

involve

a journal of mathematics

Toward a Nordhaus–Gaddum inequality
for the number of dominating sets

Lauren Keough and David Shane



Toward a Nordhaus–Gaddum inequality for the number of dominating sets

Lauren Keough and David Shane

(Communicated by Kenneth S. Berenhaut)

A dominating set in a graph G is a set S of vertices such that every vertex of G is either in S or is adjacent to a vertex in S . Nordhaus–Gaddum inequalities relate a graph G to its complement $\bar{G}$. In this spirit Wagner proved that any graph G on n vertices satisfies $\partial(G) + \partial(\bar{G}) \geq 2^n$, where $\partial(G)$ is the number of dominating sets in a graph G . In the same paper he commented that proving an upper bound for $\partial(G) + \partial(\bar{G})$ among all graphs on n vertices seems to be much more difficult. Here we prove an upper bound on $\partial(G) + \partial(\bar{G})$ and prove that any graph maximizing this sum has minimum degree at least $\lfloor n/2 \rfloor - 2$ and maximum degree at most $\lceil n/2 \rceil + 1$. We conjecture that the complete balanced bipartite graph maximizes $\partial(G) + \partial(\bar{G})$ and have verified this computationally for all graphs on at most 10 vertices.

1. Introduction

A *dominating set* in a graph G is a set of vertices S such that every vertex of G is either in S or adjacent to a vertex in S . Dominating sets, and their many variations, have long been studied [Haynes et al. 1998]. Also long-studied are Nordhaus–Gaddum inequalities, which describe the relationship between a graph parameter on G and the same graph parameter on $\bar{G}$, the complement of G , in terms of the order of the graph. The original Nordhaus–Gaddum inequalities concern the chromatic number of a graph G , denoted by $\chi(G)$. Nordhaus and Gaddum [1956] proved that if G has n vertices then

$$2\sqrt{n} \leq \chi(G) + \chi(\bar{G}) \leq n + 1$$

and

$$n \leq \chi(G) \cdot \chi(\bar{G}) \leq \left(\frac{n+1}{2}\right)^2.$$

Since then there have been several hundred papers proving similar relations for many different graph parameters [Aouchiche and Hansen 2013]. In particular, there

MSC2010: 05C35, 05C69.

Keywords: Nordhaus–Gaddum inequalities, dominating sets.

are such inequalities for the domination number (the size of a smallest dominating set) [Jaeger and Payan 1972; Borowiecki 1976]. See [Aouchiche and Hansen 2013] and [Harary and Haynes 1996] for surveys of results concerning Nordhaus–Gaddum inequalities for at least 30 types of domination numbers.

Separately, there has been interest in results concerning maximizing or minimizing the *number* of a given graph substructure, rather than its size, subject to certain conditions. For a survey on these types of problems for regular graphs see [Zhao 2017]. Recently, there have been several papers that maximize or minimize the total number of dominating sets or total dominating sets for connected graphs of a given order [Bród and Skupień 2006; Wagner 2013; Skupień 2014; Krzywkowski and Wagner 2018].

Let $\partial(G)$ be the number of dominating sets in a graph G . Uniting the ideas of Nordhaus–Gaddum inequalities and counting the number of graph substructures, Wagner [2013] proved that

$$\partial(G) + \partial(\bar{G}) \geq 2^n.$$

In the same paper, he proposed that determining the maximum of $\partial(G) + \partial(\bar{G})$ as G ranges over all possible graphs on n vertices seems to be much more difficult. We are able to prove the following theorem.

Theorem 1.1. *If G is a graph on n vertices, then*

$$\partial(G) + \partial(\bar{G}) \leq 2^{n+1} - 2^{\lfloor n/2 \rfloor} - 2^{\lceil n/2 \rceil - 1}.$$

However, this is not the least upper bound. The authors and Wagner conjecture that the extremal graph is the complete balanced bipartite graph, leading to the following conjecture.

Conjecture 1.2. *For a graph G on n vertices,*

$$\begin{aligned} \partial(G) + \partial(\bar{G}) &\leq 2(2^{\lfloor n/2 \rfloor} - 1)(2^{\lceil n/2 \rceil} - 1) + 2 \\ &= \partial(K_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}) + \partial(\bar{K}_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}). \end{aligned}$$

This conjecture has been verified up to $n = 10$ vertices. Wagner pointed out that this conjecture makes heuristic sense as both the complete balanced bipartite graph and its complement can be dominated by only two vertices (personal communication, October 3, 2017).

Throughout the paper we use $N_G(v)$ to mean the open neighborhood of the vertex v in the graph G and $N_G[v]$ for the closed neighborhood of v in G . If S is a set of vertices we define $N_G(S)$ and $N_G[S]$ similarly. In Section 2 we prove Theorem 1.1. In Section 3 we provide a maximum and minimum degree condition for the extremal graph. Finally, in Section 4 we provide some asymptotics and describe some of the difficulties in finding the least upper bound for $\partial(G) + \partial(\bar{G})$.

2. An upper bound for $\partial(G) + \partial(\bar{G})$

To prove that $\partial(G) + \partial(\bar{G}) \geq 2^n$, Wagner [2013] used the fact that if a set S does not dominate G , then $\bar{S}$ dominates $\bar{G}$. We use this same fact to express the sum of the number of dominating sets in G and $\bar{G}$ as

$$\partial(G) + \partial(\bar{G}) = 2^n + \Upsilon(G, \bar{G}),$$

where

$$\Upsilon(G, \bar{G}) = |\{A \subseteq V(G) : A \text{ dominates } G \text{ and } \bar{A} \text{ dominates } \bar{G}\}|.$$

We make use of $\Upsilon(G, \bar{G})$ to establish the following upper bound.

Lemma 2.1. *If G is a graph on n vertices and a vertex $v \in V(G)$ has $\deg_G(v) = k$, then*

$$\partial(G) + \partial(\bar{G}) \leq 2^{n+1} - 2^k - 2^{n-k-1}.$$

Proof. We bound $\Upsilon(G, \bar{G})$ in terms of n and k and thus bound $\partial(G) + \partial(\bar{G})$ in terms of n and k . It will be helpful to visualize G and $\bar{G}$ as shown in Figure 1. Note that the graphs in Figure 1 do not include any edges that are not incident with v , but every edge is in either G or $\bar{G}$.

Let's consider a set $S \subseteq V(G)$ with the following properties:

- $v \in S$.
- $N_{\bar{G}}(v) = \overline{N_G[v]} \subseteq S$.

We claim that $\bar{S}$ is not a dominating set of $\bar{G}$. Since $\bar{S} \cap N_{\bar{G}}(v) = \emptyset$ and $v \notin \bar{S}$, we have that $v \notin N_{\bar{G}}[\bar{S}]$. Thus, $\bar{S}$ is not a dominating set of $\bar{G}$. Therefore all sets satisfying the construction of S are not counted in $\Upsilon(G, \bar{G})$. Since each element of $N_G(v)$ may or may not be included in S and $|N_G(v)| = \deg_G(v) = k$, we have identified 2^k sets that are not in $\Upsilon(G, \bar{G})$.

Let's now consider a set $T \subseteq V(G)$ with the following properties:

- $v \notin T$.
- $T \cap N_G(v) = \emptyset$.

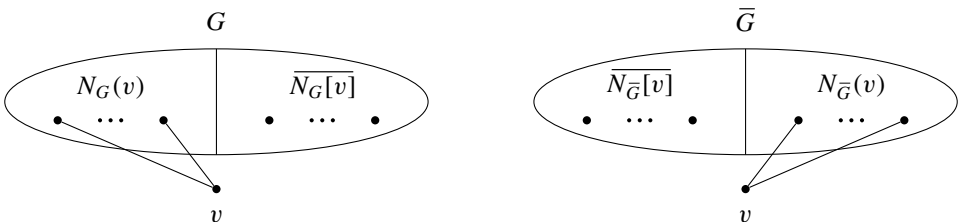


Figure 1. A drawing of G and $\bar{G}$ to aid in the proof of Lemma 2.1.

Since $v \notin N_G[T]$, we know T is not a dominating set of G and all sets satisfying the construction of T are not counted in $\Upsilon(G, \bar{G})$. Since each element of $N_{\bar{G}}(v)$ may or may not be included in T and $|N_{\bar{G}}(v)| = n - k - 1$, we have identified 2^{n-k-1} sets that are not in $\Upsilon(G, \bar{G})$.

No sets satisfy the construction of both S and T since $v \in S$ and $v \notin T$ and so we have $2^k + 2^{n-k-1}$ sets that are not counted in $\Upsilon(G, \bar{G})$. We conclude $\Upsilon(G, \bar{G}) \leq 2^n - (2^k + 2^{n-k-1})$ and thus

$$\partial(G) + \partial(\bar{G}) = 2^n + \Upsilon(G, \bar{G}) \leq 2^{n+1} - 2^k - 2^{n-k-1}. \quad \square$$

To prove [Theorem 1.1](#) we apply [Lemma 2.1](#) for a vertex of degree at least $\lfloor n/2 \rfloor$, which must exist in either G or $\bar{G}$. This eliminates the need for the knowledge of the degree of a specific vertex in G .

Proof of Theorem 1.1. Let G be a graph on n vertices. Since $\max\{\Delta(G), \Delta(\bar{G})\} \geq \lfloor n/2 \rfloor$, there exists some vertex $v \in V(G)$ such that $\deg_G(v) = \lfloor n/2 \rfloor + d$ or $\deg_{\bar{G}}(v) = \lfloor n/2 \rfloor + d$, where $d \geq 0$. Without loss of generality suppose $\deg_G(v) = \lfloor n/2 \rfloor + d$, where $d \geq 0$. From [Lemma 2.1](#) we have

$$\partial(G) + \partial(\bar{G}) \leq 2^{n+1} - 2^{\lfloor n/2 \rfloor + d} - 2^{n - (\lfloor n/2 \rfloor + d) - 1} = 2^{n+1} - 2^d \cdot 2^{\lfloor n/2 \rfloor} - \frac{2^{\lceil n/2 \rceil - 1}}{2^d}.$$

Considering the cases $d = 0$ and $d > 0$ separately we have

$$\partial(G) + \partial(\bar{G}) \leq 2^{n+1} - 2^d \cdot 2^{\lfloor n/2 \rfloor} - \frac{2^{\lceil n/2 \rceil - 1}}{2^d} \leq 2^{n+1} - 2^{\lfloor n/2 \rfloor} - 2^{\lceil n/2 \rceil - 1}. \quad \square$$

3. Degree condition

We now use [Lemma 2.1](#) and our conjectured extremal graph to get a degree condition on all possible extremal graphs.

Theorem 3.1. *If G is a graph on n vertices that maximizes $\partial(G) + \partial(\bar{G})$, then $\min\{\delta(G), \delta(\bar{G})\} \geq \lfloor n/2 \rfloor - 2$ and $\max\{\Delta(G), \Delta(\bar{G})\} \leq \lceil n/2 \rceil + 1$.*

Proof. Let G be a graph on n vertices such that G maximizes $\partial(G) + \partial(\bar{G})$. First suppose n is even. Suppose that for some $v \in V(G)$, we have $\deg_G(v) \geq n/2 + d$ for some integer $d \geq 2$. By [Lemma 2.1](#),

$$\begin{aligned} \partial(G) + \partial(\bar{G}) &\leq 2^{n+1} - 2^{n/2+d} - 2^{n-(n/2+d)-1} \\ &= 2^{n+1} - 2^{d-1} \cdot 2^{n/2+1} - \frac{2^{n/2+1}}{2^{d+2}} \\ &< 2^{n+1} - 2 \cdot 2^{n/2+1} \\ &< 2^{n+1} - 2^{n/2+2} + 4 \\ &= \partial(K_{n/2, n/2}) + \partial(\bar{K}_{n/2, n/2}). \end{aligned}$$

This contradicts that G is extremal. Therefore, $\deg_G(v) \leq n/2 + 1$. The same argument applies for $\bar{G}$, so $\deg_{\bar{G}}(v) \leq n/2 + 1$. For any vertex v , we have $\deg_G(v) + \deg_{\bar{G}}(v) = n - 1$ so these upper bounds imply

$$\deg_G(v) \geq n - \left(\frac{n}{2} + 1\right) - 1 = \frac{n}{2} - 2,$$

$$\deg_{\bar{G}}(v) \geq n - \left(\frac{n}{2} + 1\right) - 1 = \frac{n}{2} - 2.$$

These four inequalities imply the result when n is even.

Now suppose n is odd and that for some $v \in V(G)$, $\deg_G(v) \geq (n+1)/2 + d$, where $d \geq 2$. By [Lemma 2.1](#),

$$\begin{aligned} \partial(G) + \partial(\bar{G}) &\leq 2^{n+1} - 2^{(n+1)/2+d} - 2^{n-((n+1)/2+d)-1} \\ &= 2^{n+1} - 2^{d-1} \cdot 2^{(n+3)/2} - \frac{2^{(n+1)/2}}{2^{d+2}} \\ &< 2^{n+1} - 2 \cdot 2^{(n+3)/2} \\ &< 2^{n+1} - 2^{(n+3)/2} - 2^{(n+1)/2} + 4 \\ &= \partial(K_{(n+1)/2, (n-1)/2}) + \partial(\bar{K}_{(n+1)/2, (n-1)/2}). \end{aligned}$$

Again, this contradicts that G is extremal. Therefore, $\deg_G(v) \leq (n+1)/2 + 1$. As before this implies

$$\deg_{\bar{G}}(v) \leq \frac{n+1}{2} + 1,$$

$$\deg_G(v) \geq n - \left(\frac{n+1}{2} + 1\right) - 1 = \frac{n-1}{2} - 2,$$

$$\deg_{\bar{G}}(v) \geq n - \left(\frac{n+1}{2} + 1\right) - 1 = \frac{n-1}{2} - 2,$$

which imply the result when n is odd. $\square$

This theorem could be used in a future proof of [Conjecture 1.2](#), as it eliminates numerous graphs from consideration for each n .

4. Conclusion

There are several obstacles to proving [Conjecture 1.2](#) using some traditional techniques. One strategy would be to start with a graph and move edges between the graph and the complement in a way that increases $\partial(G) + \partial(\bar{G})$ at each edge move. However there are several examples that show this isn't possible. For example, $\partial(C_5) + \partial(\bar{C}_5) = 42$, but moving any edge results in only 40 dominating sets. Using a counting argument one can prove the following.

Proposition 4.1. *For any complete multipartite graph G on n vertices that is not the complete balanced bipartite graph or its complement*

$$\partial(G) + \partial(\bar{G}) < \partial(K_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}) + \partial(\bar{K}_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}).$$

If one could show that any extremal graph should be a complete multipartite graph then [Proposition 4.1](#) would complete a proof of [Conjecture 1.2](#).

A proof of [Conjecture 1.2](#) also doesn't work out nicely by induction on the number of vertices. Let H_n be the complete balanced bipartite graph on n vertices, G denote any graph on n vertices and $G + v$ mean the addition of one vertex, v , and any edges we want. We might try to prove that

$$(\partial(H_{n+1}) + \partial(\overline{H_{n+1}})) - (\partial(H_n) + \partial(\overline{H_n})) > (\partial(G + v) + \partial(\overline{G + v})) - (\partial(G) + \partial(\overline{G})).$$

That is, the step from a maximal graph to the maximal graph on one more vertex increases the Nordhaus–Gaddum sum by more than adding a vertex to any other graph would. However, as one example, $G = K_{1,3}$ does not have this property.

[Theorem 1.1](#) does give us a good result asymptotically. To see this, consider how close $\partial(G) + \partial(\overline{G})$ can be to 2^{n+1} (a trivial upper bound). The complete balanced bipartite graph shows that

$$\max\{\partial(G) + \partial(\overline{G})\} \geq 2^{n+1} - 2^{\lfloor n/2 \rfloor + 1} - 2^{\lceil n/2 \rceil + 1} + 4,$$

where the maximum is taken over all graphs G on n vertices. This shows that the gap between $\max\{\partial(G) + \partial(\overline{G})\}$ and 2^{n+1} is at most

$$\begin{aligned} (4 - o(1)) 2^{n/2} & \quad \text{if } n \text{ is even,} \\ (3\sqrt{2} - o(1)) 2^{n/2} & \quad \text{if } n \text{ is odd,} \end{aligned}$$

and we conjecture this gap is the smallest possible. From [Theorem 1.1](#) we know

$$\max(\partial(G) + \partial(\overline{G})) \leq 2^{n+1} - 2^{\lfloor n/2 \rfloor} - 2^{\lceil n/2 \rceil - 1},$$

which means that the gap is always at least

$$\begin{aligned} \left(\frac{3}{2}\right) 2^{n/2} & \quad \text{if } n \text{ is even,} \\ (\sqrt{2}) 2^{n/2} & \quad \text{if } n \text{ is odd.} \end{aligned}$$

Therefore, $2^{n/2}$ is the right order of magnitude for the gap between 2^{n+1} and $\max\{\partial(G) + \partial(\overline{G})\}$.

Acknowledgements

Shane was supported by the Alayont Undergraduate Research Fellowship in Mathematics at Grand Valley State University. We would like to thank David Galvin for his contributions to the analysis in the Conclusion and Stefan Wagner for his helpful comments on a draft of this paper.

References

- [Aouchiche and Hansen 2013] M. Aouchiche and P. Hansen, “A survey of Nordhaus–Gaddum type relations”, *Discrete Appl. Math.* **161**:4-5 (2013), 466–546. [MR](#) [Zbl](#)
- [Borowiecki 1976] M. Borowiecki, “On the external stability number of a graph and its complement”, *Prace Nauk. Inst. Mat. Politech. Wrocław* **12** (1976), 39–43. [Zbl](#)
- [Bród and Skupień 2006] D. Bród and Z. Skupień, “Trees with extremal numbers of dominating sets”, *Australas. J. Combin.* **35** (2006), 273–290. [MR](#) [Zbl](#)
- [Harary and Haynes 1996] F. Harary and T. W. Haynes, “Nordhaus–Gaddum inequalities for domination in graphs”, *Discrete Math.* **155**:1-3 (1996), 99–105. [MR](#) [Zbl](#)
- [Haynes et al. 1998] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, *Fundamentals of domination in graphs*, Monographs and Textbooks in Pure and Appl. Math. **208**, Dekker, New York, 1998. [MR](#) [Zbl](#)
- [Jaeger and Payan 1972] F. Jaeger and C. Payan, “Relations du type Nordhaus–Gaddum pour le nombre d’absorption d’un graphe simple”, *C. R. Acad. Sci. Paris Sér. A-B* **274** (1972), 728–730. [MR](#) [Zbl](#)
- [Krzywkowski and Wagner 2018] M. Krzywkowski and S. Wagner, “Graphs with few total dominating sets”, *Discrete Math.* **341**:4 (2018), 997–1009. [MR](#) [Zbl](#)
- [Nordhaus and Gaddum 1956] E. A. Nordhaus and J. W. Gaddum, “On complementary graphs”, *Amer. Math. Monthly* **63** (1956), 175–177. [MR](#) [Zbl](#)
- [Skupień 2014] Z. Skupień, “Majorization and the minimum number of dominating sets”, *Discrete Appl. Math.* **165** (2014), 295–302. [MR](#) [Zbl](#)
- [Wagner 2013] S. Wagner, “A note on the number of dominating sets of a graph”, *Util. Math.* **92** (2013), 25–31. [MR](#) [Zbl](#)
- [Zhao 2017] Y. Zhao, “Extremal regular graphs: independent sets and graph homomorphisms”, *Amer. Math. Monthly* **124**:9 (2017), 827–843. [MR](#) [Zbl](#)

Received: 2018-12-05

Revised: 2019-03-18

Accepted: 2019-03-21

keoulaur@gvsu.edu

*Department of Mathematics, Grand Valley State University,
Allendale, MI, United States*

shaned@mail.gvsu.edu

Michigan State University, East Lansing, MI, United States

INVOLVE YOUR STUDENTS IN RESEARCH

Involve showcases and encourages high-quality mathematical research involving students from all academic levels. The editorial board consists of mathematical scientists committed to nurturing student participation in research. Bridging the gap between the extremes of purely undergraduate research journals and mainstream research journals, *Involve* provides a venue to mathematicians wishing to encourage the creative involvement of students.

MANAGING EDITOR

Kenneth S. Berenhaut Wake Forest University, USA

BOARD OF EDITORS

Colin Adams	Williams College, USA	Robert B. Lund	Clemson University, USA
Arthur T. Benjamin	Harvey Mudd College, USA	Gaven J. Martin	Massey University, New Zealand
Martin Bohner	Missouri U of Science and Technology, USA	Mary Meyer	Colorado State University, USA
Amarjit S. Budhiraja	U of N Carolina, Chapel Hill, USA	Frank Morgan	Williams College, USA
Pietro Cerone	La Trobe University, Australia	Mohammad Sal Moslehian	Ferdowsi University of Mashhad, Iran
Scott Chapman	Sam Houston State University, USA	Zuhair Nashed	University of Central Florida, USA
Joshua N. Cooper	University of South Carolina, USA	Ken Ono	Univ. of Virginia, Charlottesville
Jem N. Corcoran	University of Colorado, USA	Yuval Peres	Microsoft Research, USA
Toka Diagana	Howard University, USA	Y.-F. S. Pétermann	Université de Genève, Switzerland
Michael Dorff	Brigham Young University, USA	Jonathon Peterson	Purdue University, USA
Sever S. Dragomir	Victoria University, Australia	Robert J. Plemmons	Wake Forest University, USA
Joel Foisy	SUNY Potsdam, USA	Carl B. Pomerance	Dartmouth College, USA
Erin W. Fulp	Wake Forest University, USA	Vadim Ponomarenko	San Diego State University, USA
Joseph Gallian	University of Minnesota Duluth, USA	Bjorn Poonen	UC Berkeley, USA
Stephan R. Garcia	Pomona College, USA	József H. Przytycki	George Washington University, USA
Anant Godbole	East Tennessee State University, USA	Richard Rebarber	University of Nebraska, USA
Ron Gould	Emory University, USA	Robert W. Robinson	University of Georgia, USA
Sat Gupta	U of North Carolina, Greensboro, USA	Javier Rojo	Oregon State University, USA
Jim Haglund	University of Pennsylvania, USA	Filip Saidak	U of North Carolina, Greensboro, USA
Johnny Henderson	Baylor University, USA	Hari Mohan Srivastava	University of Victoria, Canada
Glenn H. Hurlbert	Virginia Commonwealth University, USA	Andrew J. Sterge	Honorary Editor
Charles R. Johnson	College of William and Mary, USA	Ann Trenk	Wellesley College, USA
K. B. Kulasekera	Clemson University, USA	Ravi Vakil	Stanford University, USA
Gerry Ladas	University of Rhode Island, USA	Antonia Vecchio	Consiglio Nazionale delle Ricerche, Italy
David Larson	Texas A&M University, USA	John C. Wierman	Johns Hopkins University, USA
Suzanne Lenhart	University of Tennessee, USA	Michael E. Zieve	University of Michigan, USA
Chi-Kwong Li	College of William and Mary, USA		

PRODUCTION

Silvio Levy, Scientific Editor

Cover: Alex Scorpan

See inside back cover or msp.org/involve for submission instructions. The subscription price for 2019 is US \$195/year for the electronic version, and \$260/year (+\$35, if shipping outside the US) for print and electronic. Subscriptions, requests for back issues and changes of subscriber address should be sent to MSP.

Involve (ISSN 1944-4184 electronic, 1944-4176 printed) at Mathematical Sciences Publishers, 798 Evans Hall #3840, c/o University of California, Berkeley, CA 94720-3840, is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

Involve peer review and production are managed by EditFlow® from Mathematical Sciences Publishers.

PUBLISHED BY



mathematical sciences publishers

nonprofit scientific publishing

<http://msp.org/>

© 2019 Mathematical Sciences Publishers

involve

2019

vol. 12

no. 7

Asymptotic expansion of Warlimont functions on Wright semigroups	1081
MARCO ALDI AND HANQIU TAN	
A systematic development of Jeans' criterion with rotation for gravitational instabilities	1099
KOHL GILL, DAVID J. WOLLKIND AND BONNI J. DICHONE	
The linking-unlinking game	1109
ADAM GIAMBRONE AND JAKE MURPHY	
On generalizing happy numbers to fractional-base number systems	1143
ENRIQUE TREVIÑO AND MIKITA ZHYLINSKI	
On the Hadwiger number of Kneser graphs and their random subgraphs	1153
ARRAN HAMM AND KRISTEN MELTON	
A binary unrelated-question RRT model accounting for untruthful responding	1163
AMBER YOUNG, SAT GUPTA AND RYAN PARKS	
Toward a Nordhaus–Gaddum inequality for the number of dominating sets	1175
LAUREN KEOUGH AND DAVID SHANE	
On some obstructions of flag vector pairs (f_1, f_{04}) of 5-polytopes	1183
HYE BIN CHO AND JIN HONG KIM	
Benford's law beyond independence: tracking Benford behavior in copula models	1193
REBECCA F. DURST AND STEVEN J. MILLER	
Closed geodesics on doubled polygons	1219
IAN M. ADELSTEIN AND ADAM Y. W. FONG	
Sign pattern matrices that allow inertia $\mathbb{S}_n$	1229
ADAM H. BERLINER, DEREK DEBLIECK AND DEEPAK SHAH	
Some combinatorics from Zeckendorf representations	1241
TYLER BALL, RACHEL CHAISER, DEAN DUSTIN, TOM EDGAR AND PAUL LAGARDE	