involve
a journal of mathematics

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Padmapani Seneviratne and Lauren Melcher

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2018 vol. 11, no. 5

INVOLVE 11:5 (2018)
dx.doi.org/10.2140/involve.2018.11.885

On generalized MacDonald codes

Padmapani Seneviratne and Lauren Melcher

(Communicated by Joshua Cooper)

We show that the generalized q-ary MacDonald codes $C_{n,u,t}(q)$ with parameters $\left[t\left[{n \atop 1}\right]-\left[{u \atop 1}\right], n, tq^{n-1}-q^{u-1}\right]$ are two-weight codes with nonzero weights $w_1 = tq^{n-1}$, $w_2 = tq^{n-1}-q^{u-1}$ and determine the complete weight enumerator of these codes. This leads to a family of strongly regular graphs with parameters $\langle q^n, q^n-q^{n-u}, q^n-2q^{n-u}, q^n-q^{n-u}\rangle$. Further, we show that the codes $C_{n,u,t}(q)$ satisfy the Griesmer bound and are self-orthogonal for $q=2$.

1. Introduction

Two-weight codes are an interesting family of error-correcting codes. They are closely related to many other areas, including strongly regular graphs, partial geometries and finite projective spaces. The relationship between two-weight codes and projective sets was first studied by Delsarte [1972]. Calderbank and Kantor [1986], and later van Lint and Schrijver [1981], did extensive surveys on the subject. Most of these constructions used projective spaces and hence the constructed codes were projective codes. More recently, some cyclic two-weight codes were constructed in [Vega 2008; Vega and Wolfmann 2007].

The MacDonald codes, introduced in [MacDonald 1960] for binary codes, with the definition extended for q-ary codes [Bhandari and Durairajan 2003; Patel 1975], are punctured simplex codes of length $(q^n-q^u)/(q-1)$ for any n and $1 \le u \le n-1$. They have parameters $[(q^n-q^u)/(q-1), n, q^{n-1}-q^{u-1}]_q$ and are two-weight codes with nonzero words of weights $q^{n-1}-q^{u-1}$ and q^{n-1}. Following [Bhandari and Durairajan 2003], we denote these codes by $C_{n,u}(q)$.

The generalized MacDonald codes $C_{n,u,t}(q)$ were introduced in [Dodunekov and Simonis 1998] as an example of a projective multiset. The codes $C_{n,u,t}(q)$ are a direct sum of $t-1$ q-ary simplex codes $C_n(q)$ with a MacDonald code $C_{n,u}(q)$ and have parameters $\left[t\left[{n \atop 1}\right]-\left[{u \atop 1}\right], n, tq^{n-1}-q^{u-1}\right]$, where $\left[{n \atop 1}\right] = (q^n-1)/(q-1)$

MSC2010: 05C90, 94B05.

Keywords: two-weight codes, strongly regular graphs, generalized MacDonald codes, Griesmer bound.

This work was supported by the Texas A&M University–Commerce, FREP grant.

is the q-ary Gaussian coefficient. Properties of the codes $C_{n,u,t}(q)$ were hardly studied, except for the uniqueness of these codes [Tamari 1984; Dodunekov and Simonis 1998].

In this article, we find the complete weight enumerator of the codes $C_{n,u,t}(q)$ and show that they are two-weight codes. We prove that the codes $C_{n,u,t}(q)$ are maximum minimum-distance and hence satisfy the Griesmer bound. Further, we show that both classes of codes $C_{n,u}(q)$ and $C_{n,u,t}(q)$ are self-orthogonal for $q = 2$. Later, we extend these codes to $C_{n,u,s,t}(q)$ codes by taking the direct sum of t simplex codes with s MacDonald codes.

We describe our notation and provide some background definitions in Section 2 and the prove the results on properties of $C_{n,u,t}(q)$ codes in Section 3. In Section 4, we find the parameters of the $C_{n,u,s,t}(q)$ codes and prove their properties.

2. Background and terminology

Codes. A linear $[n, k, d]_q$ code C is a k-dimensional subspace of an n-dimensional vector space over a finite field $\mathbb{F}_q$, where $q = p^m$ and p is a prime. Vectors in C are called codewords. The weight $\mathrm{wt}(\boldsymbol{x})$ of a vector $\boldsymbol{x}$ in $\mathbb{F}_q^n$ is the number of nonzero entries of $\boldsymbol{x}$. The distance $d(\boldsymbol{x}, \boldsymbol{y})$ between two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ in $\mathbb{F}_q^n$ is the number of entries where $\boldsymbol{x}$ and $\boldsymbol{y}$ differ. Therefore, for a linear code $d(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{wt}(\boldsymbol{x} - \boldsymbol{y})$. A code C is said to be an $[n, k, d]_q$ code if d is the minimum nonzero weight in C. A code C is said to be t-error correcting if $t = \lfloor (d-1)/2 \rfloor$.

For vectors $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)$ in $\mathbb{F}_q^n$, the Euclidean inner product (dot product) is defined to be $\boldsymbol{x} \cdot \boldsymbol{y} = \sum_{i=1}^n x_i y_i$. The dual code $C^\perp$ of C is defined as $C^\perp = \{\boldsymbol{x} \in \mathbb{F}_q^n \mid \boldsymbol{x} \cdot \boldsymbol{c} = 0 \text{ for all } \boldsymbol{c} \in C\}$. Then $C^\perp$ is an $[n, n-k]$ linear code over $\mathbb{F}_q$. A code C is called self-orthogonal if $C \subseteq C^\perp$.

The weight enumerator $W_C(x, y)$ of C is the polynomial

$$W_C(x, y) = \sum_{i=0}^{n} A_i x^{n-i} y^i,$$

where A_i is the number of codewords of weight i. C is called a two-weight code if all nonzero codewords have weights w_1 or $w_2 (w_1 < w_2)$ for some w_1 and w_2. A linear code C over $\mathbb{F}_q$ is called a projective code if any two of its coordinates are linearly independent, i.e., if the dual code $C^\perp$ has minimum distance ≥ 3.

For a q-ary $[n, k, d]$ code, the Griesmer bound is given by

$$n_q(k, d) \geq \sum_{i=0}^{k-1} \left\lceil \frac{d}{q^i} \right\rceil, \tag{1}$$

where $n_q(k, d)$ denotes the minimum length n for which an $[n, k, d]$ linear code, over $\mathbb{F}_q$, exists.

Strongly regular graphs.

Definition 1. A simple, undirected graph Γ is called strongly regular, with parameters v, k, λ, μ, if Γ has v vertices and

(1) Γ is regular with valency k,

(2) if the vertices x and y are adjacent then there are exactly λ vertices adjacent to both x and y,

(3) if the distinct vertices x and y are not adjacent then there are exactly μ vertices adjacent to both x and y.

It is easy to verify that the complement of a strongly regular graph is strongly regular. A graph Γ is described by its (0, 1) adjacency matrix $A = (a_{i,j})$ of size v given by $a_{i,j} = 1$ if vertices i and j are adjacent and $a_{i,j} = 0$ if not. We quote the following theorems about strongly regular graphs.

Theorem 1. *If Γ is a graph with v vertices and adjacency matrix A then Γ is strongly regular if and only if there are numbers k, r, s and μ such that $AJ = kJ$ and $(A-rI)(A-sI) = \mu J$, where J is the $v \times v$ matrix of ones and I is the identity matrix of size v.*

Accordingly, it is easy to see that A has eigenvalues k (with a multiplicity of 1), r and s. We will denote the multiplicities of r and s by f and g, respectively. The following is an immediate consequence of Theorem 1.

Theorem 2. *If Γ is a regular graph with adjacency matrix A and A has only three eigenvalues then Γ is a strongly regular graph.*

The parameters of strongly regular graphs are not independent and are related.

Theorem 3. *Let Γ be a strongly regular graph with parameters $\langle v, k, \lambda, \mu\rangle$; then*

$$k(k-\lambda-1) = (v-k-1)\mu. \tag{2}$$

3. $C_{n,u,t}(q)$ codes

The MacDonald codes $C_{n,u}(q)$ can be considered as punctured simplex codes. The generator matrix of the $C_{n,u}(q)$ code can be expressed in the form

$$G_{n,u} = \left[G_n \setminus \left(\frac{\mathbf{0}}{G_u} \right) \right],$$

where $[A \setminus B]$ denotes the matrix obtained from the matrix A by deleting the columns of the matrix B and G_i is the generator matrix for the i-dimensional simplex code.

The generalized MacDonald codes $C_{n,u,t}(q)$ are defined by adding (via direct sum) $t-1$ simplex codes to a MacDonald code. Hence, we can represent the generator matrix, $G_{n,u,t}$ of a generalized MacDonald code by the generator matrix of a simplex code G_n and generator matrix of a MacDonald code $G_{n,u}$. We have

$$G_{n,u,t} = \big[\underbrace{G_n \mid G_n \mid \cdots \mid G_n}_{t-1} \mid G_{n,u}\big].$$

Theorem 4. *The binary MacDonald codes $C_{n,u}(2) = [2^n - 2^u, n, 2^{n-1} - 2^{u-1}]$ are self-orthogonal for $3 \le u \le n-1$.*

Proof. Let $\mathcal{G}_n$ be the matrix consisting of all column vectors of the vector space $\mathbb{F}_2^n$. Then $\mathcal{G}_n$ is an $n \times 2^n$ matrix. Let $\mathcal{G}_u$ be the matrix consisting of all column vectors of the vector space $\mathbb{F}_2^u$. Then $\mathcal{G}_u$ is a $u \times 2^u$ matrix. Let

$$\mathcal{G}_{u,0} = \left(\frac{\mathbf{0}}{\mathcal{G}_u}\right),$$

where $\mathbf{0}$ is the $(n-u) \times 2^u$ zero matrix with elements in $\mathbb{F}_2$. Then the generator matrix $\mathcal{G}_{n,u}$ of the binary MacDonald code is given by $[\mathcal{G}_n \setminus \mathcal{G}_{n,0}]$. Therefore, we can write $\mathcal{G}_n = [\mathcal{G}_{n,u} | \mathcal{G}_{u,0}]$. We know that $\mathcal{G}_n \mathcal{G}_n^T = \mathbf{0}$ for $n \ge 3$. Now,

$$\mathcal{G}_n \mathcal{G}_n^T = [\mathcal{G}_{n,u}|\mathcal{G}_{u,0}][\mathcal{G}_{n,u}|\mathcal{G}_{u,0}]^T = [\mathcal{G}_{n,u}|\mathcal{G}_{u,0}]\left[\frac{\mathcal{G}_{n,u}^T}{\mathcal{G}_{u,0}^T}\right] = \mathcal{G}_{n,u}\mathcal{G}_{n,u}^T + \mathcal{G}_{u,0}\mathcal{G}_{u,0}^T.$$

Therefore, we have $\mathcal{G}_{n,u}\mathcal{G}_{n,u}^T + \mathcal{G}_{u,0}\mathcal{G}_{u,0}^T = \mathbf{0}$. Further, $\mathcal{G}_{u,0}\mathcal{G}_{u,0}^T = \mathbf{0}$ for $u \ge 3$. Hence $\mathcal{G}_{n,u}\mathcal{G}_{n,u}^T = \mathbf{0}$ for $u \ge 3$. This implies that the binary $C_{n,u}(2)$ codes are self-orthogonal for $u \ge 3$. □

Similar to MacDonald codes, the generalized MacDonald codes are also self-orthogonal for $3 \le u \le n$.

Theorem 5. *The generalized MacDonald codes $C_{n,u,t}(q)$ are self-orthogonal for $3 \le u \le n-1$ and $q = 2$.*

Proof. Consider the generator matrix $G_{n,u,t}$ of the generalized MacDonald code $C_{n,u,t}(q)$. Then we have

$$G_{n,u,t} = \big[\underbrace{G_n \mid G_n \mid \cdots \mid G_n}_{t-1} \mid G_{n,u}\big],$$

where G_n is the generator matrix for a simplex code and $G_{n,u}$ is the generator matrix for a MacDonald code. Next, consider matrix product

$$\begin{aligned} G_{n,u,t}G_{n,u,t}^T &= \big[\underbrace{G_n \mid G_n \mid \cdots \mid G_n}_{t-1} \mid G_{n,u}\big]\big[\underbrace{G_n \mid G_n \mid \cdots \mid G_n}_{t-1} \mid G_{n,u}\big]^T \\ &= \underbrace{G_nG_n^T + \cdots + G_nG_n^T}_{t-1} + G_{n,u}G_{n,u}^T. \end{aligned} \tag{3}$$

Let $\mathcal{G}_n$ be the matrix obtained from the simplex matrix G_n by adding the zero column vector. That is, $\mathcal{G}_n = [\mathbf{0} \mid G_n]$. This is the same matrix obtained from column vectors of $\mathbb{F}_q^n$. Now, consider

$$\mathcal{G}_n\mathcal{G}_n^T = [\mathbf{0} \mid G_n][\mathbf{0} \mid G_n]^T = \mathbf{0}\mathbf{0}^T + G_nG_n^T = \mathbf{0}_n + G_nG_n^T = G_nG_n^T.$$

Hence, we can rewrite (3) as

$$G_{n,u,t}G_{n,u,t}^T = \underbrace{\mathcal{G}_n\mathcal{G}_n^T + \cdots + \mathcal{G}_n\mathcal{G}_n^T}_{t-1} + G_{n,u}G_{n,u}^T.$$

From the proof of Theorem 4, we have $\mathcal{G}_n\mathcal{G}_n^T = \mathbf{0}$ and $G_{n,u}G_{n,u}^T = \mathbf{0}$ for $3 \le u \le n-1$. Therefore, we have $G_{n,u,t}G_{n,u,t}^T = \mathbf{0}$ for $3 \le u \le n-1$. Hence, the generalized MacDonald codes $C_{n,u,t}(q)$ are self-orthogonal for $3 \le u \le n-1$. □

The complete weight enumerator of q-ary MacDonald codes is known [Calderbank and Kantor 1986]. Here we will state the result, as it is essential for Theorem 7.

Theorem 6. *The q-ary MacDonald code $C_{n,u}(q)$ is a $[(q^n - q^u)/(q-1), n, q^{n-1} - q^{u-1}]$ is a two-weight code with nonzero weights $w_1 = q^{n-1} - q^{u-1}$ and $w_2 = q^{n-1}$ with weight enumerator coefficients $A_{w_1} = q^n - q^{n-u}$ and $A_{w_2} = q^{n-u} - 1$.*

In the following theorem, we show that the generalized MacDonald codes are also two-weight codes with the same weight enumerator as the MacDonald codes, but with different weights.

Theorem 7. *The generalized MacDonald code $C_{n,u,t}(q)$ is a $\left[t\left[{n \atop 1}\right] - \left[{u \atop 1}\right], tq^{n-1} - q^{u-1}\right]_q$ code with nonzero weights $w_1 = tq^{n-1} - q^{u-1}$ and $w_2 = tq^{n-1}$ and weight enumerator coefficients $A_{w_1} = q^n - q^{n-u}$ and $A_{w_2} = q^{n-u} - 1$.*

Proof. Consider the generator matrix $G_{n,u,t}$ of the code $C_{n,u,t}(q)$. We can represent $G_{n,u,t}$ by

$$G_{n,u,t} = \big[\underbrace{G_n|G_n|\cdots|G_n}_{t-1}|G_{n,u}\big]. \tag{4}$$

The first $t-1$ simplex matrices contribute $(t-1)q^{n-1}$ weights to a nonzero codeword and the MacDonald matrix $G_{n,u}$ contributes q^{n-1} and $q^{n-1} - q^{u-1}$ weights. Therefore, the weights of $C_{n,u,t}(q)$ are $w_1 = tq^{n-1} - q^{u-1}$ and $w_2 = tq^{n-1}$. From (4), it is easy to see that the number of words of weights w_1 and w_2 depend only on the last MacDonald matrix $G_{n,u}$. Hence, the weight enumerator of $C_{n,u,t}(q)$ is the same as the weight enumerator of $C_{n,u}(q)$. Therefore, $A_{w_1} = q^n - q^{n-u}$ and $A_{w_2} = q^{n-u} - 1$. □

An important property of MacDonald codes is that they are maximum minimum-distance codes; i.e., they satisfy the Griesmer bound. In the next theorem, we show that the generalized MacDonald codes are also maximum minimum-distance codes.

Theorem 8. *The codes $C_{n,u,t}(q)$ satisfy the Griesmer bound.*

Proof. Let $C_{n,u,t}(q)$ be a $\left[t\begin{bmatrix}n\\1\end{bmatrix}-\begin{bmatrix}u\\1\end{bmatrix}, tq^{n-1}-q^{u-1}\right]$ code. Consider the right-hand side of the Griesmer bound (1). Then

$$\begin{aligned}
\sum_{i=0}^{k-1}\left\lceil\frac{d}{q^i}\right\rceil &= \sum_{i=0}^{k-1}\left\lceil\frac{tq^{n-1}-q^{u-1}}{q^i}\right\rceil = \sum_{i=0}^{u-1}\left\lceil\frac{tq^{n-1}-q^{u-1}}{q^i}\right\rceil + \sum_{i=u}^{k}\left\lceil\frac{tq^{n-1}-q^{u-1}}{q^i}\right\rceil \\
&= \left\{\lceil tq^{n-1}-q^{u-1}\rceil + \lceil tq^{n-2}-q^{u-2}\rceil + \cdots + \lceil tq^{n-u}-1\rceil\right\} \\
&\quad + \left\{\left\lceil tq^{n-u-1}-\frac{q^{u-1}}{q^u}\right\rceil + \left\lceil tq^{n-u-2}-\frac{q^{u-1}}{q^{u+1}}\right\rceil + \cdots + \left\lceil t-\frac{q^{u-1}}{q^{n-1}}\right\rceil\right\} \\
&= \left\{(tq^{n-1}-q^{u-1}) + (tq^{n-2}-q^{u-2}) + \cdots + (tq^{n-u}-1)\right\} \\
&\qquad\qquad + \left\{(tq^{n-u-1}) + (tq^{n-u-2}) + \cdots + (tq^0)\right\} \\
&= \left\{t(q^{n-1}+q^{n-2}+\cdots+q^{n-u}+q^{n-u-1}+\cdots+q^0)\right\} \\
&\qquad\qquad - \left\{q^{u-1}+q^{u-2}+\cdots+q^0\right\} \\
&= t\left(\frac{1-q^n}{1-q}\right) - \left(\frac{1-q^u}{1-q}\right) = t\begin{bmatrix}n\\1\end{bmatrix} - \begin{bmatrix}u\\1\end{bmatrix} = n_q(k,d). \qquad \square
\end{aligned}$$

We can obtain a strongly regular graph from a two-weight code C with weights w_1 and w_2 as follows [Calderbank and Kantor 1986]. Take codewords as vertices of Γ and join two codewords $\boldsymbol{x}$ and $\boldsymbol{y}$ by an edge if and only if $d(\boldsymbol{x}, \boldsymbol{y}) = w_1$. The strongly regular graph Γ is said be associated with C.

Theorem 9. *Let $\Gamma_{n,u,t}$ be the strongly regular graph associated with the generalized MacDonald code $C_{n,u,t}(q)$. Then $\Gamma_{n,u,t}$ has parameters $\langle q^n, q^n-q^{n-u}, q^n-2q^{n-u}, q^n-q^{n-u}\rangle$.*

Proof. The number of vertices of $\Gamma_{n,u,t}$ is equal to the number of codewords of $C_{n,u,t}(q)$; hence $v = q^n$. Let W_1 be the set of codewords of weight $w_1 = tq^{n-1}-q^{u-1}$ and W_2 be the set of codewords of weight $w_2 = tq^{n-1}$ of $C_{n,u,t}(q)$. By the construction, we know $\Gamma_{n,u,t}$ is a regular graph. The zero-vector $\mathbf{0}$, as a vertex, has degree $|W_1|$, as $d(\mathbf{0}, \boldsymbol{x}) = w_1$ for all $\boldsymbol{x} \in W_1$. Therefore, from Theorem 7, we get $k = |W_1| = q^n - q^{n-u}$.

To obtain the value of μ, consider the zero-vector $\mathbf{0}$. Pick any codeword $\boldsymbol{u}$ from W_2; then $d(\mathbf{0}, \boldsymbol{u}) = w_2$, which implies $\mathbf{0}$ is nonadjacent to all the codewords in W_2. Let $\boldsymbol{v}$ be an arbitrary codeword in W_1. Then $d(\boldsymbol{u}, \boldsymbol{v}) = w_1$; otherwise $\boldsymbol{v} \in W_2$, which contradicts our assumption that $\boldsymbol{v} \in W_1$. Since $\boldsymbol{v} \in W_1$ is arbitrary, $\boldsymbol{u} \in W_2$ is adjacent to all the codewords in W_1. Therefore, the codeword $\boldsymbol{u}$ is adjacent to $|W_1| = q^n - q^{n-u}$ vertices and hence $\mu = q^n - q^{n-u}$.

We will use (2) to determine the value of λ from the other three parameters. Consider $k(k-\lambda-1) = (v-k-1)\mu$, but $\mu = k$ implies $(k-\lambda-1) = (v-k-1)$, and then $\lambda = 2k - v = 2(q^n - q^{n-u}) - q^n = q^n - 2q^{n-u}$. This leads to $\langle v, k, \lambda, \mu\rangle = \langle q^n, q^n - q^{n-u}, q^n - 2q^{n-u}, q^n - q^{n-u}\rangle$. $\square$

4. $C_{n,u,s,t}(q)$ codes

In this section, we extend the definition of generalized MacDonald codes $C_{n,u,t}(q)$ to that of $C_{n,u,s,t}(q)$ codes. Define $C_{n,u,s,t}(q)$ codes by adding t simplex codes to s MacDonald codes.

The generator matrices of $C_{n,u,s,t}(q)$ codes can be defined similarly to generator matrices of generalized MacDonald codes $C_{n,u,t}(q)$. Let $G_{n,u,s,t}$ be the generator matrix of the code $C_{n,u,s,t}(q)$. Then

$$G_{n,u,s,t} = \big[\underbrace{G_n \mid G_n \mid \cdots \mid G_n}_{t} \mid \underbrace{G_{n,u} \mid G_{n,u} \mid \cdots \mid G_{n,u}}_{s}\big],$$

where G_n and $G_{n,u}$ are the generator matrices of simplex codes and MacDonald codes, respectively.

The parameters of the $C_{n,u,s,t}(q)$ codes can be easily deduced from that of the codes $C_{n,u}(q)$ and $C_{n,u,t}(q)$. By the form of the generator matrix $G_{n,u,s,t}$, the weight enumerator of $C_{n,u,s,t}(q)$ is the same as that of the MacDonald codes.

Theorem 10. *$C_{n,u,s,t}(q)$ is a $\big[(t+s)\big[{n \atop 1}\big] - s\big[{u \atop 1}\big], n, (t+s)q^{n-1} - sq^{u-1}\big]_q$ code with nonzero weights $w_1 = (t+s)q^{n-1} - sq^{u-1}$ and $w_2 = (t+s)q^{n-1}$ with weight enumerator coefficients $A_{w_1} = q^n - q^{n-1}$ and $A_{w_2} = q^{n-u} - 1$.*

Similar to MacDonald and generalized MacDonald codes, the binary codes $C_{n,u,s,t}(q)$ are self-orthogonal for $u \geq 3$.

Theorem 11. *$C_{n,u,s,t}$ codes are self-orthogonal for $q = 2$ and $3 \leq u \leq n-1$.*

Proof. We will show that $G_{n,u,s,t}G^T_{n,u,s,t} = \mathbf{0}$ for $3 \leq u \leq n-1$:

$$\begin{aligned}
G_{n,u,s,t}G^T_{n,u,st} &= \big[\underbrace{G_n \mid \cdots \mid G_n}_{t} \mid \underbrace{G_{n,u} \mid \cdots \mid G_{n,u}}_{s}\big]\big[\underbrace{G_n \mid \cdots \mid G_n}_{t} \mid \underbrace{G_{n,u} \mid \cdots \mid G_{n,u}}_{s}\big]^T \\
&= \underbrace{G_nG_n^T + \cdots + G_nG_n^T}_{t} + \underbrace{G_{n,u}G^T_{n,u} + \cdots + G_{n,u}G^T_{n,u}}_{s} \\
&= tG_nG_n^T + sG_{n,u}G^T_{n,u} \\
&= t\mathcal{G}_n\mathcal{G}_n^T + sG_{n,u}G^T_{n,u}.
\end{aligned}$$

For $3 \leq u \leq n-1$, from the proofs of Theorems 4 and 5, we have $\mathcal{G}_n\mathcal{G}_n^T = \mathbf{0}$ and $G_{n,u}G^T_{n,u} = \mathbf{0}$. □

Since these codes have the same weight enumerator as that of MacDonald codes, parameters of the strongly regular graphs generated by them are the same as the strongly regular graphs generated by the MacDonald codes.

5. Conclusion

In this work, we have described the weight enumerators of generalized MacDonald codes $C_{n,u,t}(q)$ and the codes $C_{n,u,s,t}(q)$ and showed that these are two-weight codes. Further, we have shown that the codes $C_{n,u}(q)$, $C_{n,u,t}(q)$ and $C_{n,u,s,t}(q)$ are self-orthogonal for $3 \leq u \leq n-1$. All three classes have the same weight enumerator and hence generate the same strongly regular graph.

All the codes in this work were constructed as a direct sum of a one-weight code (simplex code) with a two-weight code (MacDonald code). It might be interesting to study other such constructions arising from one- and two-weight codes.

Acknowledgments

The authors would like to thank the anonymous referees for their careful reading of the paper and the insightful comments and suggestions.

References

[Bhandari and Durairajan 2003] M. C. Bhandari and C. Durairajan, "A note on covering radius of MacDonald codes", pp. 221–225 in *International conference on information technology: coding and computing* (Las Vegas, 2003), IEEE, Piscataway, NJ, 2003.

[Calderbank and Kantor 1986] R. Calderbank and W. M. Kantor, "The geometry of two-weight codes", *Bull. London Math. Soc.* **18**:2 (1986), 97–122. MR Zbl

[Delsarte 1972] P. Delsarte, "Weights of linear codes and strongly regular normed spaces", *Discrete Math.* **3** (1972), 47–64. MR Zbl

[Dodunekov and Simonis 1998] S. Dodunekov and J. Simonis, "Codes and projective multisets", *Electron. J. Combin.* **5** (1998), art. id. 37. MR Zbl

[van Lint and Schrijver 1981] J. H. van Lint and A. Schrijver, "Construction of strongly regular graphs, two-weight codes and partial geometries by finite fields", *Combinatorica* **1**:1 (1981), 63–73. MR Zbl

[MacDonald 1960] J. E. MacDonald, "Design methods for maximum minimum-distance error-correcting codes", *IBM. J. Res. Develop.* **4** (1960), 43–57. MR

[Patel 1975] A. M. Patel, "Maximal q-nary linear codes with large minimum distance", *IEEE Trans. Information Theory* **21**:1 (1975), 106–110. MR Zbl

[Tamari 1984] F. Tamari, "On linear codes which attain the Solomon–Stiffler bound", *Discrete Math.* **49**:2 (1984), 179–191. MR Zbl

[Vega 2008] G. Vega, "Two-weight cyclic codes constructed as the direct sum of two one-weight cyclic codes", *Finite Fields Appl.* **14**:3 (2008), 785–797. MR Zbl

[Vega and Wolfmann 2007] G. Vega and J. Wolfmann, "New classes of 2-weight cyclic codes", *Des. Codes Cryptogr.* **42**:3 (2007), 327–334. MR Zbl

Received: 2017-08-23 Revised: 2017-12-08 Accepted: 2017-12-14

padmapani.seneviratne@tamuc.edu *Department of Mathematics, Texas A&M University–Commerce, Commerce, TX, United States*

lmelcher@leomail.tamuc.edu *Department of Mathematics, Texas A&M University–Commerce, Commerce, TX, United States*

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Involve (ISSN 1944-4184 electronic, 1944-4176 printed) at Mathematical Sciences Publishers, 798 Evans Hall #3840, c/o University of California, Berkeley, CA 94720-3840, is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

Involve peer review and production are managed by EditFLOW® from Mathematical Sciences Publishers.

PUBLISHED BY

mathematical sciences publishers

nonprofit scientific publishing

http://msp.org/

involve

2018 vol. 11 no. 5

1944-4176(2018)11:5;1-4