involve
a journal of mathematics

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2019 vol. 12, no. 7

INVOLVE 12:7 (2019)
dx.doi.org/10.2140/involve.2019.12.1163

A binary unrelated-question RRT model accounting for untruthful responding

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(Communicated by Javier Rojo)

Estimating the prevalence of a sensitive trait in a population is not a simple task due to the general tendency among survey respondents to answer sensitive questions in a way that is socially desirable. Use of randomized response techniques (RRT) is one of several approaches for reducing the impact of this tendency. However, despite the additional privacy provided by RRT models, some respondents may still provide an untruthful response. We consider the impact of untruthful responding on binary unrelated-question RRT models and observe that even if only a small number of respondents lie, a significant bias may be introduced to the model. We propose a binary unrelated-question model that accounts for those respondents who may respond untruthfully. This adds an extra layer of precaution to the estimation of the sensitive trait and decreases the importance of presurvey respondent training. Our results are validated using a simulation study.

1. Introduction

Social desirability bias (SDB) refers to the tendency among survey respondents to answer sensitive questions in a way that is viewed positively by others. SDB can interfere with estimation of the prevalence of a sensitive trait in a given population due to potential untruthful responding. There have been many methods proposed to correct for SDB such as the bogus pipeline method introduced by [Jones and Sigall 1971] and the modified Marlowe–Crowne social desirability scale investigated by [Reynolds 1982]. Here, we will focus on another method of reducing the impact of SDB — the randomized response technique (RRT).

RRT was originally introduced by [Warner 1965] and has since been generalized by many researchers including [Greenberg et al. 1969; Gupta et al. 2002; 2013; Christofides 2003; Kim et al. 2006; Nayak and Adeshiyan 2009; Barabesi and

MSC2010: 62D05.
Keywords: model efficiency, optional randomized response models, unrelated-question RRT model, untruthful responding.

Marcheselli 2010; Suarez and Gupta 2018]. This method allows respondents to provide a scrambled response to a sensitive question, where responses cannot be unscrambled at an individual level. This increases respondent privacy and encourages survey participants to respond honestly. Specifically, we will focus on the unrelated-question RRT model introduced in [Greenberg et al. 1969] and the variation of this model developed in [Sihm et al. 2016].

In the [Greenberg et al. 1969] model, a randomization device contains two questions — the sensitive question and an unrelated question. The respondent uses the randomization device and responds to whichever question they receive. The researcher does not know which question each individual respondent answered, but the proportion of respondents with the sensitive trait can still be estimated at an aggregate level. In this model, every respondent uses the randomization device.

However, a topic that is sensitive to one respondent may not be sensitive to another. Optional RRT models, introduced by [Gupta et al. 2002], allow those respondents who do not find the question sensitive to answer the sensitive question directly rather than use the randomization device. The researcher does not know whether the respondent answered directly or used the randomization device.

A variation of the unrelated-question RRT model where the use of the randomization device is optional was introduced in [Gupta et al. 2013]. Respondents are instructed to answer using the randomization device if they find the question sensitive, or respond directly to the sensitive question if they do not find it sensitive. They proposed both quantitative and binary models, and used split sampling in each case to estimate the prevalence of the sensitive trait as well as the sensitivity level of the question.

Binary and quantitative optional unrelated-question models that use a two-question approach to estimating the prevalence of the sensitive trait and the sensitivity level of the question were then developed in [Sihm et al. 2016]. The first question uses the original model of [Greenberg et al. 1969] to estimate the sensitivity level of the question, and the second question uses an optional model similar to that in [Gupta et al. 2013] to estimate the prevalence of the sensitive trait.

However, all of the aforementioned RRT models make the assumption of completely truthful responding. When this assumption is not met, a bias is introduced into these models. This can be a dangerous when asking a very sensitive question, or when the proper presurvey respondent training has not been performed because, in these situations, the proportion of untruthful responding could be relatively large. We assume that the reason for untruthful responding is a respondent's lack of trust in the randomization process — i.e., the belief that the randomization device does not completely protect their privacy.

In this paper, we propose a two-question model that accounts for untruthful responding. The first question uses the [Greenberg et al. 1969] model to estimate the proportion of respondents who trust the randomization process. The second question asks the respondents to respond using a second randomization device if they trust the randomization process, or simply answer the unrelated question if they do not. This diverts the untruthful responders from introducing bias to the estimation of the sensitive trait. Comparisons of mean squared error are used to demonstrate in which situations this model may be preferred over the models of [Greenberg et al. 1969] and [Sihm et al. 2016].

2. Background binary RRT models

2.1. The original unrelated-question RRT model was introduced in [Greenberg et al. 1969]. In this model, each respondent in a simple random sample with replacement (SRSWR) uses a randomization device and receives the sensitive or unrelated question with probabilities p or $1-p$ respectively. The prevalence of the sensitive characteristic π_x is unknown and the prevalence of the unrelated characteristic π_y is assumed to be known. Recall that in the original Greenberg model, the probability of a "yes" response is given by

$$P_y = p\pi_x + (1-p)\pi_y. \tag{2-1}$$

Rearranging for π_x, we get

$$\pi_x = \frac{P_y - (1-p)\pi_y}{p} := \pi_{\mathrm{GR}}, \tag{2-2}$$

which leads to the estimator

$$\hat{\pi}_{\mathrm{GR}} = \frac{\widehat{P}_y - (1-p)\pi_y}{p}, \tag{2-3}$$

where $\widehat{P}_y$ is the proportion of "yes" responses in the survey. The estimator variance is given by

$$\mathrm{Var}(\hat{\pi}_{\mathrm{GR}}) = \frac{P_y(1-P_y)}{np^2}. \tag{2-4}$$

2.2. A binary optional unrelated-question model using a two-question approach to estimate prevalence of the sensitive trait in a population and the sensitivity level of the trait was proposed in [Sihm et al. 2016].

In this model, respondents are asked two questions. The first question is the research question of interest, and is answered using the model of [Gupta et al. 2013]. The second question uses the [Greenberg et al. 1969] model to ask whether the respondent finds the main research question sensitive. In Question 1, π_x is the known prevalence of the sensitive trait, p is the probability of a respondent receiving the sensitive question, π_y is the prevalence of some unrelated characteristic, and W is the proportion of respondents who find the question sensitive. In Question 2, W

is again the sensitivity level, π_b is the known prevalence of a different unrelated trait, and p_b is the probability of a respondent receiving the question about whether they find the question of interest sensitive.

The probability of a "yes" response for Questions 1 and 2 respectively is represented as

$$P_{y_1} = W[p\pi_x + (1-p)\pi_y] + (1-W)\pi_x, \tag{2-5}$$

$$P_{y_2} = p_b W + (1-p_b)\pi_b. \tag{2-6}$$

Solving (2-6) for W and (2-5) for π_x we have

$$W = \frac{P_{y_2} - (1-p_b)\pi_b}{p_b} \quad \text{and} \quad \pi_x = \frac{P_{y_1} - (1-p)W\pi_y}{1-(1-p)W} := \pi_{\mathrm{SI}}. \tag{2-7}$$

This leads to the estimators

$$\widehat{W} = \frac{\widehat{P}_{y_2} - (1-p_b)\pi_b}{p_b}, \tag{2-8}$$

$$\hat{\pi}_{\mathrm{SI}} = \frac{\widehat{P}_{y_1} - (1-p)\widehat{W}\pi_y}{1-(1-p)\widehat{W}}. \tag{2-9}$$

Using a first-order Taylor approximation for $\hat{\pi}_{\mathrm{SI}}$, the variance of this estimator becomes

$$\operatorname{Var}(\hat{\pi}_{\mathrm{SI}}) \approx \frac{P_{y_1}(1-P_{y_1})}{n(1-(1-p)W)^2} + \frac{(1-p)^2(P_{y_1}-\pi_y)^2 P_{y_2}(1-P_{y_2})}{np_b^2(1-(1-p)W)^4}. \tag{2-10}$$

3. The effect of lying on existing binary unrelated question RRT models

3.1. We first consider [Greenberg et al. 1969]. Let π_a represent the probability that a respondent who has the sensitive trait (belongs to the π_x group) will give a truthful response when confronted with a question about their possession of that trait. It is assumed that those who receive the unrelated question will always provide a truthful response.

Therefore, the probability of a "yes" response in (2-1) becomes

$$P_y^* = p\pi_x\pi_a + (1-p)\pi_y, \tag{3-1}$$

and the estimate in (2-3), mistakenly assuming $\pi_a = 1$, becomes

$$\hat{\pi}_{\mathrm{GR}}^* = \frac{\widehat{P}_y^* - (1-p)\pi_y}{p}, \tag{3-2}$$

with variance

$$\operatorname{Var}(\hat{\pi}_{\mathrm{GR}}^*) = \frac{P_y^*(1-P_y^*)}{np^2}. \tag{3-3}$$

The bias in this estimate is

$$\text{Bias}(\hat{\pi}^*_{\text{GR}}) = \text{E}[\hat{\pi}^*_{\text{GR}} - \pi_x] = \pi_x(\pi_a - 1), \tag{3-4}$$

and therefore the mean squared error of the estimate is

$$\text{MSE}(\hat{\pi}^*_{\text{GR}}) = \text{Var}(\hat{\pi}^*_{\text{GR}}) + \text{Bias}^2(\hat{\pi}^*_{\text{GR}}) = \frac{P^*_y(1-P^*_y)}{np^2} + \pi_x^2(\pi_a - 1)^2. \tag{3-5}$$

3.2. We now consider [Sihm et al. 2016]. Again, let π_a represent the probability of a truthful response as described in Section 3.1. In this model, we assume that respondents who do not find the question sensitive will be honest about their possession of the trait. We also assume that there will be no dishonesty in responses to either question in Question 2, or in response to the unrelated question in Question 1. The probability of a "yes" from (2-5) becomes

$$P^*_{y_1} = W[p\pi_x\pi_a + (1-p)\pi_y] + (1-W)\pi_x, \tag{3-6}$$

and the estimate in (2-9), mistakenly assuming $\pi_a = 1$, becomes

$$\hat{\pi}^*_{\text{SI}} = \frac{\widehat{P}^*_{y_1} - (1-p)\widehat{W}\pi_y}{1-(1-p)\widehat{W}}. \tag{3-7}$$

The first-order Taylor approximation of this estimator is

$$\hat{\pi}^*_{\text{SI}} \approx \frac{P^*_{y_1} - W(1-p)\pi_y}{1-(1-p)W} + \frac{\widehat{P}^*_{y_1} - P^*_{y_1}}{1-(1-p)W} + \frac{(1-p)(P^*_{y_1} - \pi_y)(\widehat{W} - W)}{(1-(1-p)W)^2}, \tag{3-8}$$

which has an approximate variance of

$$\text{Var}(\hat{\pi}^*_{\text{SI}}) \approx \frac{P^*_{y1}(1-P^*_{y_1})}{n(1-(1-p)W)^2} + \frac{(1-p)^2(P^*_{y_1} - \pi_y)^2 P^*_{y_2}(1-P^*_{y_2})}{np_b^2(1-(1-p)W)^4}. \tag{3-9}$$

The bias for the estimate in (3-8) is

$$\text{Bias}(\hat{\pi}^*_{\text{SI}}) \approx \text{E}[\hat{\pi}^*_{\text{SI}} - \pi_x] = \frac{W\pi_x p(\pi_a - 1)}{1-(1-p)W}, \tag{3-10}$$

and therefore the mean squared error of the estimate is

$$\begin{aligned}
&\text{MSE}(\hat{\pi}^*_{\text{SI}}) \\
&= \text{Var}(\hat{\pi}^*_{\text{SI}}) + \text{Bias}^2(\hat{\pi}^*_{\text{SI}}) \\
&= \frac{P^*_{y_1}(1-P^*_{y_1})}{n(1-(1-p)W)^2} + \frac{(1-p)^2(P^*_{y_1}-\pi_y)^2 P^*_{y_2}(1-P^*_{y_2})}{np_b^2(1-(1-p)W)^4} + \left(\frac{W\pi_x p(\pi_a-1)}{1-(1-p)W}\right)^2.
\end{aligned} \tag{3-11}$$

4. The proposed model

The goal of this model is to avoid any bias introduced to the model by untruthful responding. To do this, we propose a two-question model where the first question uses the [Greenberg et al. 1969] model to ask whether respondents trust the randomization process. For the second question, respondents are asked to respond using the [Greenberg et al. 1969] model if they trust the randomization process, or simply respond to the unrelated question if they do not. This way, anyone who may be tempted to provide an untruthful answer about their involvement in the sensitive question of interest is redirected to the unrelated question.

Let π_x be the prevalence of the sensitive trait of interest, π_y be the known prevalence of some unrelated trait, π_b be the known prevalence of some other unrelated trait, p_b be the probability of receiving the question about trust in Question 1, and p be the probability of receiving the sensitive question in Question 2. Also, let π_a be the probability that a respondent will trust the randomization process (the probability someone would not give an untruthful response when faced with the sensitive question).

The probability of a "yes" response to Question i $(i = 1, 2)$ is represented as

$$P_{y_1} = p_b\pi_a + (1 - p_b)\pi_b, \tag{4-1}$$

$$P_{y_2} = \pi_a[p\pi_x + (1 - p)\pi_y] + (1 - \pi_a)\pi_y. \tag{4-2}$$

Solving (4-1) and (4-2) for π_a and π_x gives us

$$\pi_a = \frac{P_{y_1} - (1 - p_b)\pi_b}{p_b} \quad \text{and} \quad \pi_x = \frac{P_{y_2} - \pi_y(1 - \pi_a p)}{\pi_a p}, \tag{4-3}$$

which leads to the estimates

$$\hat{\pi}_a = \frac{\widehat{P}_{y_1} - (1 - p_b)\pi_b}{p_b} \quad \text{and} \quad \hat{\pi}_x = \frac{\widehat{P}_{y_2} - \pi_y(1 - \hat{\pi}_a p)}{\hat{\pi}_a p}, \tag{4-4}$$

where $\widehat{P}_{y_i}$ is the proportion of respondents who respond "yes" to Question i $(i = 1, 2)$.

Observe that $\hat{\pi}_a$ is an unbiased estimator of π_a and its variance is

$$\mathrm{Var}(\hat{\pi}_a) = \frac{P_{y_1}(1 - P_{y_1})}{np_b}. \tag{4-5}$$

Using a first-order Taylor approximation for $\hat{\pi}_x$ gives us

$$\hat{\pi}_x \approx \frac{P_{y_2} - \pi_y(1 - \pi_y p)}{\pi_a p} + \frac{\widehat{P}_{y_2} - P_{y_2}}{\pi_a p} + \frac{p(\pi_y - P_{y_2})(\hat{\pi}_a - \pi_a)}{(\pi_a p)^2} := \hat{\pi}_{\mathrm{YO}}. \tag{4-6}$$

The estimate $\hat{\pi}_{\mathrm{YO}}$ is an unbiased estimator of π_x up to a first-order Taylor approximation, and its variance is given by

$$\mathrm{Var}(\hat{\pi}_{\mathrm{YO}}) = \frac{P_{y_2}(1 - P_{y_2})}{n(\pi_a p)^2} + \frac{P_{y_1}(1 - P_{y_1})p^2(\pi_y - P_{y_2})^2}{np_b^2(\pi_a p)^4}. \tag{4-7}$$

5. Simulation results

We now present simulation results for our estimator $\hat{\pi}_{\text{YO}}$ and compare it to the estimators $\hat{\pi}^*_{\text{GR}}$ and $\hat{\pi}^*_{\text{SI}}$ as detailed in Sections 3.1 and 3.2, respectively. Table 1 details the simulation results using 10,000 iterations at $n = 500$. We allow π_a (the

			π_a					
			1.00	0.99	0.95	0.90	0.85	0.80
proposed model		$\hat{\pi}_{\text{YO}}$	0.199906	0.199673	0.199971	0.200081	0.199688	0.200349
		$\widehat{\text{Var}}(\hat{\pi}_{\text{YO}})$	0.000540	0.000548	0.000602	0.000684	0.000778	0.000879
		$\text{Var}(\hat{\pi}_{\text{YO}})$	0.000541	0.000553	0.000608	0.000686	0.000780	0.000891
		$\hat{\pi}_a$	0.999841	0.989917	0.949589	0.900047	0.850556	0.799957
		$\widehat{\text{Var}}(\hat{\pi}_a)$	0.000450	0.000479	0.000536	0.000599	0.000666	0.000690
		$\text{Var}(\hat{\pi}_a)$	0.000461	0.000477	0.000536	0.000601	0.000656	0.000701
simple unrelated		$\hat{\pi}^*_{\text{GR}}$	0.199769	0.198214	0.189576	0.180255	0.170079	0.160274
		$\widehat{\text{Var}}(\hat{\pi}^*_{\text{GR}})$	0.000540	0.000517	0.000517	0.000507	0.000489	0.000473
		$\text{Var}(\hat{\pi}^*_{\text{GR}})$	0.000536	0.000533	0.000522	0.000507	0.000492	0.000477
		$\text{Bias}(\hat{\pi}^*_{\text{GR}})$	0.000000	0.002000	0.010000	0.020000	0.030000	0.040000
		$\text{MSE}(\hat{\pi}^*_{\text{GR}})$	0.000536	0.000537	0.000622	0.000907	0.001392	0.002077
optional two-question	$W = 0.70$	$\hat{\pi}^*_{\text{SI}}$	0.200002	0.198722	0.193519	0.187040	0.180684	0.174260
		$\widehat{\text{Var}}(\hat{\pi}^*_{\text{SI}})$	0.000457	0.000440	0.000444	0.000431	0.000427	0.000414
		$\text{Var}(\hat{\pi}^*_{\text{SI}})$	0.000455	0.000454	0.000447	0.000438	0.000429	0.000420
		$\text{Bias}(\hat{\pi}^*_{\text{SI}})$	0.000000	0.001302	0.006512	0.013023	0.019535	0.026047
		$\text{MSE}(\hat{\pi}^*_{\text{SI}})$	0.000455	0.000455	0.000489	0.000607	0.000810	0.001098
	$W = 0.80$	$\hat{\pi}^*_{\text{SI}}$	0.199866	0.198278	0.191730	0.184722	0.176871	0.169212
		$\widehat{\text{Var}}(\hat{\pi}^*_{\text{SI}})$	0.000487	0.000487	0.000456	0.000461	0.000448	0.000435
		$\text{Var}(\hat{\pi}^*_{\text{SI}})$	0.000480	0.000478	0.000470	0.000459	0.000449	0.000438
		$\text{Bias}(\hat{\pi}^*_{\text{SI}})$	0.000000	0.001524	0.007619	0.015238	0.022857	0.030476
		$\text{MSE}(\hat{\pi}^*_{\text{SI}})$	0.000480	0.000481	0.000528	0.000692	0.000971	0.001366
	$W = 0.90$	$\hat{\pi}^*_{\text{SI}}$	0.200340	0.198214	0.191183	0.182637	0.173468	0.165007
		$\widehat{\text{Var}}(\hat{\pi}^*_{\text{SI}})$	0.000501	0.000510	0.000505	0.000485	0.000482	0.000452
		$\text{Var}(\hat{\pi}^*_{\text{SI}})$	0.000507	0.000505	0.000495	0.000483	0.000470	0.000457
		$\text{Bias}(\hat{\pi}^*_{\text{SI}})$	0.000000	0.001756	0.008780	0.017561	0.026341	0.035122
		$\text{MSE}(\hat{\pi}^*_{\text{SI}})$	0.000507	0.000508	0.000572	0.000791	0.001164	0.001690

Table 1. Simulation results for all models under untruthful responding: iterations $= 10,000$, $n = 500$, $p = 0.8$, $p_b = 0.8$, $\pi_y = 0.3$, $\pi_b = 0.1$, $\pi_x = 0.2$.

π_a	π_x 0.10	0.20	0.30
1.00	0.9533	0.9915	**1.0000**
0.99	0.9224	0.9711	0.9890
0.95	0.8519	**1.0234**	**1.1907**
0.90	0.8519	**1.3219**	**1.8816**
0.85	0.9117	**1.7858**	**2.8979**
0.80	**1.0003**	**2.3302**	**4.0853**

Table 2. Percent relative efficiency $\mathrm{PRE}(\hat{\pi}_{\mathrm{YO}}, \hat{\pi}^*_{\mathrm{GR}})$ under untruthful responding: $n = 500,\ p = 0.8,\ p_b = 0.8,\ \pi_y = 0.3,\ \pi_b = 0.1$.

proportion of truthful responding) to vary and fix other parameters at $\pi_x = 0.2$, $p = 0.8$, $p_b = 0.8$, $\pi_y = 0.3$, and $\pi_b = 0.1$. Note that the proposed model's estimate for the proportion of truthful responding ($\hat{\pi}_a$) is also included in Table 1.

Notice that $\hat{\pi}^*_{\mathrm{GR}}$ and $\hat{\pi}^*_{\mathrm{SI}}$ underestimate the prevalence of the sensitive trait when the proportion of truthful responding is less than 1. This is due to the bias introduced to these models under untruthful responding. To compare the efficiency of the proposed model to those of existing binary RRT models when some untruthful responding is suspected, we use the *percent relative efficiency* (PRE), where

$$\mathrm{PRE}(\hat{\pi}_{\mathrm{YO}}, \hat{\pi}^*_{\mathrm{GR}}) = \frac{\mathrm{MSE}(\hat{\pi}^*_{\mathrm{GR}})}{\mathrm{MSE}(\hat{\pi}_{\mathrm{YO}})}, \tag{5-1}$$

$$\mathrm{PRE}(\hat{\pi}_{\mathrm{YO}}, \hat{\pi}^*_{\mathrm{SI}}) = \frac{\mathrm{MSE}(\hat{\pi}^*_{\mathrm{SI}})}{\mathrm{MSE}(\hat{\pi}_{\mathrm{YO}})}. \tag{5-2}$$

A PRE value of 1 or greater favors the proposed model over the existing model. The proposed model is unbiased under untruthful responding, therefore

$$\mathrm{MSE}(\hat{\pi}_{\mathrm{YO}}) = \mathrm{Var}(\hat{\pi}_{\mathrm{YO}}). \tag{5-3}$$

The comparison of the proposed model with the [Greenberg et al. 1969] model with untruthful responding can be seen in Table 2. We can see that when the prevalence of the sensitive trait is at least 20% ($\pi = 0.20$), and as low as only 5% of respondents give untruthful responses ($\pi_a = 0.95$), the proposed model is generally preferred over the original Greenberg model.

The comparison of the proposed model to that of [Sihm et al. 2016] can be found in Table 3. We can see that when the sensitivity level of the question, the prevalence of the sensitive trait, or the proportion of respondents who give an untruthful response increases, the proposed model tends to be more efficient than that of [Sihm et al. 2016].

W	π_a	π_x 0.10	0.20	0.30
0.70	1.00	0.7689	0.8417	0.8653
	0.99	0.7439	0.8226	0.8511
	0.95	0.6700	0.8045	0.8994
	0.90	0.6224	0.8847	**1.1486**
	0.85	0.6069	**1.0392**	**1.5386**
	0.80	0.6093	**1.2318**	**2.0041**
0.80	1.00	0.8268	0.8882	0.9070
	0.99	0.8000	0.8685	0.8935
	0.95	0.7260	0.8688	0.9829
	0.90	0.6901	**1.0075**	**1.3514**
	0.85	0.6942	**1.2454**	**1.9110**
	0.80	0.7192	**1.5330**	**2.5721**
0.90	1.00	0.8891	0.9382	0.9518
	0.99	0.8603	0.9181	0.9394
	0.95	0.7873	0.9416	**1.0794**
	0.90	0.7671	**1.1525**	**1.5940**
	0.85	0.7959	**1.4927**	**2.3607**
	0.80	0.8493	**1.8966**	**3.2605**

Table 3. Percent relative efficiency PRE($\hat{\pi}_{\mathrm{YO}}, \hat{\pi}^*_{\mathrm{SI}}$) under untruthful responding: $n = 500,\ p = 0.8,\ p_b = 0.8,\ \pi_y = 0.3,\ \pi_b = 0.1$.

6. Conclusion

We propose a binary unrelated-question RRT model that accounts for untruthful responding. This model provides an unbiased estimator, whereas existing models are biased under untruthful responding. This provides an additional layer of precaution to the estimation of a sensitive trait. We found that there are many scenarios in which this model would be preferred over the model of [Greenberg et al. 1969] and even when it would be preferred over an optional binary RRT model as in [Sihm et al. 2016].

For instance, when the prevalence of the sensitive trait is high or the proportion of untruthful responding is high, we found that the proposed model has a higher efficiency than that of [Greenberg et al. 1969] and [Sihm et al. 2016]. However, when the proportion of untruthful responding is low and the prevalence of the sensitive trait is also low, it may not be worth expending the extra energy in estimating π_a. It also may not be worth expending the energy when comparing the proposed model to [Sihm et al. 2016] when the sensitivity level of the question is low.

In examining the advantage of the proposed method over the existing methods, we have relied on the commonly used approach of looking at the percent relative

π_a	π_x 0.20	π_x 0.30
1.00	587	546
0.99	599	552
0.95	558	451
0.90	415	277
0.85	298	177
0.80	224	125

Table 4. Sample size needed for proposed model to achieve the same efficiency as Greenberg model under untruthful responding (MSE($\hat{\pi}^*_{\mathrm{GR}}$)) with a fixed sample size of $n = 500$, $p = 0.8$, $p_b = 0.8$, $\pi_y = 0.3$, $\pi_b = 0.1$.

efficiency, as seen in Tables 2 and 3. In this approach, we keep the sample size fixed and look at the mean squared error (MSE) of one model as compared to the other. An alternative approach could be to look at the MSE of one model with a fixed sample size, and then see what sample size would be necessary for the proposed model to achieve the same efficiency. Limited results are presented in Table 4 to give the reader some idea as to how much reduction in sample size can be achieved by the proposed method. It is clear by these results that when the proportion of untruthful responding is high or the prevalence of the sensitive trait is high, the proposed model can offer a large reduction in sample size while achieving the same efficiency of other models. However, when either of these values is very low, it again may not be worth the energy to estimate π_a.

It is also important to note that, because the proposed model is unbiased under untruthful responding, it eliminates the need for extensive presurvey training of respondents, as the purpose of presurvey training is to minimize untruthful responding.

Acknowledgements

The authors would like to express their sincere appreciation for the careful reading of the original version of this paper by the reviewer and the editor. Their comments have helped improve the presentation of our work considerably.

This material is based upon work supported by the National Science Foundation, grant no. DMS-1560332.

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Received: 2018-11-02 Revised: 2019-03-23 Accepted: 2019-05-01

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Involve (ISSN 1944-4184 electronic, 1944-4176 printed) at Mathematical Sciences Publishers, 798 Evans Hall #3840, c/o University of California, Berkeley, CA 94720-3840, is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

Involve peer review and production are managed by EditFlow® from Mathematical Sciences Publishers.

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2019 vol. 12 no. 7

1944-4176(2019)12:7;1-Z