.

3. Research problems

Ranking. Ever since players and teams started playing against each other in competitions, a timeless question has been: "Who is the best?" In some sports, such as college football or basketball, it may not be feasible for every team to play every other team. As a result, rating systems have been developed to help compare teams. These systems can be derived from complicated mathematics, such as the Colley [2013] and Massey rating systems (masseyratings.com/theory), which are based on ideas from linear algebra. Variants of each of these ranking systems are currently used as part of the computer ranking for the college football Bowl Championship Series (see bcsfootball.org/news/story?id=4765872). Another interesting ranking method is from Keener [1993] who works with eigenvectors of a set of scores to determine a ranking. One example of recent undergraduate and faculty collaboration in ranking is between Furman University faculty members John Harris and Kevin Hutson and their students. They ranked baseball players using linear algebra-based methods (the project, not yet published, is mentioned in [Chartier 2012b]). The technique they describe might help to find effective players who are "diamonds in the rough" and may not have a high salary. Another recent REU project at Duke University investigated different ways to rank basketball teams using the so-called BODGE model [Barrow et al. 2012]. Ranking techniques that have traditionally been used for baseball, figure skating, football and basketball have also been applied to other sports, such as golf [Minton 2010], and there is still plenty of room for improvement.

One challenge in the paper of Harris et al. is that the data used for their analysis was not readily available. Instead, the authors had to find ways to collect information off publicly available websites. One way to obtain data is by using an application programming interface (API) that allows users to search specific company data in a specific manner. For instance, the API allowing users to search within Twitter (see dev.twitter.com) is very popular. Another way to obtain data is to "scrape" it — that is, collect data via the actual HTML output of websites. This technique may take some additional knowledge, but it is sufficiently common that there are many programs and blog posts to help users, such as scrapy.org and [Brody 2012]. What makes scraping challenging is that often the scraper has to reverse engineer how the data of interest is stored on a particular website. Given the ever-increasing amount of data in today's world, it may be worth a student's time to gain familiarity with scraping as data collection has become a field in itself. In particular, for sports, companies such as STATS, Inc. and the Elias Sports Bureau specialize in providing statistical information to sports-related clients.

If students or faculty do not have the time or expertise to obtain data via scraping, there are still many data sets ripe for exploration available online. One interesting project is outlined by Davidson College's Tim Chartier [2012a], who essentially uses data from Ken Massey's website (masseyratings.com/data.php) where data is stored in text file format for a variety of sports and seasons. A project like Chartier's makes the analysis more approachable to students who may have minimal programming experience. Another source of data that contains play-by-play and other forms of data for NBA games is publicly available for download from (basketballvalue.com). The NBA also has statistical information available on its website (nba.com). As time progresses, more and more data sets will likely be available in formats that are easy for students to manipulate.

Statistical modeling. Regression analysis is another tool that is widely used in many settings, including sports analytics. The basic premise is to develop a best-possible model for a particular situation based on a set of prior data. This technique is commonly employed in sports-related articles from economics journals, but practitioners in all fields have used this as an important modeling technique. For instance, in my mathematical statistics course at Davidson College, one of my students, Beau Reese, used regression analysis to determine how different factors (both on and off the field) affect the attendance of a Philadelphia Phillies baseball game. Among the characteristics he tested, factors that were significant were a Phillies' starting pitcher's earned run average, the number of "star" players starting in a given game, and whether the game in question was played on a Friday, Saturday or Sunday.

Game theory. Another area of research ripe with questions related to sports analytics is game theory. This field examines optimal strategies for players in structured situations. One example of an already studied area is penalty kicks in soccer. The penalty-kick situation has been modeled as a two-player zero-sum game. Recent empirical studies show that when examining some years of European soccer league [Palacios-Huerta 2003] penalty kicks, the average of the strategies is very close to the optimal minimax strategy predicted by game theory. Questions that are good for game theoretic analysis usually involve one or two players who have the option to make a small number of choices. In the penalty kick situation, the goalie can either decide to move left, move right, or stay in the center position. Similarly, the player may choose to kick the ball to the left, right or center of the goal. In the study, the author found that collectively soccer players play very close to a minimax strategy for penalty kick shooting.

Infographics. One additional area of interest in many settings is finding novel ways to present data. In an age where we are inundated with data, presenting results in an understandable way to potential customers, student-athletes, or donors is very valuable. *Infographics* is the study of data visualization. A simple example of an infographic is the USA Today series "Snapshots" (usatoday30.usatoday.com/news/snapshot.htm), where a graphic is presented to highlight a trend or describe survey results. Infographics may be useful for marketing, but more importantly, are a way to highlight patterns. For example, an illustrative computer program might help tennis players find locations on the court where they are more than usually prone to make a mistake. It might help a baseball manager analyze a team's hitters quickly to understand their strengths and weaknesses, and be able to make the snap decision to bring in a particular pitcher. For some analytic tools to be useful, they need to be implementable in real time. For instance, a computer program may be helpful by informing a NASCAR crew chief the optimal pit stop strategy at any given point in the race. A further resource is David McCandless' TED talk, "The beauty of data visualization" [2010].

4. Ways to find problems

There are numerous methods for finding interesting research problems in this area. Consulting the academic literature, such as the *Journal of Quantitative Analysis in Sports* or other journals may be a good starting point. Interesting questions may also arise from local coaches who may have questions or coaching strategies that could form the basis of a paper. For example, a conversation with the basketball coach at Davidson College spurred the author to write an article [Britton and Yerger 2013] related to a coaching strategy that involved partitioning the game based on television time-outs. At Davidson College, we have also developed a relationship with Michael Waltrip Racing, and students are working on projects with applications to the racing teams there. Students may be more motivated when there are local people very interested in the outcome of their research. This also could help to develop relationships between colleges/universities and outside community organizations. pt

5. Presenting results

An advantage of working in sports analytics is that the diversity of skills needed to solve problems allows for a wide variety of venues where work can be presented. One natural place is in research journals related to sports analytics such as the *Journal of Quantitative Analysis in Sports*, the *Journal of Sports Economics* or the *Journal of Sports Sciences*. Undergraduate research journals such as *Involve* or the *Rose-Hulman Undergraduate Mathematics Journal* are also venues for presentation. Other more general purpose journals such as *The Statistician* or journals published by the MAA may also be appropriate, depending upon the problem. There are also many conferences where sports research would be of great interest. The best known conference in this area is the MIT Sloan Sports Analytics Conference. Other potential venues include the Wharton Sports Innovation Conference, the New England Symposium on Statistics in Sports, a Joint Mathematics Meetings special session, or an MAA section meeting. Internationally, the IMA sponsors an International Conference on Mathematics in Sport held biannually. From personal experience, presentations involving sports tend to be well attended because conference participants from many specialties are attracted to sports research.

As a final note, I want to reemphasize that a deep knowledge of sports-related background information is not required to be successful in this area of research. Students have the ability to make contributions to a wide range of problems and develop and use a variety of skills that will be useful in their future endeavors. Another added bonus is that students can be the experts in sports-related topical knowledge and this can provide an enriching experience for both students and faculty.

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