Appendix A. *Mathematica* code and output for $E\tau_m$

Equation (3.5) gives a closed form representation for $E\tau_m$ in terms of the q-gamma function ψ_m. In this section, using *Mathematica*, we compare the calculated results applying equation (3.5) with the results of simulations.

Here, we input m, generate 100,000 extinction times, and output four quantities: the empirical mean (based on the 100,000 simulations), the theoretical mean, the bounding interval described by Theorem 3.2, and the maximum time to extinction. We do this for three different values of m : .2, .5, and .999. The empirical results based on the simulations are very close to the theoretical results.

```
In[∘]:= f[m_] := Module[{a}, p = 1 / (1 + m);
          a := NestWhileList[Total[RandomVariate[GeometricDistribution[p], #]] &,
            1, (# ≠ 0) &];
          sims = Table[Length[a] - 1, 100 000];
          Print["Empirical Mean", "=", Mean[sims] // N];
          Print["Theoretical Mean", "=",
           (1 - 1 / m) (QPolyGamma[1, m] + Log[1 - m]) / Log[1 / m]];
          Print["Lower/Upper bound interval", "=",
           {-Log[1 - m] / m, -Log[1 - Sqrt[m]] / Sqrt[m]}];
          Print["SD", "=", StandardDeviation[sims] // N];
          Print["MaximumObservation", "=", Max[sims]]]

In[∘]:= f[.2]
        Empirical Mean=1.20841
        Theoretical Mean=1.20694
        Lower/Upper bound interval={1.11572, 1.3255}
        SD=0.514896
        MaximumObservation=7

In[∘]:= f[.5]
        Empirical Mean=1.60843
        Theoretical Mean=1.6067
        Lower/Upper bound interval={1.38629, 1.73658}
        SD=1.14559
        MaximumObservation=18

In[∘]:= f[.999]
        Empirical Mean=7.46778
        Theoretical Mean=7.48847
        Lower/Upper bound interval={6.91467, 7.60446}
        SD=58.6799
        MaximumObservation=4307
```

Appendix B. The variance of τ_m

Here we include *Mathematica*-generated graphic of the $\operatorname{Var}\tau_m$ as a function of m based on *Mathematica*'s numerical/functional approximations. A question that arose in the context of this research was whether or not, like the expected value of the extinction time, the variance of τ_m also increases with m. The computer output strongly suggests that it does, but we were not successful in proving it analytically.

```
In[ ]:= e[m_] := (1 - 1 / m) (QPolyGamma[0, 1, m] + Log[1 - m]) / Log[1 / m] // N (*Expected value of τm*)

In[ ]:= var[m_] := NSum[(2 n - 1) m ^ (n - 1) (1 - m) / (1 - m ^ n), {n, 1, Infinity}] -
          e[m] ^ 2 (*Variance of τm as defined by equation 29*)

In[ ]:= Plot[var[m], {m, 0, .95}]
```

Out[ ]=