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A systematic development of Jeans' criterion with rotation for gravitational instabilities

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An inviscid fluid model of a self-gravitating infinite expanse of a uniformly rotating adiabatic gas cloud consisting of the continuity, Euler's, and Poisson's equations for that situation is considered. There exists a static homogeneous density solution to this model relating that equilibrium density to the uniform rotation. A systematic linear stability analysis of this exact solution then yields a gravitational instability criterion equivalent to that developed by Sir James Jeans in the absence of rotation instead of the slightly more complicated stability behavior deduced by Subrahmanyan Chandrasekhar for this model with rotation, both of which suffered from the same deficiency in that neither of them actually examined whether their perturbation analysis was of an exact solution. For the former case, it was not and, for the latter, the equilibrium density and uniform rotation were erroneously assumed to be independent instead of related to each other. Then this gravitational instability criterion is employed in the form of Jeans' length to show that there is very good agreement between this theoretical prediction and the actual mean distance of separation of stars formed in the outer arms of the spiral galaxy Andromeda M31. Further, the uniform rotation determined from the exact solution relation to equilibrium density and the corresponding rotational velocity for a reference radial distance are consistent with the spectroscopic measurements of Andromeda and the observational data of the spiral Milky Way galaxy.

1. Introduction and formulation of the problem

Consider the governing equations for a self-gravitational adiabatic inviscid fluid of infinite extent undergoing uniform rotation [Chandrasekhar 1961]:

$$\text{continuity equation:}\quad \frac{D\rho}{Dt}+\rho\nabla\cdot\boldsymbol{v}=0, \tag{1-1a}$$

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$$\text{Euler's equation:}\quad \frac{D\boldsymbol{v}}{Dt}+2\boldsymbol{\Omega}\times\boldsymbol{v}+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{r}) = -\frac{1}{\rho}\mathscr{P}'(\rho)\nabla\rho+\boldsymbol{g}, \tag{1-1b}$$

$$\text{Poisson's equation:}\quad \nabla\cdot\boldsymbol{g} = -4\pi G_0\rho. \tag{1-1c}$$

Here $t \equiv$ time, $\boldsymbol{r} = (x, y, z) \equiv$ position vector, $\boldsymbol{\Omega} = (0, 0, \Omega_0) \equiv$ uniform rotation vector, $\rho \equiv$ density (mass/[unit volume]), $\boldsymbol{v} = (u, v, w) \equiv$ velocity vector with respect to the rotating frame, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z) \equiv$ gradient operator, $D/Dt = \partial/\partial t+\boldsymbol{v}\cdot\nabla \equiv$ material derivative, $\mathscr{P}(\rho) = p_0(\rho/\rho_0)^{\gamma_0} \equiv$ adiabatic pressure, $\boldsymbol{g} = -\nabla\varphi \equiv$ gravitational acceleration vector with $\varphi \equiv$ self-gravitating potential, and $G_0 \equiv$ universal gravitational constant. The continuity and Euler's equations follow from the conservation of mass and momentum for an inviscid fluid [Lin and Segel 1974] with the addition of the extra second and third terms on the left-hand side of (1-1b), which represent the Coriolis effect and centrifugal force, respectively, due to the rotation [Greenspan 1968]. Poisson's equation follows from the divergence theorem and Newton's law of universal gravitation [Binney and Tremaine 1987; Lin and Segel 1974]. Since

$$\begin{gathered}\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{r}) = (\boldsymbol{\Omega}\cdot\boldsymbol{r})\boldsymbol{\Omega}-(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega})\boldsymbol{r} = -\Omega_0^2(x, y, 0),\\ \boldsymbol{\Omega}\times\boldsymbol{v} = \Omega_0(-v, u, 0), \quad \nabla\cdot\boldsymbol{g} = -\nabla^2\varphi\end{gathered} \tag{1-1d}$$

[Segel 1977], the Euler's and Poisson's equations become

$$\frac{D\boldsymbol{v}}{Dt}+2\Omega_0(-v,u,0)-\Omega_0^2(x,y,0) = -\frac{1}{\rho}\mathscr{P}'(\rho)\nabla\rho-\nabla\varphi, \tag{1-1e}$$

$$\nabla^2\varphi = 4\pi G_0\rho, \quad \text{where } \nabla^2 \equiv \nabla\cdot\nabla. \tag{1-1f}$$

Sir James Jeans [1902; 1928] proposed that a gravitational instability mechanism occurring in the spiral arms of protogalactic nebulae could result in the formation of chains of condensations, which eventually developed into those stars visible in the outer regions of fully evolved galaxies. He suggested that a nonrotating self-gravitating unbounded interstellar cloud of adiabatic gas, which is initially uniform in density and quiescent, should undergo an instability mechanism of this sort when acted on by random infinitesimal perturbations. Jeans [1902; 1928] deduced a criterion for which such an interstellar cloud would exhibit a gravitational instability by performing a linear stability analysis on what he assumed to be an exact solution to his governing inviscid gas dynamical model system that was equivalent to equations (1-1) in the absence of rotation, arriving at the following secular equation satisfied by σ and λ, the growth rate and wavelength, respectively, of his small density fluctuations:

$$\sigma^2 = 4\pi\left(G_0\rho_0 - \pi\frac{c_0^2}{\lambda^2}\right), \tag{1-2a}$$

where c_0 is the speed of sound in an adiabatic medium of uniform density ρ_0. This relation differed from that for the propagation of sound in a homogeneous medium only due to the presence of the gravity term in (1-2a). Then, from (1-2a), Jeans concluded that there would be instability corresponding to $\sigma^2 > 0$ provided

$$\lambda > \lambda_J = c_0\sqrt{\frac{\pi}{G_0\rho_0}} \equiv \text{Jeans' length}, \tag{1-2b}$$

which is known as Jeans' criterion for gravitational instabilities.

The only problem with this derivation is that Jeans represented his exact static solution to those governing equations symbolically as $\boldsymbol{v} \equiv \mathbf{0} = (0, 0, 0)$, $\rho = \rho_0$, $\varphi = \varphi_0$. Since this analysis was for a nonrotating system with $\Omega_0 = 0$, when he assumed in addition that ρ_0 was uniform to make his perturbation equations constant coefficient this implicitly required $\nabla\varphi_0 = \mathbf{0}$, which implied $\nabla^2\varphi_0 = 0 = 4\pi G_0\rho_0$ or $\rho_0 = 0$ and hence is termed Jeans' swindle by Binney and Tremaine [1987]. Kiessling [2003] refutes their claim that Jeans' derivation represents a swindle because it can be justified by taking the proper limit of the appropriate cosmological model.

Since spectroscopic evidence (reviewed by Rubin and Ford [1970]) ultimately showed these nebulae to be rotating, Subrahmanyan Chandrasekhar [1961] considered the effect of adding rotation to Jeans' governing system of perturbation equations and repeated that analysis, demonstrating in the process that its stability behavior was slightly more complicated in that it involved an extra instability condition as well as Jeans' criterion. Chandrasekhar's perturbation analysis suffered from the same deficiency as Jeans' in that he did not develop a parameter relationship for his implicit exact solution and thus treated ρ_0 and Ω_0 as independent. We shall demonstrate that the proper relationship between these parameters eliminates this extra condition and only yields Jeans' instability criterion. Many subsequent linear stability analyses of similar problems influenced by the methodology of these works have treated their associated perturbation systems independently of the actual exact solution of the governing equations and thus replicate this deficiency including recent studies and reviews of gravitational instabilities [Stahler and Palla 2004]. Hence, we believe there is some merit in performing a systematic linear stability analysis of the relevant exact solution for Chandrasekhar's problem and toward that end present an investigation of this sort in the next section.

2. The exact static homogeneous density solution and its linear stability

There exists an exact static homogeneous density solution of our basic equations of the form

$$\boldsymbol{v} \equiv \mathbf{0} = (0, 0, 0), \quad \rho \equiv \rho_0, \quad \varphi = \varphi_0, \tag{2-1a}$$

where φ_0 satisfies

$$\nabla\varphi_0 = \Omega_0^2(x, y, 0), \quad \nabla^2\varphi_0 = 4\pi G_0\rho_0 \tag{2-1b}$$

or

$$\varphi_0(x, y) = \tfrac{1}{2}\Omega_0^2(x^2 + y^2), \quad \text{with } \Omega_0^2 = 2\pi G_0\rho_0 > 0. \tag{2-1c}$$

Now seeking a linear perturbation solution of these basic equations of the form

$$\begin{gathered} \boldsymbol{v} = \varepsilon \boldsymbol{v}_1 + \boldsymbol{O}(\varepsilon^2), \quad \text{where } \boldsymbol{v}_1 = (u_1, v_1, w_1), \\ \rho = \rho_0[1 + \varepsilon s + O(\varepsilon^2)], \quad \varphi = \varphi_0 + \varepsilon\varphi_1 + O(\varepsilon^2), \end{gathered} \tag{2-2}$$

with $|\varepsilon| \ll 1$, substituting (2-2) into those equations, neglecting terms of $O(\varepsilon^2)$, and canceling the resulting ε common factor; we deduce that the perturbation quantities to this exact solution satisfy

$$\frac{\partial s}{\partial t} + \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \tag{2-3a}$$

$$\frac{\partial u_1}{\partial t} - 2\Omega_0 v_1 + c_0^2\frac{\partial s}{\partial x} + \frac{\partial \varphi_1}{\partial x} = 0, \quad \text{where } c_0^2 = \mathscr{P}'(\rho_0) = \gamma_0\frac{p_0}{\rho_0} > 0, \tag{2-3b}$$

$$\frac{\partial v_1}{\partial t} + 2\Omega_0 u_1 + c_0^2\frac{\partial s}{\partial y} + \frac{\partial \varphi_1}{\partial y} = 0, \tag{2-3c}$$

$$\frac{\partial w_1}{\partial t} + c_0^2\frac{\partial s}{\partial z} + \frac{\partial \varphi_1}{\partial z} = 0, \tag{2-3d}$$

$$2\Omega_0^2 s - \nabla^2\varphi_1 = 0. \tag{2-3e}$$

Then assuming a normal mode solution for these perturbation quantities of the form

$$[u_1, v_1, w_1, s, \varphi_1](x, y, z, t) = [A, B, C, E, F]e^{i(k_1x+k_2y+k_3z)+\sigma t}, \tag{2-4}$$

where $|A|^2 + |B|^2 + |C|^2 + |E|^2 + |F|^2 \neq 0$, $i = \sqrt{-1}$, and $k_{1,2,3} \in \mathbb{R}$ satisfy the implicit far-field boundedness property for those quantities, and substituting (2-4) into (2-3), we obtain the following equations for $[A, B, C, E, F]$ upon cancellation of the exponential common factor:

$$ik_1A + ik_2B + ik_3C + \sigma E = 0, \tag{2-5a}$$

$$\sigma A - 2\Omega_0 B + ic_0^2k_1E + ik_1F = 0, \tag{2-5b}$$

$$2\Omega_0 A + \sigma B + ic_0^2k_2E + ik_2F = 0, \tag{2-5c}$$

$$\sigma C + ic_0^2k_3E + ik_3F = 0, \tag{2-5d}$$

$$2\Omega_0^2E + k^2F = 0, \quad \text{where } k^2 = k_1^2 + k_2^2 + k_3^2. \tag{2-5e}$$

Setting the determinant of the 5×5 coefficient matrix for the linear homogeneous system (2-5) of constants equal to zero to satisfy their nontriviality property, we obtain

$$k^2[\sigma^4 + (c_0^2k^2 + 2\Omega_0^2)\sigma^2] + 4\Omega_0^2(c_0^2k^2 - 2\Omega_0^2)k_3^2 = 0. \tag{2-6}$$

Defining the wavenumber vector $\boldsymbol{k} = (k_1, k_2, k_3)$, its dot product with $\boldsymbol{\Omega}$ satisfies

$$\boldsymbol{k} \cdot \boldsymbol{\Omega} = k_3\Omega_0 = |\boldsymbol{k}||\boldsymbol{\Omega}| \cos(\theta) = k\Omega_0 \cos(\theta), \tag{2-7a}$$

θ being the azimuthal angle between $\boldsymbol{k}$ and $\boldsymbol{\Omega}$, which implies

$$k_3 = k \cos(\theta). \tag{2-7b}$$

Then, substitution of (2-7b) into (2-6) and cancellation of k^2 yields the secular equation

$$\sigma^4 + (c_0^2k^2 + 2\Omega_0^2)\sigma^2 + 4\Omega_0^2(c_0^2k^2 - 2\Omega_0^2) \cos^2(\theta) = 0. \tag{2-8}$$

Since this secular equation is a quadratic in σ^2, we first demonstrate that $\sigma^2 \in \mathbb{R}$ by showing that its discriminant $\mathcal{D}$ satisfies

$$\mathcal{D} = (c_0^2k^2 + 2\Omega_0^2)^2 - 16\Omega_0^2(c_0^2k^2 - 2\Omega_0^2) \cos^2(\theta) \geq 0. \tag{2-9a}$$

Consider the two cases of $c_0^2k^2 - 2\Omega_0^2 \leq 0$ and $c_0^2k^2 - 2\Omega_0^2 > 0$ separately. For the former case it is obvious, while for the latter one it can be deduced by noting that

$$\mathcal{D} \geq (c_0^2k^2 + 2\Omega_0^2)^2 - 16\Omega_0^2(c_0^2k^2 - 2\Omega_0^2) = (c_0^2k^2 - 6\Omega_0^2)^2. \tag{2-9b}$$

For $\theta = \frac{\pi}{2}$, we can conclude from (2-8) that

$$\sigma^2 = 0 \quad \text{or} \quad \sigma^2 = -(c_0^2k^2 + 2\Omega_0^2) < 0, \tag{2-10a}$$

while for $\theta \neq \frac{\pi}{2}$, the stability criteria governing such quadratics, namely,

$$\text{given } \omega^2 + a\omega + b = 0 \text{ with } \mathcal{D} = a^2 - 4b \geq 0, \quad \omega < 0 \iff a, b > 0 \tag{2-10b}$$

[Uspensky 1948], implies that

$$\sigma^2 < 0 \iff c_0^2k^2 - 2\Omega_0^2 > 0. \tag{2-10c}$$

Making an interpretation of these results, we can deduce from (2-10) and (2-1c) that there will only be $\sigma^2 > 0$ and hence unstable behavior provided

$$c_0^2k^2 - 4\pi G_0\rho_0 < 0, \tag{2-11a}$$

which is equivalent to Jeans' gravitational instability criterion (1-2b)

$$\lambda > \lambda_J = c_0\sqrt{\frac{\pi}{G_0\rho_0}} \equiv \text{Jeans' length} \tag{2-11b}$$

since

$$\lambda = \frac{2\pi}{k}. \tag{2-11c}$$

Figure 1. A Galaxy Evolution Explorer image of the Andromeda galaxy M31, courtesy NASA/JPL-Caltech.

3. Comparisons

Let us return to Jeans' analysis. In writing (1-2), one must implicitly assume that $\rho_0 > 0$, which seems plausible in that $\rho_0 = 0$ corresponds to a completely empty space [Scheffler and Elsässer 1988]. When gravity is taken into account in the absence of rotation, however, such an assumption is not strictly compatible with the equations of hydrostatic equilibrium, as we have seen. Thus, Jeans' uniform density solution, as mentioned above, was not exact. The problem under examination demonstrates that adding rotation to the system as Chandrasekhar did and again performing a standard linear stability analysis of its exact static solution yields Jeans' instability criterion but in a systematic manner and such a model also has the added advantage of being more astrophysically realistic. Jeans got the right answer for the wrong reason, as was shown in [Kiessling 2003] by taking the proper limit of the appropriate cosmological model to fix that analysis. In his review of hydrodynamic stability theory, the renowned comprehensive applied mathematical modeler Lee Segel [1966] stated that "Anyone can get the right answer for the right reason. It takes a genius or a physicist to get the right answer for the wrong reason." In this context, Sir James Jeans was both.

The formula for λ_J in (2-11b) is of fundamental importance in astrophysics and cosmology where many significant deductions concerning the formation of galaxies and stars have been based upon it. In particular, Jeans' interpretation of the criterion, now bearing his name, was that a gas cloud of characteristic dimension much greater than λ_J would tend to form condensations with mean distance of separation comparable to λ_J that then developed into those protostars observable in the outer arms of spiral galaxies such as Andromeda M31 (see Figure 1). Sekimura

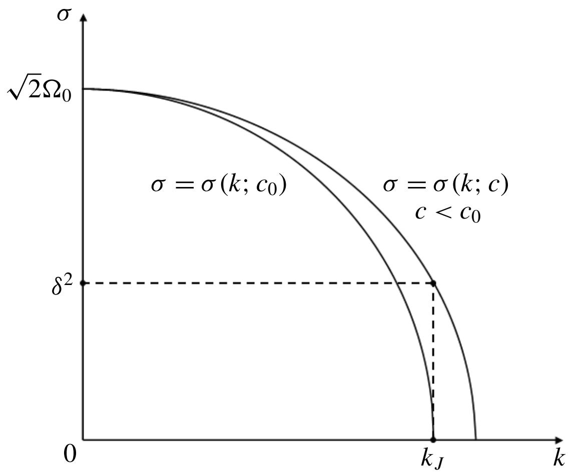

Figure 2. Schematic plots in the k-σ plane depicting the methodology employed by [Sekimura et al. 1999] applied to the Jeans' secular equation $\sigma = \sigma(k; c) = \sqrt{2\Omega_0^2 - c^2k^2}$. That curve is plotted for both a general speed of sound c and our specific speed $c_0 > c$ in this figure where $k_J = 2\pi/\lambda_J$ is such that $\sigma(k_J; c_0) = 0$. In a weakly nonlinear stability analysis one takes the disturbance wavenumber $k \equiv k_J$ and its growth rate to be equal to $\sigma_J(c) = \sigma(k_J; c) = \delta^2 > 0$ where c is close enough to c_0 so that δ is a small parameter. Then in the $\lim_{c\to c_0} \sigma_J(c) = 0$ which is a requirement for the application of weakly nonlinear stability theory and any re-equilibrated pattern will exhibit a wavelength of λ_J. Here $c^2 = \gamma(p_0/\rho_0)$ with $\gamma < \gamma_0$ and hence the operation $\lim_{c\to c_0}$ is equivalent to $\lim_{\gamma\to\gamma_0}$.

et al. [1999] have demonstrated that, for a secular equation similar in form to (1-2a), λ_J actually corresponds to the so-called critical wavelength λ_c of linear stability theory associated with $\sigma = 0$ (see Figure 2), while nonlinear stability analyses of physical phenomena involving related secular equations have shown that the observed wavelengths are determined to a close approximation by that λ_c rather than by the dominant wavelength λ_d at which σ achieves its maximum value from linear theory (see, e.g., [Tian and Wollkind 2003]). Hence, Jeans' interpretation, although unusual for linear stability theory (where it is often presumed that such a disturbance associated with the largest growth rate predominates), both anticipated and is consistent with these nonlinear results, since, by the time perturbations have grown enough for the effect of the maximum growth rate to be observed, the neglected nonlinearities may have rendered that linear analysis inaccurate [Segel and Stoeckly 1972]. In this context, note that for a typical value of $\theta \neq \frac{\pi}{2}$, namely $\theta = 0$, we can factor our secular equation (2-8) to obtain the roots $\sigma^2 = -4\Omega_0^2$ and

$\sigma^2 = 2\Omega_0^2 - c_0^2k^2$. Observe that this last condition, which yields our instability, is equivalent to the Jeans' secular equation (1-2a).

Thus using the formula for Jeans' length λ_J with the parameters c_0 and ρ_0 assigned the values

$$c_0 = \tfrac{2}{3} \times 10^4 \, \frac{\text{cm}}{\text{sec}} \quad \text{and} \quad \rho_0 = 10^{-22} \, \frac{\text{gm}}{\text{cm}^3} \tag{3-1a}$$

employed by [Jeans 1928] for this purpose but when the polytropic index γ_0 is $\frac{4}{3}$ [Bonnor 1957], while taking

$$G_0 = 6.67 \times 10^{-8} \, \frac{\text{cm}^3}{\text{gmsec}^2} \tag{3-1b}$$

in cgs units yields

$$\lambda_J = 4.58 \times 10^{18} \, \text{cm} = 1.48 \, \text{pc}, \tag{3-1c}$$

where 1 pc $\equiv 3.09 \times 10^{18}$ cm, which compares quite favorably with the mean distance between actual adjacent condensations originally formed in the outer arms of Andromeda since, in those parts of M31, the averaged observed distance between protostars in such chains is about 1.4 pc or somewhat more if allowances are made for foreshortening [Jeans 1928].

Given the small size of ρ_0 in (3-1a), Chandrasekhar [1961] was one of those individuals who regarded Jeans' analysis as a close approximation to reality [Scheffler and Elsässer 1988]. Although he oriented his axes so that $\boldsymbol{\Omega} = (0, \Omega_y, \Omega_z)$ with $|\boldsymbol{\Omega}| = \Omega_0$ and $\boldsymbol{k} = (0, 0, k)$, using our more general orientation Chandrasekhar, in effect, considered uniform rotation Ω_0 in his perturbation equations through the Coriolis force terms of (2-3b) and (2-3c) in order to make the model more realistic while retaining the coefficient $4\pi\rho_0G_0$ for s in (2-3e). In so doing, he implicitly assumed that Ω_0 and ρ_0 were independent rather than related parameters. Chandrasekhar plotted σ^2 versus k for $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$ and $\Lambda^2 \equiv \Omega_0^2/(\pi G_0\rho_0) = 0.5, 1.0, 2.0$. Besides Jeans' criterion for $\theta \neq \frac{\pi}{2}$, this yielded an extra extraneous instability criterion for the case of $\theta = \frac{\pi}{2}$, namely,

$$c_0^2k^2 < 4(\pi G_0\rho_0 - \Omega_0^2) \quad \text{should } \Omega_0^2 < \pi G_0\rho_0. \tag{3-2}$$

In point of fact, $\Lambda^2 = 0.5$ is a representative value of that quantity for this instability condition of (3-2), while $\Lambda^2 = 2.0$, his upper bound, actually corresponds to its value as per our formula of (2-1c) relating these parameters, which implies

$$\Omega_0 = \sqrt{2\pi\rho_0G_0}. \tag{3-3a}$$

Let us examine the plausibility of (3-3a), which violates (3-2) identically. In conjunction with the values for ρ_0 and G_0 of (3-1), (3-3a) yields the uniform rotation

$$\Omega_0 = 6.47 \times 10^{-15}/\text{sec} \tag{3-3b}$$

and the corresponding rotational velocity

$$V_0 = r_0\Omega_0 = 200\,\frac{\text{km}}{\text{sec}} \tag{3-3c}$$

for the reference radial distance of

$$r_0 = 1\,\text{kpc} = 10^3\,\text{pc} = 3.09 \times 10^{21}\,\text{cm} = 3.09 \times 10^{16}\,\text{km}, \tag{3-3d}$$

both of which are consistent with the spectroscopic measurements of the Andromeda nebula and the observational data of the spiral Milky Way galaxy [Rubin and Ford 1970].

In conclusion our development presents a systematic linear stability analysis of Chandrasekhar's [1961] gravitational instability model in the presence of uniform rotation. We close by noting that Binney and Tremaine [1987] considered this gravitational instability model in a cylindrical rotating system as a problem in Chapter 5 of their book *Galactic Dynamics*. They observed that rotation allowed the Jeans' instability to be analyzed exactly. Since the first part of their problem was to find the condition on Ω_0 so that the homogeneous quiescent gas would be in equilibrium, Binney and Tremaine did not examine the plausibility of this condition. Further, the last part of their problem was to show, upon finding the resulting secular equation from its linear stability analysis, that waves propagating perpendicular to the rotation vector were always stable, while those propagating parallel to it were unstable if and only if the usual Jeans' criterion without rotation was satisfied. Although the latter conclusion for $\theta = 0$ agrees with our predictions, the former does not since, when $\theta = \frac{\pi}{2}$, we predicted $\sigma^2 = 0$, as well as those $\sigma^2 < 0$ which only implies a condition of neutral stability. Our results demonstrate that the best way to test the validity of a model for a natural science phenomenon is to compare its theoretical predictions with observable data of this phenomenon. Sir Arthur Conan Doyle characterized that philosophy probably as well as anyone by a Sherlock Holmes quote from "A scandal in Bohemia" in his 1891 collection entitled *The adventures of Sherlock Holmes*:

> It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts.

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