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Jagan Nath Sharma and Naveen Thakur

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PLANE HARMONIC ELASTO-THERMODIFFUSIVE WAVES IN SEMICONDUCTOR MATERIALS

JAGAN NATH SHARMA AND NAVEEN THAKUR

The aim of this article is to give a detailed account of the plane harmonic generalized elasto-thermo-diffusive (ETNP) waves in semiconductive materials. The shear (purely transverse) waves get decoupled from the rest of the motion and remain independent of the influence of other fields. These waves propagate without dispersion and attenuation in semiconductors. The coupled system of partial differential equations, governing the rest of the interacting fields, has been solved to obtain a complex secular equation. According to the frequency equation, four coupled longitudinal waves, namely, the quasithermoelastic (QTE), quasielastodiffusive (QEN/QEP), quasithermodiffusive (QTN/QTP), and quasithermal (T-mode), can exist and propagate in an infinite semiconductor. The complex secular equation of plane harmonic waves in semiconductors is solved by using Descartes' algorithm and the irreducible case of Cardan's method in order to obtain phase velocities and attenuation coefficients of all possible coupled waves. The thermoelastic (ET), elastodiffusive (EN/EP) and thermodiffusive (TN/TP) waves have also been investigated as special cases. The derived theoretical results have been illustrated and verified numerically for germanium (Ge) and silicon (Si) semiconductors. The computed phase velocity and attenuation profiles have been presented graphically.

1. Introduction

Certain substances like germanium, silicon, carbon etc. are neither good conductors like copper nor insulators like glass. In other words, the resistivity (10^{-4} to $0.5 \Omega\text{m}$) of these materials lies between conductors and insulators. Such substances are classified as semiconductors. Semiconductors have some useful properties and are being extensively used in electronic circuits. For instance, transistor – a semiconductor device is fast replacing bulky vacuum tubes in almost all applications. A semiconductor has negative temperature coefficient of resistance i.e. the resistance of a semiconductor decreases with increase in temperature and vice-versa. Wave motion is a form of disturbance, which travels through a medium due to the repeated periodic motion of particles about their mean positions. The motion being handed over from one particle to the other. The waves which can be produced or propagated in a material medium, are called elastic (mechanical) waves. In case of large frictional forces present in the medium, the wave motion dies out soon. [Maruszewski 1986a; 1986b; 1987a] presented theoretical considerations of the simultaneous interactions of elastic, thermal and diffusion of charge carriers' fields in semiconductors. The problems of interaction of various fields were formulated mathematically by [Maruszewski 1989; 1986c; 1987b] based on the following assumptions:

- (i) All the considerations are made in the framework of the phenomenological model.
- (ii) The electric neutrality of the semiconductor is satisfied.

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- (iii) The magnetic field effect can be ignored.
- (iv) The mass of the charge carrier fields is negligible.
- (v) The surface heat sources are neglected.
- (vi) The electric field with in the boundary layer is very weak and can be neglected.
- (vii) The recombination functions of electrons and holes are selected on the basis of facts that take care of defects and hence the concentration values of the charge carrier fields according to [Many et al. 1965].
- (viii) The surface of the semiconductor is free of mechanical loading.
- (ix) The temperature $T_0 = \text{constant}$ is the uniform reference temperature and $\theta = T - T_0$, is the temperature change of the body.
- (x) The concentrations of electrons and holes satisfy the conditions $N = n - n_0$, $P = p - p_0$, where n , p and n_0 , p_0 are respectively the nonequilibrium and equilibrium values of electrons and holes concentrations.

Maruszewski [1989] studied the propagation of thermodiffusive surface waves in the semiconductors with relaxation of heat and charge carrier fields. The secular equation of coupled elastic, thermal and diffusive waves is obtained and illustrated by considering two particular cases of elastodiffusive (EN) and thermodiffusive (TN) waves.

The governing equations in classical dynamic coupled thermoelasticity are wave-type (hyperbolic) equations of motion and a diffusion type (parabolic) equation of heat conduction. Therefore, it is seen that a part of the solution of energy equation extends to infinity implying that if a homogeneous isotropic elastic medium is subjected to thermal or mechanical disturbance, the effects of temperature and displacement fields are experienced at an infinite distance from the source of disturbance. This shows that part of the disturbance has an infinite velocity of propagation, a physically impossible phenomenon. With this drawback in mind, some researches such as [Lord and Shulman 1967; Green and Lindsay 1972; Dhaliwal and Sherief 1980] and [Chandrasekharaiah 1986], modified the Fourier law of heat conduction and constitutive relations to obtain a hyperbolic equation for heat conduction. These models include the time needed for acceleration of heat flow in addition to the coupling between temperature and strain fields. According to the investigations of [Ackerman et al. 1966; Guyer and Krumhansl 1966; Ackerman and Overton 1969], these theories have also been supported with the experimental exhibition of actual occurrence of ‘second sound’ at low temperature and small intervals of time. [Banerjee and Pao 1974] investigated the propagation of plane harmonic waves in infinitely extended anisotropic thermoelastic solids by taking into account the thermal relaxation time. Extensive studies of wave propagation in heat conducting elastic solids under the influence of thermal relaxation time in “infinite velocity” and “finite velocity” descriptions, have been carried out by many investigators such as [Scott 1989; Chadwick 1979; Sharma et al. 2000; Sharma 1986] and [Sharma and Singh 1989; 1990].

The present article is aimed at giving a detailed account of the plane harmonic generalized thermoelastic waves in infinite semiconductor materials in context of the mathematical model formulated by [Maruszewski 1989]. The basic equations are first nondimensionalized and then solved by adopting the approach of [Achenbach 1973] after decoupling the shear waves’ (purely transverse) motion. The shear waves are not affected by thermal and charge carrier fields and also remain independent from the

rest of the motion. The frequency equation for rest of the motion reveals that, in general, there are four longitudinal waves namely, a quasithermoelastic (QTE), a quasithermal (T-mode) and two quasidiffusive (elastodiffusive QEN/QEP and thermodiffusive QTN/QTP waves), which can propagate in such semiconductor media. These waves are coupled and get modified due to thermal variations, thermal relaxation time, and life/relaxation times of electron and hole fields. The complex secular equation of coupled waves is then solved by using Descartes' algorithm along with irreducible Cardan's method. The analytical results so obtained have been verified numerically and illustrated graphically in case of Ge and Si materials.

2. Formulation of the problem

We consider an unbounded, homogeneous, isotropic, thermoelastic semiconductor at a uniform temperature T_0 in an undisturbed state. Let $\vec{u}(x_1, x_2, x_3, t) = (u_1, u_2, u_3)$ and $\theta(x_1, x_2, x_3, t)$ be the displacement vector and temperature change of the medium at any time t , respectively. The basic governing equations of motion and heat conduction, in the absence of body forces and heat sources, for such materials as given by [Maruszewski 1989] are

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \rho \ddot{u}_i - \lambda^n N_{,i} - \lambda^p P_{,i} - \lambda^T \theta_{,i} = 0, \quad (1)$$

$$\begin{aligned} K\theta_{,ii} + m^{nq} N_{,ii} + m^{pq} P_{,ii} \\ - \left(1 + t^Q \frac{\partial}{\partial t}\right) (\rho C_e \dot{\theta} + \rho T_0 \alpha^n \dot{N} + \rho T_0 \alpha^p \dot{P} + T_0 \lambda^T \dot{u}_{k,k}) - \rho (a_1^n \dot{N} + a_1^p \dot{P}) \\ = \left(a_1^n \left(\frac{\rho}{t_n^+}\right) N + a_1^p \left(\frac{\rho}{t_p^+}\right) P\right), \end{aligned} \quad (2)$$

$$\begin{aligned} \rho D^n N_{,ii} + m^{qn} \theta_{,ii} - \rho \left(1 - a_2^n T_0 \alpha^n + t^n \frac{\partial}{\partial t}\right) \dot{N} - a_2^n (\rho C_e \dot{\theta} + \rho T_0 \alpha^p \dot{P} + T_0 \lambda^T \dot{u}_{k,k}) \\ = - \left(1 + t^n \frac{\partial}{\partial t}\right) \left(\frac{\rho}{t_n^+}\right) N, \end{aligned} \quad (3)$$

$$\begin{aligned} \rho D^p P_{,ii} + m^{qp} \theta_{,ii} - \rho \left(1 - a_2^p T_0 \alpha^p + t^p \frac{\partial}{\partial t}\right) \dot{P} - a_2^p (\rho C_e \dot{\theta} + \rho T_0 \alpha^n \dot{N} + T_0 \lambda^T \dot{u}_{k,k}) \\ = - \left(1 + t^p \frac{\partial}{\partial t}\right) \left(\frac{\rho}{t_p^+}\right) P, \end{aligned} \quad (4)$$

where the notation

$$\begin{aligned} a_1^n = \frac{a^{Qn}}{a^Q}, \quad a_1^p = \frac{a^{Qp}}{a^Q}, \quad a_2^n = \frac{a^{Qn}}{a^n}, \quad a_2^p = \frac{a^{Qp}}{a^p}, \\ P = p - p_0, \quad N = n - n_0, \quad \lambda^T = (3\lambda + 2\mu)\alpha_T, \end{aligned}$$

is used. The field variables have been subjected to only those assumptions (except (v), (vi) and (viii)) of [Maruszewski 1989] that are applicable and relevant in the present context of an infinite description

of semiconductors. Here λ, μ are Lamé parameters; ρ is the density of the semiconductor; λ^n, λ^p are the elastodiffusive constants of electrons and holes; α_T is the coefficient of linear thermal expansion of the material; K is the thermal conductivity; α^p, α^n are thermodiffusive constants of holes and electrons; $a^{Qn}, a^{Qp}, a^Q, a^n, a^p$ are the flux-like constants; and D^n, D^p are the diffusion coefficients of electron and holes. The quantities $m^{nq}, m^{pq}, m^{qn}, m^{qp}$ are the Peltier–Seebeck–Dufour–Soret-like constants; t^Q, t^n and t^p are the relaxation times of heat, electron and hole fields, respectively; C_e is the specific heat; t_n^+, t_p^+ denotes the life times of the carriers' fields; and n, p and n_0, p_0 are the nonequilibrium and equilibrium values of electrons and holes, respectively. The comma notation is used for spatial derivatives and a superposed dot represents differentiation with respect to time.

We define the quantities

$$\begin{aligned}
 x'_i &= \frac{\omega^* x_i}{c_1}, & t'l &= \omega^* t, & \theta' &= \frac{\theta}{T_0}, & P' &= \frac{P}{p_0}, \\
 N' &= \frac{N}{n_0}, & u'_i &= \frac{\rho \omega^* c_1}{\lambda^T T_0} u_i, & t^{Q'} &= t^Q \omega^*, & t^{p'} &= t^p \omega^*, \\
 t^{n'} &= t^n \omega^*, & t_n^{+'} &= t_n^+ \omega^*, & t_p^{+'} &= t_p^+ \omega^*, & \delta^2 &= \frac{c_2^2}{c_1^2}, \\
 \varepsilon_T &= \frac{\lambda^{T^2} T_0}{\rho C_e (\lambda + 2\mu)}, & \omega^* &= \frac{C_e (\lambda + 2\mu)}{K}, & c_1^2 &= \frac{\lambda + 2\mu}{\rho}, \\
 c_2^2 &= \frac{\mu}{\rho}, & k &= \frac{K}{\rho C_e}, & \bar{\lambda}_n &= \frac{\lambda^n n_0}{\lambda^T T_0}, & \bar{\lambda}_p &= \frac{\lambda^p p_0}{\lambda^T T_0}, \\
 \varepsilon^{qn} &= \frac{m^{qn} T_0}{\rho D^n n_0}, & \varepsilon^{qp} &= \frac{m^{qp} T_0}{\rho D^p p_0}, & \varepsilon_n &= \frac{a_2^n K T_0}{\rho n_0 D^n}, & \varepsilon_p &= \frac{a_2^p K T_0}{\rho p_0 D^p}, \\
 \varepsilon^{pq} &= \frac{m^{pq} p_0}{K T_0}, & \varepsilon^{nq} &= \frac{m^{nq} n_0}{K T_0}, & \alpha_0^n &= \frac{a_1^n n_0}{C_e T_0}, & \alpha_0^p &= \frac{a_1^p p_0}{C_e T_0}, \\
 & & \alpha_0^n &= \frac{\alpha^n n_0}{C_e}, & \alpha_0^p &= \frac{\alpha^p p_0}{C_e}.
 \end{aligned} \tag{5}$$

Here ε_T and k are the thermoelastic coupling parameter and thermal diffusivity. Upon introducing the scalar point potential function ϕ and vector point potential function $\vec{\psi}$ defined by the relations

$$\vec{u} = \nabla \phi + \nabla \times \vec{\psi}, \quad \nabla \cdot \vec{\psi} = 0 \tag{6}$$

into Equations (1)–(4), along with the quantities in (5), we obtain

$$\nabla^2 \phi - \ddot{\phi} - \bar{\lambda}_n N - \bar{\lambda}_p P - \theta = 0, \tag{7a}$$

$$\begin{aligned}
 -\varepsilon_T \nabla^2 (\dot{\phi} + t^Q \ddot{\phi}) + \varepsilon^{nq} \nabla^2 N - \left\{ \frac{\alpha_0^n t^Q \partial^2}{\partial t^2} + (a_0^n + \alpha_0^n) \frac{\partial}{\partial t} + \frac{a_0^n}{t_n^+} \right\} N \\
 + \varepsilon^{pq} \nabla^2 P - \left\{ \frac{\alpha_0^p t^Q \partial^2}{\partial t^2} + (a_0^p + \alpha_0^p) \frac{\partial}{\partial t} + \frac{a_0^p}{t_p^+} \right\} P + \nabla^2 \theta - (\dot{\theta} + t^Q \ddot{\theta}) = 0,
 \end{aligned} \tag{7b}$$

$$\begin{aligned}
 -\varepsilon_n \varepsilon_T \nabla^2 \dot{\phi} + \nabla^2 N - \frac{K}{\rho C_e D^n} \left(-\frac{1}{t_n^+} + \left(1 - \frac{\varepsilon_n \alpha_0^n D^n}{k} - \frac{t^n}{t_n^+} \right) \frac{\partial}{\partial t} + \frac{t^n \partial^2}{\partial t^2} \right) N \\
 -\varepsilon_n \alpha_0^p \dot{P} - \varepsilon_n \dot{\theta} + \varepsilon^{qn} \nabla^2 \theta = 0,
 \end{aligned} \tag{7c}$$

$$\begin{aligned}
 -\varepsilon_p \varepsilon_T \nabla^2 \dot{\phi} + \nabla^2 P - \frac{K}{\rho C_e D^p} \left(-\frac{1}{t_p^+} + \left(1 - \frac{\varepsilon_p \alpha_0^p D^p}{k} - \frac{t^p}{t_p^+} \right) \frac{\partial}{\partial t} + \frac{t^p \partial^2}{\partial t^2} \right) P \\
 -\varepsilon_p \alpha_0^n \dot{N} - \varepsilon_p \dot{\theta} + \varepsilon^{qp} \nabla^2 \theta = 0,
 \end{aligned} \tag{7d}$$

$$\nabla^2 \vec{\psi} = \frac{1}{\delta^2} \ddot{\psi}. \tag{7e}$$

The last equation of (7e) corresponds to purely transverse waves which get decoupled from rest of the motion and are not affected by the thermal and charge carrier fields. These waves travel with nondimensional velocity δ without dispersion, attenuation, or damping. We drop this motion in the following analysis unless stated otherwise. Equations (7c) and (7d) can be further simplified under the assumption that the considered semiconductor is of relaxation type. For such materials, according to [Maruszewski 1989], the diffusion approximation of the physical process ceases to be obligatory and the relaxation/life times t^n, t_n^+, t^p, t_p^+ become comparable to each other in their values ($t^n = t_n^+, t^p = t_p^+$).

3. Solution of the problem

We may take plane harmonic wave solutions as

$$(\phi, \theta, N, P) = (\overline{\phi}, \overline{\theta}, \overline{N}, \overline{P}) \exp\{i\omega(v^{-1}x_r n_r - t)\}, \quad r = 1, 2, 3. \tag{8}$$

The use of solution (8) in the coupled system of equations (7a)–(7d), after straightforward algebraic reductions and manipulations, leads to the following characteristic equation

$$\zeta^4 - A\zeta^3 + B\zeta^2 - C\zeta + D = 0, \quad \zeta = v^{-2}, \tag{9}$$

where

$$\begin{aligned} A &= \frac{1 + (\Delta_1 + \Delta_2 + (1 + \varepsilon_T)\Delta_3)}{\Delta}, \\ B &= \frac{(\Delta'_1 + (1 + \varepsilon_T)(\Delta'_2 + \Delta'_3) + \Delta'_4 + \Delta'_5 + \Delta'_6)}{\Delta}, \\ C &= \frac{((1 + \varepsilon_T)\Delta''_1 + \Delta''_2 + \Delta''_3 + \Delta''_4)}{\Delta}, \\ D &= \frac{\Delta'''_1}{\Delta}. \end{aligned} \tag{10}$$

Here the quantities $\Delta_i (i = 1, 2, 3)$, $\Delta'_i (i = 1, 2, 3, 4, 5, 6)$, $\Delta''_i (i = 1, 2, 3, 4)$, Δ'''_1 and Δ are defined in the [Appendix. Equation \(9\)](#), being a fourth degree polynomial equation in ζ , has four roots, and hence in general, there are four ETNP waves: a QTE, a T-mode, and two quasidiffusive waves (QEN/QEP and QTN/QTP), in addition to purely transverse waves which can propagate in such semiconductive materials. The secular equation (9) with complex coefficients A , B , C , and D contains complete information regarding the wave number, frequency, phase velocity and attenuation coefficient of these waves. In order to solve the complex secular equation (9) we use Descartes' algorithm outlined below:

Shifting the roots of secular equation (9) by a factor of $\frac{A}{4}$ to eliminate the second term, we obtain

$$\zeta^4 + H\zeta^2 + G\zeta + I = 0, \tag{11}$$

where

$$\zeta = \zeta - \frac{A}{4}, \quad H = B - \frac{3A^2}{8}, \quad G = \frac{AB}{2} - \frac{A^3}{8} - C, \quad I = D + \frac{A^2B}{16} - \frac{3A^4}{256} - \frac{AC}{4}.$$

Factoring [Equation \(11\)](#) into two quadratic factors, we have

$$\zeta^4 + H\zeta^2 + G\zeta + I = (\zeta^2 + l\zeta + m)(\zeta^2 - l\zeta + n). \tag{12}$$

Comparing the coefficients of various powers of ζ in (12) on both sides, we get

$$m + n = l^2 + H, \quad n - m = \frac{G}{l}, \quad mn = I. \tag{13}$$

Eliminating m and n from [Equation \(13\)](#), we obtain

$$Z^3 + 2HZ^2 + (H^2 - 4I)Z - G^2 = 0, \tag{14}$$

where $Z = l^2$. Being cubic with complex coefficients, [Equation \(14\)](#) can be solved by using the irreducible case of Cardan's method with the help of De Moivre's theorem. We again shift the roots of (14) by a factor of $\frac{-2H}{3}$ in order to obtain the standard cubic as

$$Y^3 - 3H^*Y - G^* = 0, \tag{15}$$

where

$$Y = Z + \frac{2H}{3}, \quad H^* = \frac{(H^2 + 12I)}{9}, \quad G^* = G^2 - \frac{8HI}{3} + \frac{2H^3}{27}. \tag{16}$$

Let the roots of Equation (15) be of the type

$$Y = U + V \tag{17}$$

so that $U^3 + V^3 = G^*$, $U^3 V^3 = H^{*3}$.

We may find the cube roots with the help of De Moivre’s theorem, as shown below:

Let

$$U^3 = \frac{G^* + \sqrt{G^* - 4H^{*3}}}{2} = L + iM, \quad L, M \in R. \tag{18}$$

Then the values of U are given by

$$U_k = r^{1/3} \left(\cos \frac{2k\pi + \Phi}{3} + i \sin \frac{2k\pi + \Phi}{3} \right), \quad k = 0, 1, 2, \tag{19}$$

where $r = \sqrt{L^2 + M^2}$ and $\Phi = \tan^{-1}(M/L)$. Having determined U , the values of V can be obtained from the relation $UV = H^*$ which further leads to the required values of Y and hence to the values of $l^2 = Z = Y - 2H/3$. One of the (convenient) values of l so obtained is then used to evaluate m and n by Equation (13). Using the values of m, n and l , the reduced secular Equation (11) is factored into two quadratic factors of the type (12), which are further solved to obtain the four roots $\zeta_i, i = 1, 2, 3, 4$. The complex roots of secular equation (9) are obtained from the relation $\xi_i = \zeta_i + A/4$, for $i = 1, 2, 3, 4$. This leads to the determination of the complex phase velocities as

$$v_i = \frac{1}{\sqrt{\xi_i}}, \quad \text{for } i = 1, 2, 3, 4. \tag{20}$$

In general, v is complex, and hence we may write

$$v^{-1} = V^{-1} + i\omega^{-1}Q \tag{21}$$

so that the exponent part $\exp[i\omega(v^{-1}x_p n_p - t)]$ of solutions (8) can be rewritten as

$$\exp\{i\omega(V^{-1}x_p n_p - t) - Qx_p n_p\}.$$

This implies that V is the phase speed and Q the attenuation coefficient of ETNP waves.

Upon using representation (21) in Equation (20) we can obtain the phase velocity (V_i) and the attenuation coefficient (Q_i) of different modes of wave propagation. We obtain

$$V_i = \frac{1}{\text{Re} \sqrt{\xi_i}}, \quad Q_i = \omega I_m(\sqrt{\xi_i}), \quad i = 1, 2, 3, 4. \tag{22}$$

4. Special cases of wave solutions

In this section we consider some special cases of wave propagation such as EN/EP, TN/TP and ET waves, in semiconductor materials.

4.1. Elastodiffusive (EN/EP) waves. If we confine our discussion concerning the reciprocal dynamical interactions of elastic and electron diffusion fields to the propagation of EN waves, the system of equations (7) in the absence of thermal and hole charge carrier fields ($P = \theta = 0$, $\varepsilon_T = 0 = \varepsilon^{nq}$, $\alpha_0^n = 0 = a_0^n$) along with the solution to (8) leads to the secular equation

$$(\zeta - p_1^2)(\zeta - p_2^2) = 0, \quad (23)$$

where $p_1^2 + p_2^2 = 1 + \tau_n^{*'}$, $p_1^2 p_2^2 = \tau_n^{*'}$.

Here $\tau_n^{*'}$ is obtained from τ_n^* defined in the Appendix, on setting $\alpha_0^n = 0$. In general, the roots $\zeta = p_1^2, p_2^2$ are complex and hence waves are attenuated in space. The corresponding phase velocity depends directly on the relaxation and life times of the electrons. For relaxation type semiconductors ($t^n = t_n^+$), the quantity $\tau_n^{*'}$ becomes real and so are the roots $\zeta = p_1^2, p_2^2$. Therefore for such semiconductors EN waves propagate without dispersion, damping, and attenuation, which is in agreement with [Maruszewski 1989]. The amplitude ratios of the waves in this case are related by

$$\frac{\bar{N}}{\bar{\phi}} = k^2(v^2 - 1)/\bar{\lambda}_n. \quad (24)$$

Upon using representation (21), the phase velocities and attenuation coefficients of EN waves are obtained as

$$V_i = \frac{1}{\text{Re}(p_i)}, \quad Q_i = \omega \text{Im}(p_i), \quad i = 1, 2. \quad (25)$$

The EP-waves can also be discussed in a similar manner by omitting the influence of thermal and electron fields ($N = \theta = 0$, $\varepsilon_T = 0 = \varepsilon^{pq}$, $\alpha_0^p = 0 = a_0^p$). The corresponding results can be obtained by replacing (N, n) with (P, p) in the above analysis.

4.2. TN/TP waves. Here we confine our discussion concerning the reciprocal dynamical interactions of the thermal and electron diffusion fields to the propagation of TN waves, and omit the elastic and hole charge carrier fields ($\phi = 0 = P$, $\alpha_0^p = 0 = \varepsilon_p$, $\varepsilon^{qp} = 0 = \varepsilon_T$). The system of equations (7) with the help of solution (8) in this case leads to the secular equation

$$(\zeta - q_1^2)(\zeta - q_2^2) = 0, \quad (26)$$

where

$$q_1^2 + q_2^2 = \frac{\tau^Q + \tau_n^* - \varepsilon^{qn} \tau_n' - i\omega^{-1} \varepsilon_n \varepsilon^{nq}}{1 - \varepsilon^{nq} \varepsilon^{qn}}, \quad q_1^2 q_2^2 = \frac{\tau^Q \tau_n^* - i\omega^{-1} \varepsilon_n \tau_n'}{1 - \varepsilon^{nq} \varepsilon^{qn}}. \quad (27)$$

The complex phase velocity can be obtained from Equation (26) as

$$v_i^{-1} = q_i, \quad i = 1, 2. \quad (28)$$

In this case the waves are attenuated in space and damped with time even for relaxation type of semiconductors. Upon using representation (21) the real phase velocities and attenuation coefficients of TN waves are obtained as

$$V_i = \frac{1}{\text{Re}(q_i)}, \quad Q_i = \omega \text{Im}(q_i), \quad i = 1, 2. \quad (29)$$

Here the amplitude ratios are given by

$$\frac{\bar{\theta}}{\bar{N}} = \frac{\tau'_n v^2 - \varepsilon^{nq}}{1 - \tau Q v^2} = \frac{\tau_n^* v^2 - 1}{\varepsilon^{qn} - i\omega^{-1} \varepsilon_n v^2}. \tag{30}$$

The results pertaining to TP waves can be obtained from the above analysis by replacing N, n with P, p after setting ($\phi = 0 = N, \alpha_0^n = 0 = \varepsilon_n, \varepsilon^{qn} = 0 = \varepsilon_T$) in the governing equations (7).

4.3. ET waves. When a complete equilibrium state of electron and hole concentration is established the system becomes charge-free. Here, we confine our discussion concerning the reciprocal dynamical interactions of elastic and thermal fields in the absence of electron and hole fields ($N = 0 = P, \varepsilon_n = \varepsilon_p = 0, \varepsilon^{qn} = 0 = \varepsilon^{qp}$) to the propagation of ET waves.

In this case the system of equations (7) governing the interaction along with solution (8) leads to the secular equation for ET waves as under

$$(\xi - a_1^2)(\xi - a_2^2) = 0, \tag{31}$$

where

$$a_1^2 + a_2^2 = 1 + (1 + \varepsilon_T)\tau Q, \quad a_1^2 a_2^2 = \tau Q \tag{32}$$

and τQ is defined in the Appendix. The secular equation (31) gives us $v_i^{-1} = \pm a_i, i = 1, 2$.

These quantities are also complex so the waves are attenuated in space and damped with time. Upon using representation (21) the real phase velocities and attenuation coefficients of ET waves are obtained as

$$V_i = \frac{1}{\text{Re}(a_i)}, \quad Q_i = \omega \text{Im}(a_i), \quad i = 1, 2. \tag{33}$$

In this case the amplitude ratios are given by

$$\frac{\bar{\theta}}{\bar{\phi}} = k^2(v^2 - 1). \tag{34}$$

This type of wave motion has already been discussed by many authors such as [Chandrasekharaiah 1986; Chadwick 1979; Chadwick and Seet 1970; Sharma et al. 2000; Sharma 1986; Sharma and Singh 1989; 1990; Scott 1989].

5. Numerical results and discussion

In this section the values of phase velocity and attenuation coefficient of various partial wave modes have been computed numerically from the analytical results obtained above for Ge and Si materials under the assumption that the semiconductor considered is of relaxation type. In such a case, the diffusion approximation of the physical processes ceases to be obligatory, and t_n, t_n^+, t_p, t_p^+ become comparable to each other in their values so that $t_n = t_n^+, t_p = t_p^+$. The physical data for these materials is given in Table 1. To understand the interactions of various fields considered in thermoelastic semiconductors, the nondimensional phase velocities and attenuation coefficients of different modes of wave propagation have been obtained and computed numerically for Ge and Si materials, and their profiles are plotted on log-linear scale against nondimensional frequency (ω) in Figures 1, 2–7, 8. The phase velocity and attenuation coefficient profiles in special cases of ET, EN/EP and TN/TP waves have also been computed

Coefficient	Unit	Value (Ge)	Value (Si)	Reference
λ	Nm^{-2}	0.48×10^{11}	0.64×10^{11}	[Maruszewski 1989]
μ	Nm^{-2}	0.53×10^{11}	0.65×10^{11}	[Maruszewski 1989]
ρ	Kgm^{-3}	5.3×10^3	2.3×10^3	[Maruszewski 1989]
t_n^+	s	$< 10^{-5}$	$< 1.4 \times 10^{-6}$	[Maruszewski 1989]
t_p^+	s	$< 10^{-5}$	$< 10^{-5}$	[Maruszewski 1989]
D^n	m^2s^{-1}	10^{-2}	0.35×10^{-2}	[Maruszewski 1989]
D^p	m^2s^{-1}	0.5×10^{-2}	0.125×10^{-2}	[Maruszewski 1989]
K	$\omega\text{m}^{-1}\text{K}^{-1}$	60	150	[Sze 1981]
C_e	$\text{JKg}^{-1}\text{K}^{-1}$	310	700	[Sze 1981]
α_T	K^{-1}	5.8×10^{-6}	2.6×10^{-6}	[Sze 1981]
$n_0 = p_0$	m^{-3}	10^{20}	10^{20}	[Zambuto 1989]
α^p	m^2 / s	1.3×10^{-3}	5×10^{-3}	[Zambuto 1989]
α^n	m^2 / s	3.4×10^{-3}	1×10^{-2}	[Zambuto 1989]
m^{nq}	vk^{-1}	1.4×10^{-5}	1.4×10^{-5}	[Lal 1995]
m^{pq}	vk^{-1}	-0.004×10^{-6}	-0.004×10^{-6}	[Lal 1995]
m^{qn}	vk^{-1}	1.4×10^{-5}	1.4×10^{-5}	[Lal 1995]
m^{qp}	vk^{-1}	-0.004×10^{-6}	-0.004×10^{-6}	[Lal 1995]

Table 1. Physical data of germanium and silicon.

and represented graphically on log-linear scales in Figures 9–15. The numerical results are found to be in close agreement with the theoretical analysis.

Figure 1 shows the phase velocity (V_1) profiles of QTE waves with frequency in Ge and Si semiconductor materials. The variations of phase velocity at low frequency ($\omega \ll 1$) limits are quite small as compared to that at high-frequency ($\omega \gg 1$). The phase velocity in both the cases increases sharply in the frequency range $0.3 \leq \omega \leq 100$ to attain its maximum value at $\omega = 100$ for Ge and at $\omega = 30$ in case of Si, and then becomes steady after a slight decline in its value for $\omega \geq 100$. Although the effect of relaxation time of heat transportation is negligibly small, it is still more significant at high frequency (isothermal) conditions than at low frequency (isentropic) limits, especially when $\omega \geq 10$. This also shows that thermal relaxation (second sound) effects are short lived. The behavior of dispersion curves for Ge and Si materials is similar except that the magnitude of velocity in the latter one is quite small. Figure 2 shows nondimensional attenuation-frequency curves of QTE waves. As evident from Figures 1 and 2, the behavior of nondimensional attenuation coefficient (Q_1) is similar to that of nondimensional velocity (V_1) for Ge. It assumes maximum value at $\omega = 100$ in higher frequency regime and is significantly affected by thermal relaxation time of heat transportation. The attenuation profile of silicon (Si) is linear everywhere except in the range $1 \leq \omega \leq 100$, where it has Gaussian character with mean value at $\omega = 10$.

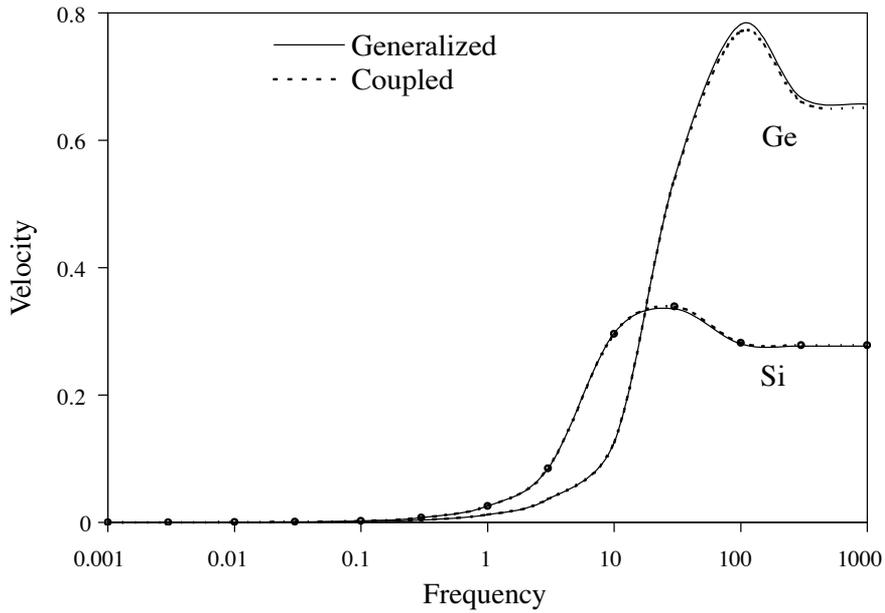


Figure 1. Phase velocity profile of QTE waves.

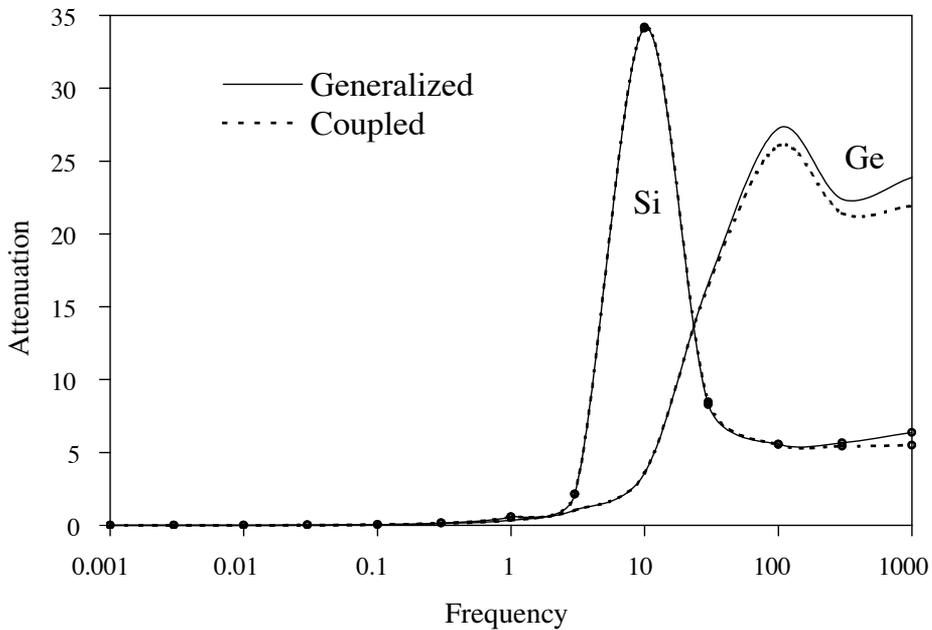


Figure 2. Attenuation coefficient profile of QTE waves.

Figure 3 shows nondimensional phase velocity (V_2) profiles of QEN/QEP waves with nondimensional frequency (ω) in Ge and Si semiconductor materials. For the $0 \leq \omega \leq 10$ frequency range, the variations

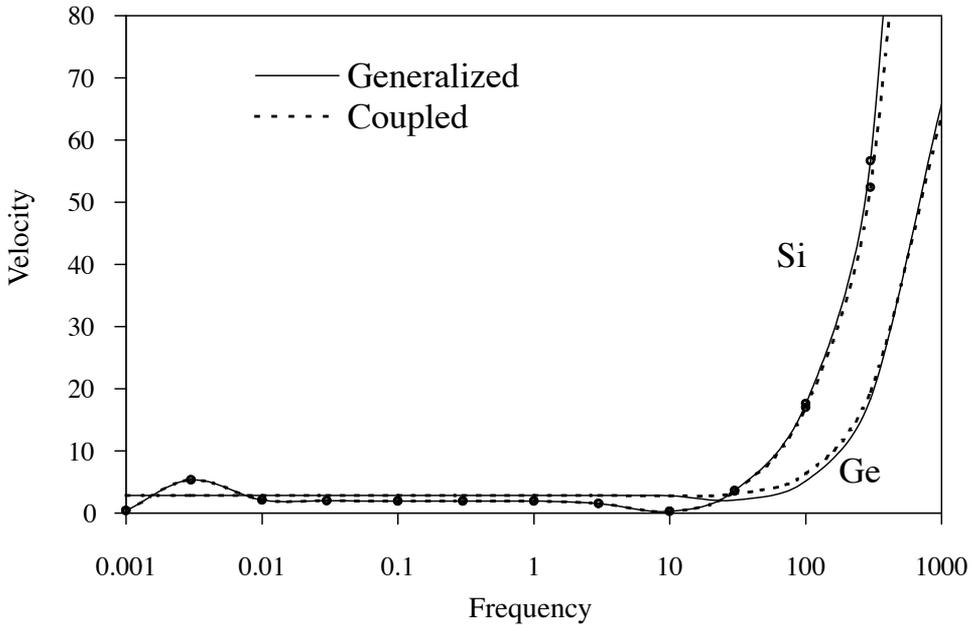


Figure 3. Phase velocity profile of QEN/QEP waves.

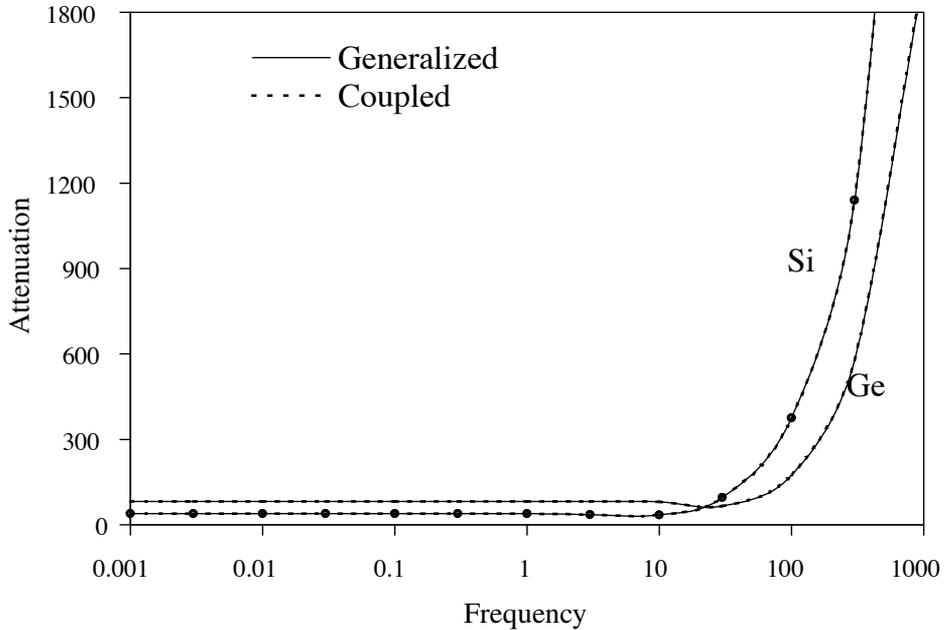


Figure 4. Attenuation coefficient profile of QEN/QEP waves.

of phase velocity are almost linear in Ge material and increase logarithmically at higher values of the frequency afterwards. The phase velocity profile for silicon (Si) semiconductor is slightly dispersive for $0 \leq \omega \leq 10$ in contrast to that of Ge which is linear and hence nondispersive in this range of frequency

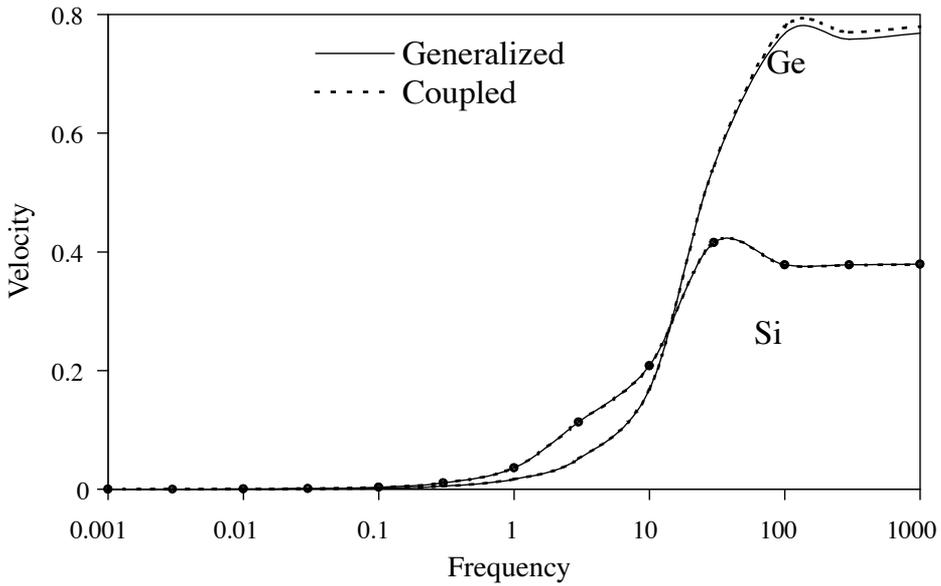


Figure 5. Phase velocity profile of QTN/QTP waves.

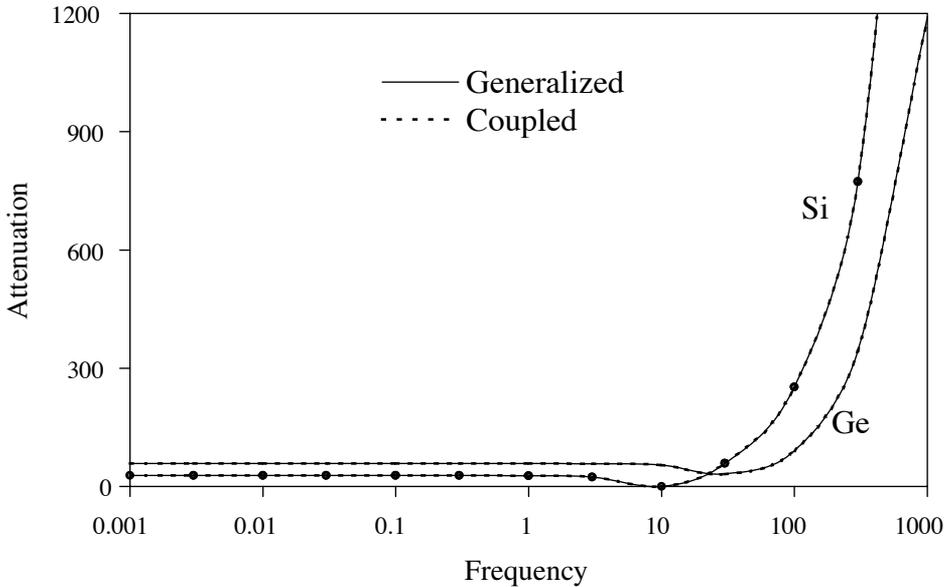


Figure 6. Attenuation coefficient profile of QTN/QTP waves.

values. The effect of thermal relaxation on the transportation of charge carrier fields is observed to be prominent for higher values ($\omega \geq 10$) of the frequency. From [Figure 4](#), we see that the behavior of attenuation coefficient (Q_2) is same as that of the phase velocity in [Figure 3](#), except that the thermal relaxation has negligibly small effect in this case. The variations of the attenuation coefficient are

linear for the frequency range $0 \leq \omega \leq 10$ and increase logarithmically afterwards in both Ge and Si semiconductors. Here the attenuation profiles for thermal relaxation and nonrelaxation cases overlap each other, meaning that thermal relaxation phenomenon does not affect the attenuation coefficient. The critical value of frequency pertaining to the change of this behavior of phase velocity and attenuation coefficient in both the semiconductors (Ge and Si) is observed to be at $\omega = 30$ rather than at $\omega = 1.0$.

Figure 5 shows the phase velocity (V_3) profile of QTN/QTP waves in Ge and Si semiconductors. It is observed that the phase velocity is almost negligible in the frequency range $0 \leq \omega \leq 0.3$, and suffers a sharp increase in its value in $0.3 < \omega \leq 10$. This slightly decreases after attaining its maximum value at $\omega = 100$ for Ge and at $\omega = 30$ in the case of Si to become steady afterwards. The effect of thermal relaxation time is quite pertinent to phase velocity at higher frequencies in the case of Ge, but it has virtually no effect on Si semiconductors. The magnitude of phase velocity in the germanium semiconductor is much higher than that of the silicon one. Figure 6 represents attenuation coefficient profiles of QTN/QTP waves for Ge and Si materials. The behavior of the attenuation coefficient (Q_3) in this case is more or less similar to that of Q_2 as represented by Figure 4 except for the variations in magnitude.

Figure 7 shows the phase velocity (V_4) profiles of T-mode waves in Ge and Si semiconductor materials with respect to nondimensional frequency. The phase velocity in silicon (Si) material has Gaussian behavior with mean value at $\omega = 10$ in the frequency range $0.3 \leq \omega \leq 30$. The phase velocity profile of Ge has linear variations in $0 \leq \omega \leq 3$ but is subject to dispersion beyond $\omega \geq 3$. The effect of thermal relaxation on phase velocity is clearly visible in the case of Si, but is quite small for the Ge semiconductor. Figure 8 represents the attenuation coefficient profile of T-mode waves for Ge and Si materials. The behavior of attenuation coefficient (Q_4) profiles in this case is more or less similar to that of Q_3 represented by the profiles in Figure 6 except for certain variations in magnitude and its prominence in this case.

Figure 9 shows phase velocity and attenuation coefficient profiles for EN/EP waves with respect to frequency. The nondimensional phase velocity in Ge and Si increases sharply in the frequency range $0 \leq \omega \leq 1$ and becomes linear for $\omega \geq 1$ in both materials. This means that the elasto-diffusive waves are significantly influenced by electron and hole charge carrier fields in the low frequency limit, but remain unaffected by such fields at higher frequencies in both materials. The variations of attenuation coefficient are noticed to be significant, but quite small, in the frequency range $0 \leq \omega \leq 0.1$ for Ge and Si materials. The magnitude of the attenuation coefficient in the silicon (Si) semiconductor is greater than that of the Ge, but it varies in Gaussian manner with mean value at $\omega = 0.01$ in both materials. The attenuation coefficient profiles disappear for $\omega \geq 0.1$ in the materials seen in Figure 9.

Figure 10 represents the nondimensional phase velocity and attenuation coefficient profiles of electron/hole diffusive (N/P) waves with frequency. We see that the phase velocity varies linearly with the frequency in the range $0 \leq \omega \leq 0.1$, and then increases logarithmically for $0.1 \leq \omega \leq 10$ before becoming steady/fixed at $\omega \geq 10$. The amplitude of phase velocity in Ge is larger than that of the case of Si material. The behavior of the attenuation coefficient of electron/hole diffusive (N/P) waves is similar to that of phase velocity for electron/hole diffusive waves except for the fact that the magnitude of the former has larger values in the case of Si as compared to those of the Ge semiconductor. Thus, in the Si semiconductor these waves are subjected to more attenuation than in the case of Ge.

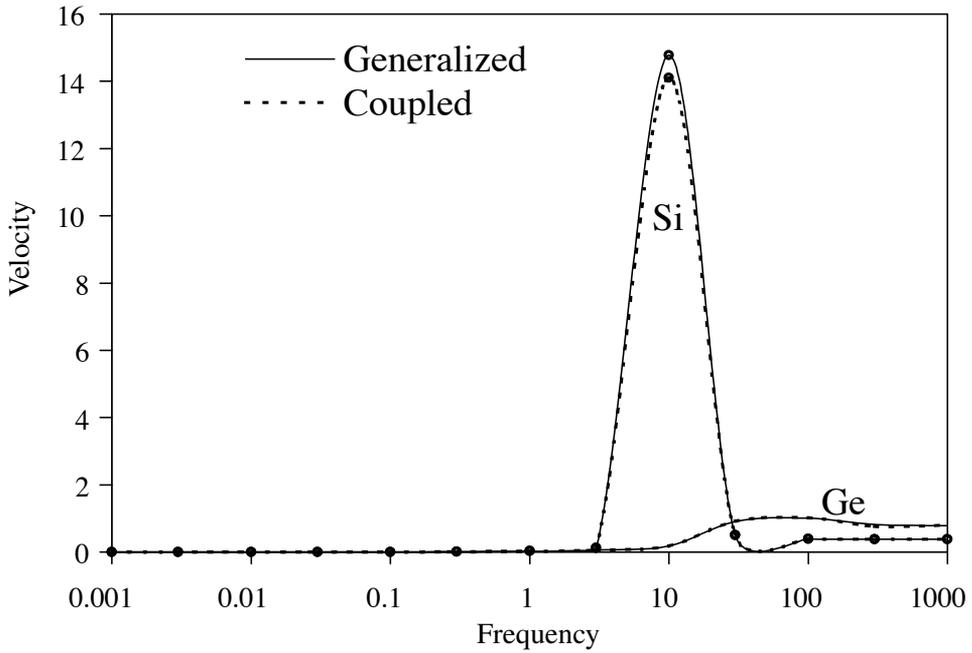


Figure 7. Phase velocity profile of T-mode waves.

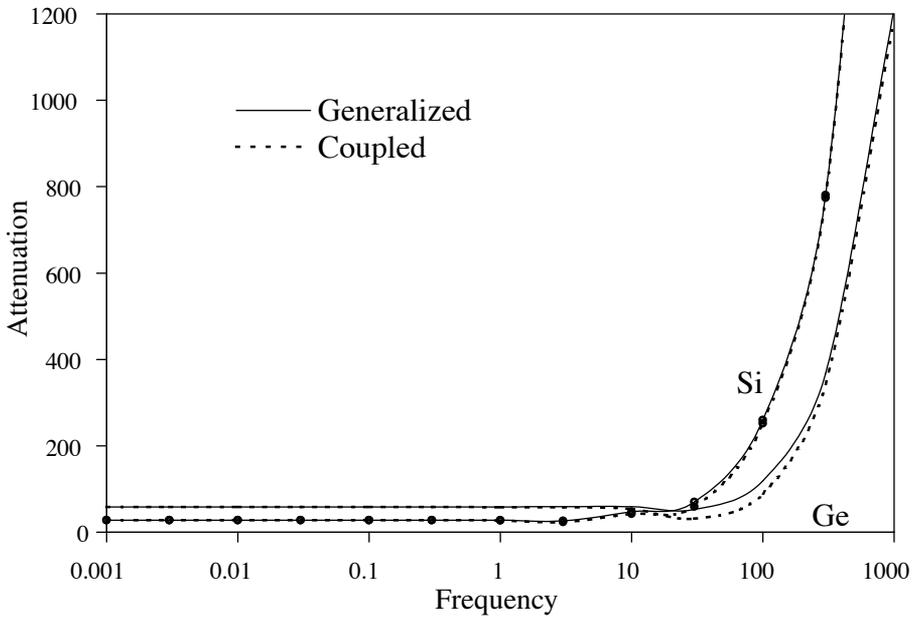


Figure 8. Attenuation coefficient profile of T-mode waves.

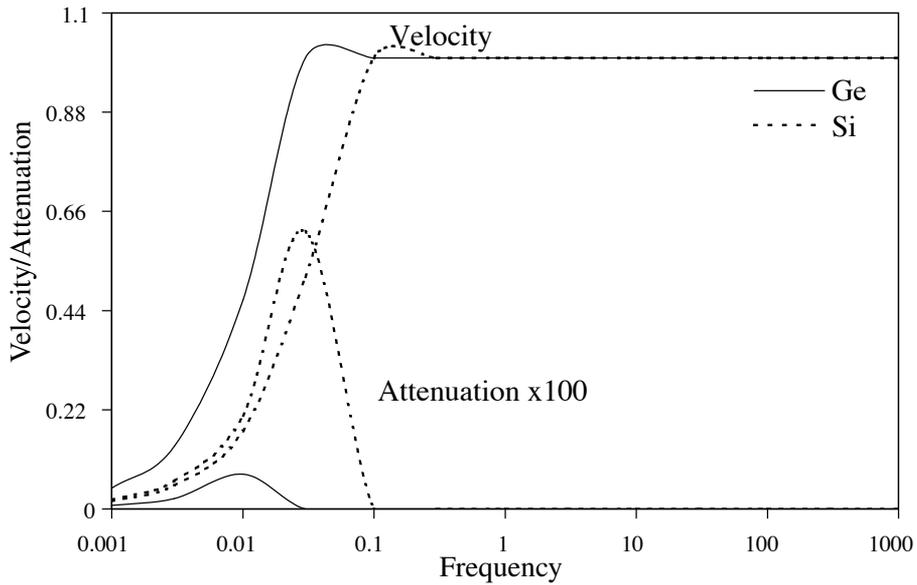


Figure 9. Phase velocity and attenuation coefficient profiles of elasto-diffusive (EN/EP) waves.

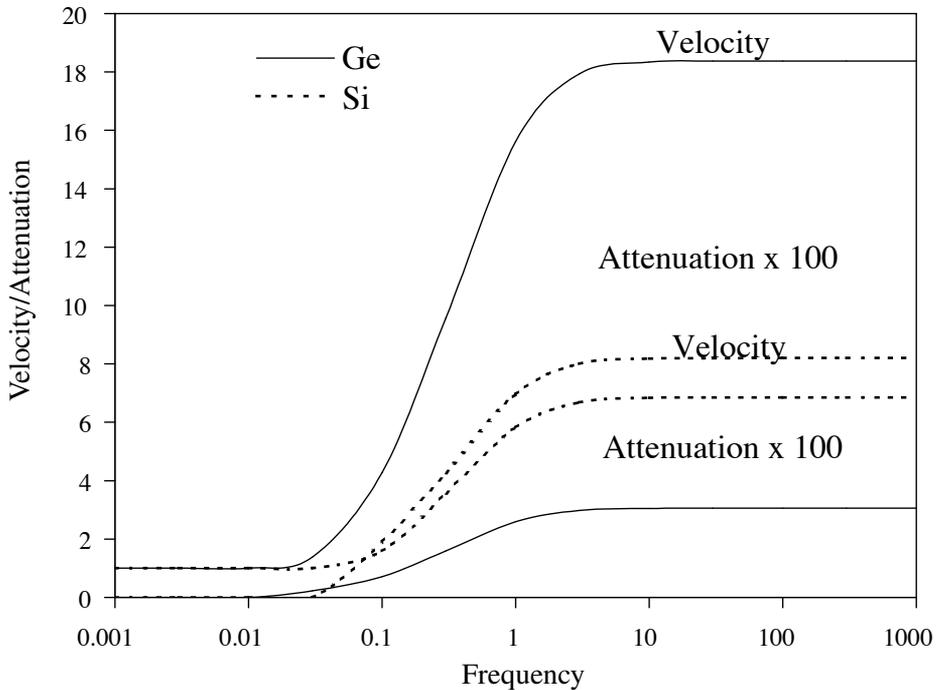


Figure 10. Phase velocity and attenuation coefficient profiles of electron/hole diffusive (N/P) waves.

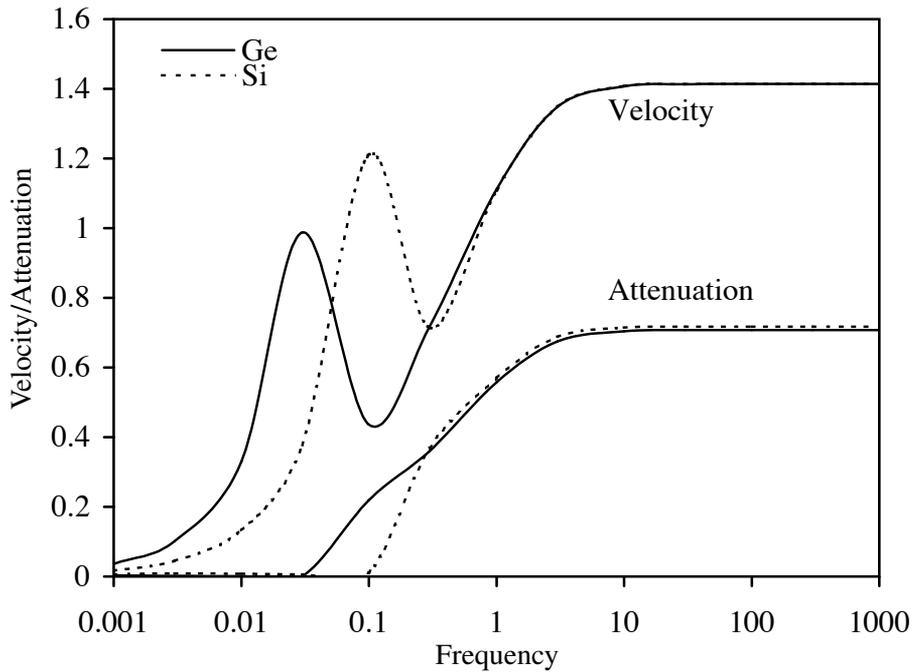


Figure 11. Phase velocity and attenuation coefficient profiles of thermodiffusive (TN/TP) waves.

Figure 11 depicts phase velocity and attenuation coefficient profiles of N/P waves. In the frequency range $0 \leq \omega \leq 10$, a zigzag type of behavior of phase velocity profiles is noticed and it varies linearly for $\omega \geq 10$. For higher frequency ($\omega \geq 10$) velocity, the profiles for both materials overlap each other, meaning that there is no physical distinction between Ge and Si profiles. It is further observed that there is a sharp increase in the values of attenuation coefficient in the frequency range $0.01 \leq \omega \leq 10$, which become steady/fixed at $\omega \geq 10$ in both semiconductors. Figure 12 shows the phase velocity and attenuation coefficient profiles of electron and hole (N/P) diffusive waves with respect to nondimensional frequency. The behavior of these profiles is more or less similar to that in Figure 9, except for some variations in the behavior of phase velocity in $0 \leq \omega \leq 0.03$ and that in the magnitude of attenuation coefficient in addition to its existence beyond $\omega \geq 0.1$ here. The peak values of attenuation are observed to be at $\omega = 0.03$ and $\omega = 0.1$ for Ge and Si, respectively, instead of $\omega = 0.01$ in Figure 9.

Figure 13 deals with the phase velocity profiles of ET waves in Ge and Si materials. The nondimensional phase velocity in Ge materials has value unity at low frequency ($\omega \ll 1$) range, which comes down to its isothermal value at high-frequency ($\omega \gg 1$) limits. Thus in the Ge semiconductor, the phase velocity lies between isentropic and isothermal values as already established by many authors, such as [Chadwick 1979; Chadwick and Seet 1970; Sharma et al. 2000] and [Sharma and Singh 1989]. For the Si material the phase velocity varies linearly throughout with value unity at all frequencies and hence ET waves in this case travel without dispersion, irrespective of isentropic or isothermal conditions. Figure 14 represents attenuation coefficient profiles of ET waves. The nature of the attenuation profiles is the same

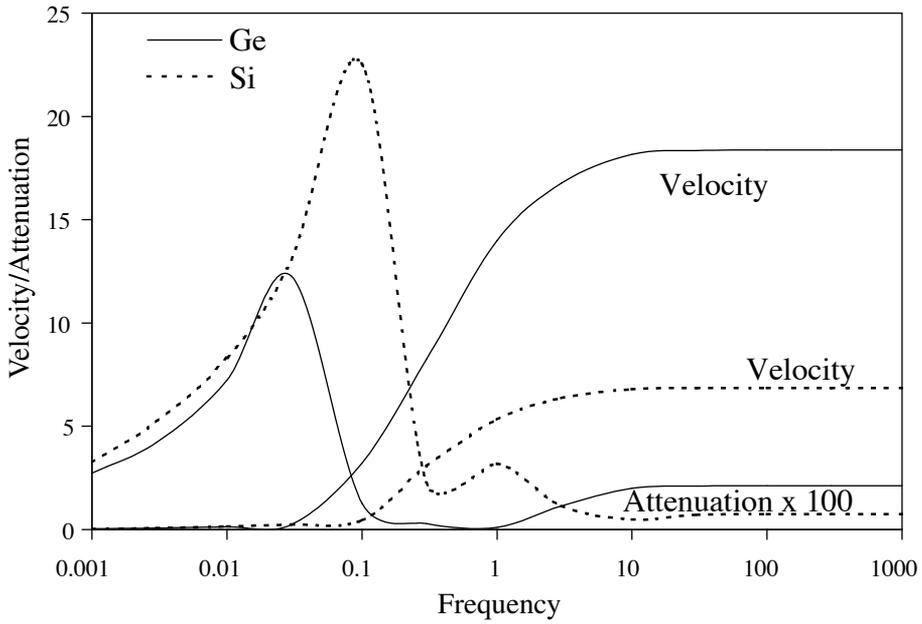


Figure 12. Phase velocity and attenuation coefficient profiles of electron/hole diffusive (N/P) waves.

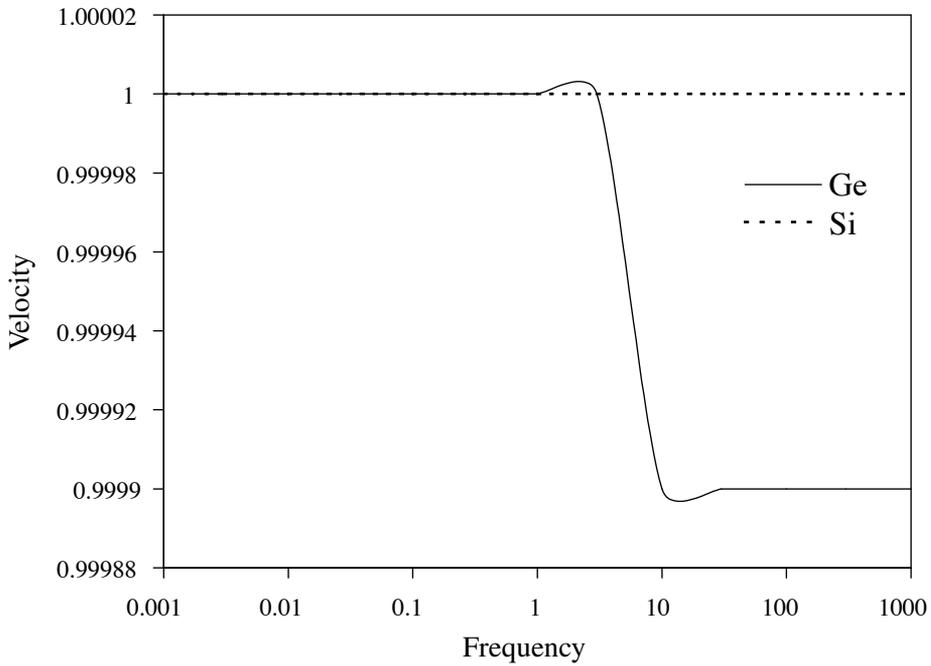


Figure 13. Phase velocity profile of ET waves.

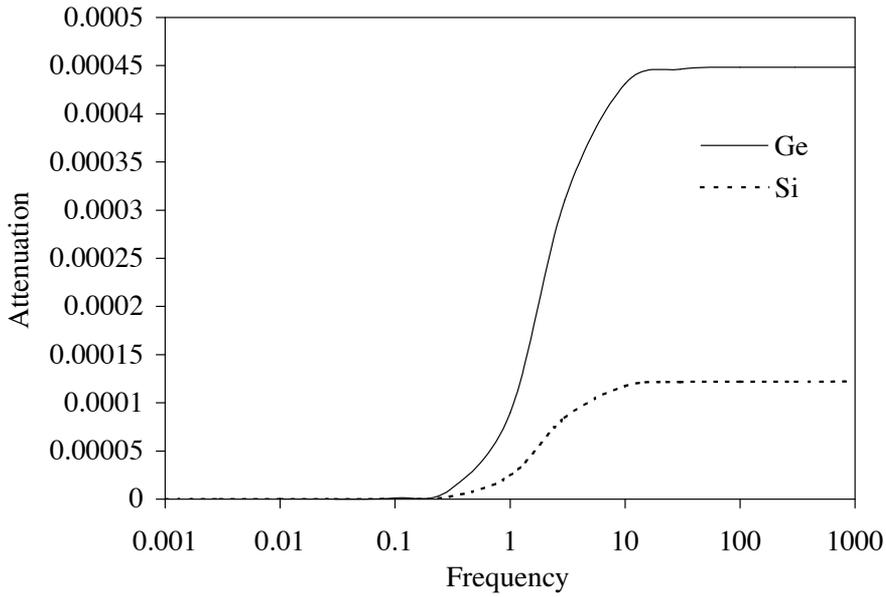


Figure 14. Attenuation coefficient profile of ET waves.

as that in [Figure 10](#) except that waves have less attenuation in the case of Si than Ge here, in addition to negligible magnitude of attenuation coefficient. The nondimensional phase velocity and attenuation coefficient profiles in the case of Ge and Si semiconductors are given in [Figure 15](#). Both the quantities

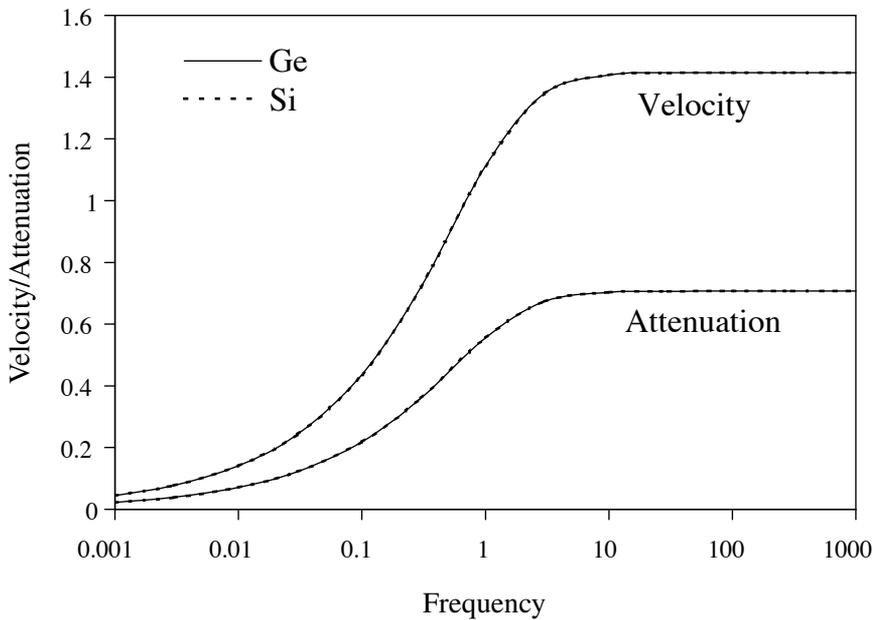


Figure 15. Phase velocity and attenuation coefficient profiles of T-mode waves.

are observed to vary logarithmically and the phenomenon is closer to wave motion than diffusion one. The profiles in case of Ge and Si materials overlap each other because of negligibly small distinction in their behavior and difference in their values.

The comparison of Figures 1, 2, 9, 13, and 14 suggests that the interactions of mechanical, thermal and electron/hole charge carrier fields have attributed to significant modifications in the values of phase velocity and attenuation coefficients of elastic, thermal and diffusive waves in the low and high frequency ranges. While the phase velocity has been lower down in its value in both germanium and silicon semiconductors, the attenuation coefficient has increased manifold and shifts in the value of critical frequency (ω^c) from $\omega^c = 1$ to $\omega^c = 10$ in case of Si and to $\omega^c = 100$ for Ge semiconductor. Similarly, the comparison of Figures 3, 4, 10 and 12 leads to the conclusion that the interaction of all the above fields with each other results in a fourfold increase in the phase velocity and an increase of four orders of magnitude in the attenuation coefficient of QEN/QEP waves in addition to phase shifts/changes. Figures 5, 6, 7, 8, 11, and 15 reveal that the magnitude of nondimensional phase velocity and attenuation coefficient of thermal waves has increased tenfold to that of T-mode due to the considered effect of various fields. The nature of this quantity has become closer to wave phenomena in contrast to diffusion as in Figure 15.

Appendix

The quantities

$$\begin{aligned}\Delta_i, \quad i &= 1, 2, 3, \\ \Delta'_i, \quad i &= 1, 2, 3, 4, 5, 6, \\ \Delta''_i, \quad i &= 1, 2, 3, 4, \\ \Delta'''_1 \quad \text{and} \quad \Delta,\end{aligned}$$

used in equations (10) are defined as

$$\begin{aligned}\Delta &= 1 - \varepsilon^{nq} \varepsilon^{qn} - \varepsilon^{pq} \varepsilon^{qp}, \quad \Delta_3 = \tau^Q - \varepsilon_n \varepsilon^{nq} - \varepsilon_p \varepsilon^{pq}, \\ \Delta_1 &= (1 - \varepsilon^{qn} \varepsilon^{nq})(\tau_p^* + i\omega^{-1} \varepsilon_p \varepsilon_T \bar{\lambda}_p) \\ &\quad - \varepsilon^{qp} \{ \tau'_p + \bar{\lambda}_p \tau^Q \varepsilon_T - i\omega^{-1} \varepsilon_n \varepsilon^{nq} (\alpha_0^n + \bar{\lambda}_p \varepsilon_T) \}, \\ \Delta'_2 &= (\tau^Q - \varepsilon_n \varepsilon^{nq})(\tau_p^* + i\omega^{-1} \varepsilon_p \varepsilon_T \bar{\lambda}_p) \\ &\quad - \varepsilon_p \{ \tau'_p + \bar{\lambda}_p \tau^Q \varepsilon_T - i\omega^{-1} \varepsilon_n \varepsilon^{nq} (\alpha_0^n + \bar{\lambda}_p \varepsilon_T) \}, \\ \Delta'_4 &= \tau_p^* (1 - \varepsilon^{qn} \varepsilon^{nq}) - \varepsilon^{qp} (\tau'_p - i\omega^{-1} \varepsilon_n \varepsilon^{nq} \alpha_0^n), \\ \Delta'_6 &= \tau^Q - i\omega^{-1} (\varepsilon_n \varepsilon^{nq} + \varepsilon_p \varepsilon^{pq}), \\ \Delta''_3 &= \tau_p^* (\tau^Q - i\omega^{-1} \varepsilon_n \varepsilon^{nq}) - i\omega^{-1} \varepsilon_p (\tau'_p - i\omega^{-1} \varepsilon_n \varepsilon^{nq} \alpha_0^n), \\ \Delta''_2 &= (\tau_n^* - \varepsilon^{qn} \tau'_n) (\tau_p^* - \varepsilon^{qp} \tau'_p) \\ &\quad - \{ (i\omega^{-1} \varepsilon_p \alpha_0^p - \varepsilon^{qp} \tau'_n) (i\omega^{-1} \varepsilon_n \alpha_0^n - \varepsilon^{qn} \tau'_p) \},\end{aligned}$$

$$\begin{aligned} \Delta'_1 &= \{(\tau_p^* + i\omega^{-1}\varepsilon_p\varepsilon_T\bar{\lambda}_p) - \varepsilon^{qp}(\tau'_p + \varepsilon_T\bar{\lambda}_p\tau^Q)\} \\ &\quad \times \{(\tau_n^* + i\omega^{-1}\varepsilon_n\varepsilon_T\bar{\lambda}_n) - \varepsilon^{qn}(\tau'_n + \varepsilon_T\bar{\lambda}_n\tau^Q)\} \\ &\quad - \{i\omega^{-1}\varepsilon_p(\alpha_0^p + \bar{\lambda}_n\varepsilon_T) - \varepsilon^{qp}(\tau'_n + \varepsilon_T\bar{\lambda}_n\tau^Q)\} \\ &\quad \times \{(i\omega^{-1}\varepsilon_n(\alpha_0^n + \bar{\lambda}_p\varepsilon_T) - \varepsilon^{qn}(\tau'_p + \varepsilon_T\bar{\lambda}_p\tau^Q)\}, \\ \Delta'''_1 &= (\tau_n^* - i\omega^{-1}\varepsilon_n\frac{\tau'_n}{\tau^Q})(\tau_p^* - i\omega^{-1}\varepsilon_p\frac{\tau'_p}{\tau^Q}) + \omega^{-2}\varepsilon_n\varepsilon_p(\alpha_0^n - \frac{\tau'_p}{\tau^Q})(\alpha_0^p - \frac{\tau'_n}{\tau^Q}), \\ \Delta''_1 &= \left((\tau_p^* + i\omega^{-1}\varepsilon_p\varepsilon_T\bar{\lambda}_p) - \frac{\varepsilon_p(\tau'_p + \varepsilon_T\bar{\lambda}_p\tau^Q)}{\tau^Q} \right) \\ &\quad \times \left((\tau_n^* + i\omega^{-1}\varepsilon_n\varepsilon_T\bar{\lambda}_n) - \frac{\varepsilon_n(\tau'_n + \varepsilon_T\bar{\lambda}_n\tau^Q)}{\tau^Q} \right) \\ &\quad - \left(i\omega^{-1}\varepsilon_p(\alpha_0^p + \varepsilon_T\bar{\lambda}_n) - \frac{\varepsilon_p(\tau'_n + \varepsilon_T\bar{\lambda}_n\tau^Q)}{\tau^Q} \right) \\ &\quad \times \left(i\omega^{-1}\varepsilon_n(\alpha_0^n + \varepsilon_T\bar{\lambda}_p) - \frac{\varepsilon_n(\tau'_p + \varepsilon_T\bar{\lambda}_p\tau^Q)}{\tau^Q} \right), \end{aligned}$$

where

$$\begin{aligned} \tau^Q &= t^Q + i\omega^{-1}, \\ \varepsilon_n' &= \frac{\varepsilon_n}{\varepsilon^{qn}}, \\ \varepsilon_p' &= \frac{\varepsilon_p}{\varepsilon^{qp}}, \\ \tau_n' &= t^Q\alpha_0^n + i\omega^{-1}(\alpha_0^n + a_0^n) - a_0^n\omega^{-2}/t_n^+, \\ \tau_p' &= t^Q\alpha_0^p + i\omega^{-1}(\alpha_0^p + a_0^p) - a_0^p\omega^{-2}/t_p^+, \\ \tau_n^* &= \frac{K}{\rho C_e D^n} \left(t^n + i\omega^{-1} \left(1 - \frac{\varepsilon_n \alpha_0^n D^n}{k} - \frac{t^n}{t_n^+} \right) + \frac{1}{\omega^2 t_n^+} \right), \\ \tau_p^* &= \frac{K}{\rho C_e D^p} \left(t^p + i\omega^{-1} \left(1 - \frac{\varepsilon_p \alpha_0^p D^p}{k} - \frac{t^p}{t_p^+} \right) + \frac{1}{\omega^2 t_p^+} \right). \end{aligned}$$

The other coefficients Δ_2 , Δ'_3 , Δ'_5 and Δ''_4 can be written from Δ_1 , Δ'_2 , Δ'_4 and Δ''_3 respectively, by replacing n with p and vice-versa.

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JAGAN NATH SHARMA: jns@recham.ernet.in

Department of Applied Sciences, National Institute of Technology, Hamirpur 177 005, Himachal Pradesh, India

NAVEEN THAKUR: rinkalthakur@yahoo.com

Department of Applied Sciences, National Institute of Technology, Hamirpur 177 005, Himachal Pradesh, India

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